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Title Truncated Positive Normal Model and Extensions

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Description Provide data generation and estimation tools for the truncated positive normal (tpn) model discussed in Gomez, Olmos, Varela and Bolfarine (2018) <[doi:10.1007/s11766-018-3354-x](https://doi.org/10.1007/s11766-018-3354-x)>, the slash tpn distribution discussed in Gomez, Gallardo and Santoro (2021) <[doi:10.3390/sym13112164](https://doi.org/10.3390/sym13112164)>, the bimodal tpn distribution discussed in Gomez et al. (2022) <[doi:10.3390/sym14040665](https://doi.org/10.3390/sym14040665)> and the flexible tpn model.

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btpn	<i>Bimodal truncated positive normal</i>
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Description

Density, distribution function and random generation for the bimodal truncated positive normal (btpn) discussed in Gomez et al. (2022).

Usage

```
dbtpn(x, sigma, lambda, eta, log = FALSE)
pbtpn(x, sigma, lambda, eta, lower.tail=TRUE, log=FALSE)
rbtpn(n, sigma, lambda, eta)
```

Arguments

x	vector of quantiles
n	number of observations
sigma	scale parameter for the distribution
lambda	shape parameter for the distribution
eta	shape parameter for the distribution
log	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

Random generation is based on the stochastic representation of the model, i.e., the product between a tpn (see Gomez et al. 2018) and a dichotomous variable assuming values $-(1 + \epsilon)$ and $1 - \epsilon$ with probabilities $(1 + \epsilon)/2$ and $(1 - \epsilon)/2$, respectively.

Value

dbtpn gives the density, pbtpn gives the distribution function and rbtpn generates random deviates.

The length of the result is determined by n for rbtpn, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

A variable have btpr distribution with parameters $\sigma > 0$, $\lambda \in \mathbb{R}$ and $\eta \in \mathbb{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \frac{\phi\left(\frac{x}{\sigma(1+\epsilon)} + \lambda\right)}{2\sigma\Phi(\lambda)}, y < 0,$$

and

$$f(y; \sigma, \lambda, q) = \frac{\phi\left(\frac{x}{\sigma(1-\epsilon)} - \lambda\right)}{2\sigma\Phi(\lambda)}, y \geq 0,$$

where $\epsilon = \eta/\sqrt{1+\eta^2}$ and $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function and the cumulative distribution function for the standard normal distribution, respectively.

Author(s)

Gallardo, D.I., Gomez, H.J. and Gomez, Y.M.

References

Gomez, H.J., Caimanque, W., Gomez, Y.M., Magalhaes, T.M., Concha, M., Gallardo, D.I. (2022) Bimodal Truncation Positive Normal Distribution. *Symmetry*, 14, 665.

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. *Applied Mathematical Journal of Chinese Universities*, 33, 163-176.

Examples

```
dbtpr(c(1,2), sigma=1, lambda=-1, eta=2)
pbtp(c(1,2), sigma=1, lambda=-1, eta=2)
rbtp(n=10, sigma=1, lambda=-1, eta=2)
```

choose.fts

Choose a distribution in the flexible truncated positive class of models

Description

Provide model selection for a given data set in the flexible truncated positive class of models

Usage

```
choose.fts(y, criteria = "AIC")
```

Arguments

y positive vector of responses
 criteria model criteria for the selection: AIC (default) or BIC.

Details

The function fits the truncated positive normal, truncated positive laplace, truncated positive Cauchy and truncated positive logistic models and select the model which provides the lower criteria (AIC or BIC).

Value

A list with the following components

AIC	a vector with the AIC for the different truncated positive fitted models: normal, laplace, cauchy and logistic.
selected	the selected model
estimate	the estimated for sigma and lambda and the respective standard errors (s.e.)
conv	the code related to the convergence for the optim function. 0 if the convergence was attached.
logLik	log-likelihood function evaluated in the estimated parameters.
AIC	Akaike's criterion.
BIC	Schwartz's criterion.

Author(s)

Gallardo, D.I., Gomez, H.J. and Gomez, Y.M.

References

Gomez, H.J., Gomez, H.W., Santoro, K.I., Venegas, O., Gallardo, D.I. (2022). A Family of Truncation Positive Distributions. Submitted.

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. Applied Mathematical Journal of Chinese Universities, 33, 163-176.

Examples

```
set.seed(2021)
y=rfts(n=100,sigma=10,lambda=1,dist="logis")
choose.ft(y)
```

 est.btpn

Parameter estimation for the btpn model

Description

Perform the parameter estimation for the bimodal truncated positive normal (btpn) discussed in Gomez et al. (2022). Estimated errors are computed based on the hessian matrix.

Usage

```
est.btpn(y)
```

Arguments

`y` the response vector. All the values must be positive.

Details

A variable have btpn distribution with parameters $\sigma > 0$, $\lambda \in \mathbb{R}$ and $\eta \in \mathbb{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \frac{\phi\left(\frac{x}{\sigma(1+\epsilon)} + \lambda\right)}{2\sigma\Phi(\lambda)}, y < 0,$$

and

$$f(y; \sigma, \lambda, q) = \frac{\phi\left(\frac{x}{\sigma(1-\epsilon)} - \lambda\right)}{2\sigma\Phi(\lambda)}, y \geq 0,$$

where $\epsilon = \eta/\sqrt{1+\eta^2}$ and $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function and the cumulative distribution function for the standard normal distribution, respectively.

Value

A list with the following components

estimate	A matrix with the estimates and standard errors
iter	Iterations in which the convergence were attached.
logLik	log-likelihood function evaluated in the estimated parameters.
AIC	Akaike's criterion.
BIC	Schwartz's criterion.

Note

A warning is presented if the estimated hessian matrix is not invertible.

Author(s)

Gallardo, D.I., Gomez, H.J. and Gomez, Y.M.

References

Gomez, H.J., Caimanque, W., Gomez, Y.M., Magalhaes, T.M., Concha, M., Gallardo, D.I. (2022) Bimodal Truncation Positive Normal Distribution. *Symmetry*, 14, 665.

Examples

```
set.seed(2021)
y=rbtnp(n=100,sigma=10,lambda=1,eta=1.5)
est.btpn(y)
```

 est.fts

Parameter estimation for the ftp class of distributions

Description

Perform the parameter estimation for the Flexible truncated positive (fts) class discussed in Gomez et al. (2022) based on maximum likelihood estimation. Estimated errors are computed based on the hessian matrix.

Usage

```
est.fts(y, dist="norm")
```

Arguments

y	the response vector. All the values must be positive.
dist	standard symmetrical distribution. Available options: norm (default), logis, cauchy and laplace.

Details

A variable has fts distribution with parameters $\sigma > 0$ and $\lambda \in \mathbb{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \frac{g_0\left(\frac{y}{\sigma} - \lambda\right)}{\sigma G_0(\lambda)}, y > 0,$$

where $g_0(\cdot)$ and $G_0(\cdot)$ denote the pdf and cdf for the specified distribution. The case where $g_0(\cdot)$ and $G_0(\cdot)$ are from the standard normal model is known as the truncated positive normal model discussed in Gomez et al. (2018).

Value

A list with the following components

estimate	A matrix with the estimates and standard errors
dist	distribution specified
conv	the code related to the convergence for the optim function. 0 if the convergence was attached.
logLik	log-likelihood function evaluated in the estimated parameters.
AIC	Akaike's criterion.
BIC	Schwartz's criterion.

Note

A warning is presented if the estimated hessian matrix is not invertible.

Author(s)

Gallardo, D.I. and Gomez, H.J.

References

Gomez, H.J., Gomez, H.W., Santoro, K.I., Venegas, O., Gallardo, D.I. (2022). A Family of Truncation Positive Distributions. Submitted.

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. Applied Mathematical Journal of Chinese Universities, 33, 163-176.

Examples

```
set.seed(2021)
y=rfts(n=100,sigma=10,lambda=1,dist="logis")
est.fts(y,dist="logis")
```

 est.stpn

Parameter estimation for the stpn model

Description

Perform the parameter estimation for the slash truncated positive normal (stpn) discussed in Gomez, Gallardo and Santoro (2021) based on the EM algorithm. Estimated errors are computed based on the Louis method to approximate the hessian matrix.

Usage

```
est.stpn(y, sigma0=NULL, lambda0=NULL, q0=NULL, prec = 0.001,
         max.iter = 1000)
```

Arguments

y	the response vector. All the values must be positive.
sigma0, lambda0, q0	initial values for the EM algorithm for sigma, lambda and q. If they are omitted, by default sigma0 is defined as the root of the mean of the y^2, lambda as 0 and q as 3.
prec	the precision defined for each parameter. By default is 0.001.
max.iter	the maximum iterations for the EM algorithm. By default is 1000.

Details

A variable has stpn distribution with parameters $\sigma > 0$, $\lambda \in \mathbb{R}$ and $q > 0$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \int_0^1 t^{1/q} \sigma \phi(yt^{1/q} \sigma - \lambda) dt, y > 0,$$

where $\phi(\cdot)$ denotes the density function for the standard normal distribution.

Value

A list with the following components

estimate	A matrix with the estimates and standard errors
iter	Iterations in which the convergence were attached.
logLik	log-likelihood function evaluated in the estimated parameters.
AIC	Akaike's criterion.
BIC	Schwartz's criterion.

Note

A warning is presented if the estimated hessian matrix is not invertible.

Author(s)

Gallardo, D.I. and Gomez, H.J.

References

Gomez, H., Gallardo, D.I., Santoro, K. (2021) Slash Truncation Positive Normal Distribution: with application using the EM algorithm. *Symmetry*, 13, 2164.

Examples

```
set.seed(2021)
y=rstp(n=100,sigma=10,lambda=1,q=2)
est.stpn(y)
```

est.tpn

Parameter estimation for the tpn

Description

Perform the parameter estimation for the truncated positive normal (tpn) discussed in Gomez et al. (2018) based on maximum likelihood estimation. Estimated errors are computed based on the hessian matrix.

Usage

```
est.tpn(y)
```

Arguments

y the response vector. All the values must be positive.

Details

A variable have tpn distribution with parameters $\sigma > 0$ and $\lambda \in \mathbf{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \frac{\phi\left(\frac{y}{\sigma} - \lambda\right)}{\sigma\Phi(\lambda)}, y > 0,$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the density and cumulative distribution functions for the standard normal distribution.

Value

A list with the following components

estimate	A matrix with the estimates and standard errors
logLik	log-likelihood function evaluated in the estimated parameters.
AIC	Akaike's criterion.
BIC	Schwartz's criterion.

Note

A warning is presented if the estimated hessian matrix is not invertible.

Author(s)

Gallardo, D.I. and Gomez, H.J.

References

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. Applied Mathematical Journal of Chinese Universities, 33, 163-176.

Examples

```
set.seed(2021)
y=rtpn(n=100,sigma=10,lambda=1)
est.tpn(y)
```

 est.utpn

Parameter estimation for the utpn model

Description

Perform the parameter estimation for the unit truncated positive normal (utpn) type 1, 2, 3 or 4, parameterized in terms of the quantile based on maximum likelihood estimation. Estimated errors are computed based on the hessian matrix.

Usage

```
est.utpn(y, x=NULL, type=1, link="logit", q=0.5)
```

Arguments

y	the response vector. All the values must be positive.
x	the covariates vector.
type	to distinguish the type of the utpn model: 1 (default), 2, 3 or 4.
link	link function to be used for the covariates: logit (default).
q	quantile of the distribution to be modelled.

Value

A list with the following components

estimate	A matrix with the estimates and standard errors
logLik	log-likelihood function evaluated in the estimated parameters.
AIC	Akaike's criterion.
BIC	Schwartz's criterion.

Note

A warning is presented if the estimated hessian matrix is not invertible.

Author(s)

Gallardo, D.I.

References

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. *Applied Mathematical Journal of Chinese Universities*, 33, 163-176.

Examples

```
set.seed(2021)
y=rutpn(n=100,sigma=10,lambda=1)
est.utpn(y)
```

fts *Flexible truncated positive normal*

Description

Density, distribution function and random generation for the flexible truncated positive (ftp) class discussed in Gomez et al. (2022).

Usage

```
dfts(x, sigma, lambda, dist="norm", log = FALSE)
pfts(x, sigma, lambda, dist="norm", lower.tail=TRUE, log.p=FALSE)
qfts(p, sigma, lambda, dist="norm")
rfts(n, sigma, lambda, dist="norm")
```

Arguments

x	vector of quantiles
p	vector of probabilities
n	number of observations
sigma	scale parameter for the distribution
lambda	shape parameter for the distribution
dist	standard symmetrical distribution. Available options: norm (default), logis, cauchy and laplace.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x] otherwise, P[X > x].

Details

Random generation is based on the inverse transformation method.

Value

dfts gives the density, pfts gives the distribution function, qfts gives the quantile function and rfts generates random deviates.

The length of the result is determined by n for rftpn, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

A variable have fts distribution with parameters $\sigma > 0$ and $\lambda \in \mathbb{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \frac{g_0\left(\frac{y}{\sigma} - \lambda\right)}{\sigma G_0(\lambda)}, y > 0,$$

where $g_0(\cdot)$ and $G_0(\cdot)$ denote the pdf and cdf for the specified distribution. The case where $g_0(\cdot)$ and $G_0(\cdot)$ are from the standard normal model is known as the truncated positive normal model discussed in Gomez et al. (2018).

Author(s)

Gallardo, D.I., Gomez, H.J. and Gomez, Y.M.

References

Gomez, H.J., Gomez, H.W., Santoro, K.I., Venegas, O., Gallardo, D.I. (2022). A Family of Truncation Positive Distributions. Submitted.

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. Applied Mathematical Journal of Chinese Universities, 33, 163-176.

Examples

```
dfts(c(1,2), sigma=1, lambda=1, dist="logis")
pfts(c(1,2), sigma=1, lambda=1, dist="logis")
rfts(n=10, sigma=1, lambda=1, dist="logis")
```

 stpn

Slash truncated positive normal

Description

Density, distribution function and random generation for the slash truncated positive normal (stpn) discussed in Gomez, Gallardo and Santoro (2021).

Usage

```
dstpn(x, sigma, lambda, q, log = FALSE)
pstpn(x, sigma, lambda, q, lower.tail=TRUE, log=FALSE)
rstpn(n, sigma, lambda, q)
```

Arguments

x	vector of quantiles
n	number of observations
sigma	scale parameter for the distribution
lambda	shape parameter for the distribution
q	shape parameter for the distribution
log	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x] otherwise, P[X > x].

Details

Random generation is based on the stochastic representation of the model, i.e., the quotient between a tpn (see Gomez et al. 2018) and a beta random variable.

Value

dstpn gives the density, pstpn gives the distribution function and rstpn generates random deviates.

The length of the result is determined by n for rstpn, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

A variable has stpn distribution with parameters $\sigma > 0$, $\lambda \in \mathbb{R}$ and $q > 0$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \int_0^1 t^{1/q} \sigma \phi(yt^{1/q} \sigma - \lambda) dt, y > 0,$$

where $\phi(\cdot)$ denotes the density function for the standard normal distribution.

Author(s)

Gallardo, D.I. and Gomez, H.J.

References

Gomez, H., Gallardo, D.I., Santoro, K. (2021) Slash Truncation Positive Normal Distribution: with application using the EM algorithm. *Symmetry*, 13, 2164.

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. *Applied Mathematical Journal of Chinese Universities*, 33, 163-176.

Examples

```
dstpn(c(1,2), sigma=1, lambda=-1, q=2)
pstpn(c(1,2), sigma=1, lambda=-1, q=2)
rstpn(n=10, sigma=1, lambda=-1, q=2)
```

 tpn

Truncated positive normal

Description

Density, distribution function and random generation for the slash truncated positive normal (stpn) discussed in Gomez, Gallardo and Santoro (2021).

Usage

```
dtpn(x, sigma, lambda, log = FALSE)
ptpn(x, sigma, lambda, lower.tail=TRUE, log=FALSE)
rtpn(n, sigma, lambda)
```

Arguments

<code>x</code>	vector of quantiles
<code>n</code>	number of observations
<code>sigma</code>	scale parameter for the distribution
<code>lambda</code>	shape parameter for the distribution
<code>log</code>	logical; if TRUE, probabilities p are given as $\log(p)$.
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

Random generation is based on the inverse transformation method.

Value

`dtpn` gives the density, `ptpn` gives the distribution function and `rtpn` generates random deviates.

The length of the result is determined by `n` for `rtpn`, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than `n` are recycled to the length of the result. Only the first elements of the logical arguments are used.

A variable have tpn distribution with parameters $\sigma > 0$ and $\lambda \in \mathbb{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda, q) = \frac{\phi\left(\frac{y}{\sigma} - \lambda\right)}{\sigma\Phi(\lambda)}, y > 0,$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the density and cumulative distribution functions for the standard normal distribution.

Author(s)

Gallardo, D.I. and Gomez, H.J.

References

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. *Applied Mathematical Journal of Chinese Universities*, 33, 163-176.

Examples

```
dtpn(c(1,2), sigma=1, lambda=-1)
ptpn(c(1,2), sigma=1, lambda=-1)
rtpn(n=10, sigma=1, lambda=-1)
```

utpn	<i>Truncated positive normal</i>
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Description

Density, distribution function and random generation for the unit truncated positive normal (utpn) type 1 or 2 discussed in Gomez, Gallardo and Santoro (2021).

Usage

```

dutpn(x, sigma = 1, lambda = 0, type = 1, log = FALSE)
putpn(x, sigma = 1, lambda = 0, type = 1, lower.tail = TRUE, log = FALSE)
qutpn(p, sigma = 1, lambda = 0, type = 1)
rutpn(n, sigma = 1, lambda = 0, type = 1)

```

Arguments

<code>x</code>	vector of quantiles
<code>n</code>	number of observations
<code>p</code>	vector of probabilities
<code>sigma</code>	scale parameter for the distribution
<code>lambda</code>	shape parameter for the distribution
<code>type</code>	to distinguish the type of the utpn model: 1 (default) or 2.
<code>log</code>	logical; if TRUE, probabilities <code>p</code> are given as $\log(p)$.
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

Random generation is based on the inverse transformation method.

Value

`dutpn` gives the density, `putpn` gives the distribution function, `qutpn` provides the quantile function and `rutpn` generates random deviates.

The length of the result is determined by `n` for `rutpn`, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than `n` are recycled to the length of the result. Only the first elements of the logical arguments are used.

A variable has utpn distribution with scale parameter $\sigma > 0$ and shape parameter $\lambda \in \mathbb{R}$ if its probability density function can be written as

$$f(y; \sigma, \lambda) = \frac{\phi\left(\frac{1-y}{\sigma} - \lambda\right)}{\sigma y^2 \Phi(\lambda)}, y > 0, \text{ (type 1),}$$

$$f(y; \sigma, \lambda) = \frac{\phi\left(\frac{y}{\sigma(1-y)} - \lambda\right)}{\sigma(1-y)^2\Phi(\lambda)}, y > 0, \text{ (type 2),}$$

$$f(y; \sigma, \lambda) = \frac{\phi\left(\frac{\log(y)}{\sigma} + \lambda\right)}{\sigma y \Phi(\lambda)}, y > 0, \text{ (type 3),}$$

$$f(y; \sigma, \lambda) = \frac{\phi\left(\frac{\log(1-y)}{\sigma} + \lambda\right)}{\sigma(1-y)\Phi(\lambda)}, y > 0, \text{ (type 4),}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the density and cumulative distribution functions for the standard normal distribution.

Author(s)

Gallardo, D.I.

References

Gomez, H.J., Olmos, N.M., Varela, H., Bolfarine, H. (2018). Inference for a truncated positive normal distribution. Applied Mathematical Journal of Chinese Universities, 33, 163-176.

Examples

```

dutpn(c(0.1,0.2), sigma=1, lambda=-1)
putpn(c(0.1,0.2), sigma=1, lambda=-1)
rutpn(n=10, sigma=1, lambda=-1)

```

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