# Package: tgp (via r-universe)

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Title Bayesian Treed Gaussian Process Models

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Imports maptree

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Description Bayesian nonstationary, semiparametric nonlinear regression and design by treed Gaussian processes (GPs) with jumps to the limiting linear model (LLM). Special cases also implemented include Bayesian linear models, CART, treed linear models, stationary separable and isotropic GPs, and GP single-index models. Provides 1-d and 2-d plotting functions (with projection and slice capabilities) and tree drawing, designed for visualization of tgp-class output. Sensitivity analysis and multi-resolution models are supported. Sequential experimental design and adaptive sampling functions are also provided, including ALM, ALC, and expected improvement. The latter supports derivative-free optimization of noisy black-box functions. For details and tutorials, see Gramacy (2007) <doi:10.18637/jss.v019.i09> and Gramacy & Taddy (2010) <doi:10.18637/jss.v033.i06>.

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# Description

A Bayesian nonstationary nonparametric regression and design package implementing an array of models of varying flexibility and complexity.

# **Details**

This package implements Bayesian nonstationary, semiparametric nonlinear regression with "treed Gaussian process models" with jumps to the limiting linear model (LLM). The package contains functions which facilitate inference for seven regression models of varying complexity using Markov chain Monte Carlo (MCMC): linear model, CART (Classification and Regression Tree), treed linear model, Gaussian process (GP), GP with jumps to the LLM, GP single-index models, treed GPs, treed GP LLMs, and treed GP single-index models. R provides an interface to the C/C++ backbone, and a serves as mechanism for graphically visualizing the results of inference and posterior predictive surfaces under the models. A Bayesian Monte Carlo based sensitivity analysis is implemented, and multi-resolution models are also supported. Sequential experimental design and adaptive sampling functions are also provided, including ALM, ALC, and expected improvement. The latter supports derivative-free optimization of noisy black-box functions.

For a fuller overview including a complete list of functions, demos and vignettes, please use help(package="tgp").

#### Author(s)

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#### References

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btgp

Bayesian Nonparametric & Nonstationary Regression Models

### **Description**

The seven functions described below implement Bayesian regression models of varying complexity: linear model, linear CART, Gaussian process (GP), GP with jumps to the limiting linear model (LLM), treed GP, and treed GP LLM.

#### Usage

```
blm(X, Z, XX = NULL, meanfn = "linear", bprior = "bflat",
    BTE = c(1000, 4000, 3), R = 1, m0r1 = TRUE, itemps = NULL,
    pred.n = TRUE, krige = TRUE, zcov = FALSE, Ds2x = FALSE,
    improv = FALSE, sens.p = NULL, trace = FALSE, verb = 1, ...)
```

```
btlm(X, Z, XX = NULL, meanfn = "linear", bprior = "bflat",
       tree = c(0.5, 2), BTE = c(2000, 7000, 2), R = 1, m0r1 = TRUE,
itemps = NULL, pred.n = TRUE, krige = TRUE, zcov = FALSE,
       Ds2x = FALSE, improv = FALSE, sens.p = NULL, trace = FALSE,
       verb = 1, \ldots
bcart(X, Z, XX = NULL, bprior = "bflat", tree = c(0.5, 2),
       BTE = c(2000, 7000, 2), R = 1, m0r1 = TRUE, itemps = NULL,
       pred.n = TRUE, krige = TRUE, zcov = FALSE, Ds2x = FALSE,
       improv=FALSE, sens.p = NULL, trace = FALSE, verb = 1, ...)
bgp(X, Z, XX = NULL, meanfn = "linear", bprior = "bflat",
       corr = "expsep", BTE = c(1000, 4000, 2), R = 1, m0r1 = TRUE,
itemps = NULL, pred.n = TRUE, krige = TRUE, zcov = FALSE,
       Ds2x = FALSE, improv = FALSE, sens.p = NULL, nu = 1.5,
       trace = FALSE, verb = 1, ...)
bgpllm(X, Z, XX = NULL, meanfn = "linear", bprior = "bflat",
       corr = "expsep", gamma=c(10,0.2,0.7), BTE = c(1000, 4000, 2),
       R = 1, m0r1 = TRUE, itemps = NULL, pred.n = TRUE,
       krige = TRUE, zcov = FALSE, Ds2x = FALSE, improv = FALSE,
       sens.p = NULL, nu = 1.5, trace = FALSE, verb = 1, ...)
btgp(X, Z, XX = NULL, meanfn = "linear", bprior = "bflat";
       corr = "expsep", tree = c(0.5, 2), BTE = c(2000, 7000, 2),
       R = 1, m0r1 = TRUE, linburn = FALSE, itemps = NULL,
pred.n = TRUE, krige = TRUE, zcov = FALSE, Ds2x = FALSE,
       improv = FALSE, sens.p = NULL, nu = 1.5, trace = FALSE,
       verb = 1, \ldots)
btgpllm(X, Z, XX = NULL, meanfn = "linear", bprior = "bflat",
       corr = "expsep", tree = c(0.5, 2), gamma=c(10, 0.2, 0.7),
BTE = c(2000, 7000, 2), R = 1, m0r1 = TRUE, linburn = FALSE,
       itemps = NULL, pred.n = TRUE, krige = TRUE, zcov = FALSE,
       Ds2x = FALSE, improv = FALSE, sens.p = NULL, nu = 1.5,
       trace = FALSE, verb = 1, ...)
```

### **Arguments**

Each of the above functions takes some subset of the following arguments...

X data.frame, matrix, or vector of inputs X

Z Vector of output responses Z of length equal to the leading dimension (rows) of X, i.e., length(Z) == nrow(X)

XX Optional data.frame, matrix, or vector of predictive input locations with the same number of columns as X, i.e., ncol(XX) == ncol(X)

meanfn A choice of mean function for the process. When meanfn = "linear" (default), then we have the process

$$Z = (\mathbf{1} \ \mathbf{X})\beta + W(\mathbf{X})$$

where  $W(\mathbf{X})$  represents the Gaussian process part of the model (if present). Otherwise, when meanfn = "constant", then

$$Z = \beta_0 + W(\mathbf{X})$$

bprior

Linear (beta) prior, default is "bflat"; alternates include "b0" hierarchical Normal prior, "bmle" empirical Bayes Normal prior, "b0not" Bayesian treed LM-style prior from Chipman et al. (same as "b0" but without tau2), "bmzt" a independent Normal prior (mean zero) with inverse-gamma variance (tau2), and "bmznot" is the same as "bmznot" without tau2. The default "bflat" gives an "improper" prior which can perform badly when the signal-to-noise ratio is low. In these cases the "proper" hierarchical specification "b0" or independent "bmzt" or "bmznot" priors may perform better

tree

a 2-vector containing the tree process prior parameterization c(alpha, beta) specifying

$$p_{\text{split}}(\eta, \mathcal{T}) = \alpha * (1 + \eta)^{\beta}$$

automatically giving zero probability to trees with partitions containing less than  $\min(c(10, nrow(X)+1))$  data points. You may also specify a longer vector, writing over more of the components of the \$tree output from tgp.default.params

gamma

Limiting linear model parameters c(g, t1, t2), with growth parameter g > 0 minimum parameter t1 >= 0 and maximum parameter t1 >= 0, where t1 + t2 <= 1 specifies

$$p(b|d) = t_1 + \exp\left\{\frac{-g(t_2 - t_1)}{d - 0.5}\right\}$$

corr

Gaussian process correlation model. Choose between the isotropic power exponential family ("exp") or the separable power exponential family ("expsep", default); the current version also supports the isotropic Matern ("matern") and single-index Model ("sim") as "beta" functionality.

BTE

3-vector of Monte-carlo parameters (B)urn in, (T)otal, and (E)very. Predictive samples are saved every E MCMC rounds starting at round B, stopping at T.

R

Number of repeats or restarts of BTE MCMC rounds, default R=1 is no restarts If TRUE (default) the responses Z will be scaled to have a mean of zero and a

m0r1

If TRUE (default) the responses 2 will be scaled to have a mean of zero and a range of 1

linburn

If TRUE initializes MCMC with B (additional) rounds of Bayesian Linear CART (btlm); default is FALSE

itemps

Importance tempering (IT) inverse temperature ladder, or powers to improve mixing. See default.itemps. The default is no IT itemps = NULL

pred.n

TRUE (default) value results in prediction at the inputs X; FALSE skips prediction at X resulting in a faster implementation

krige

TRUE (default) value results in collection of kriging means and variances at predictive (and/or data) locations; FALSE skips the gathering of kriging statistics giving a savings in storage

zcov

If TRUE then the predictive covariance matrix is calculated—can be computationally (and memory) intensive if X or XX is large. Otherwise only the variances (diagonal of covariance matrices) are calculated (default). See outputs Zp.s2, ZZ.s2, etc., below

Ds2x

TRUE results in ALC (Active Learning–Cohn) computation of expected reduction in uncertainty calculations at the XX locations, which can be used for adaptive sampling; FALSE (default) skips this computation, resulting in a faster implementation

improv

TRUE results in samples from the improvement at locations XX with respect to the observed data minimum. These samples are used to calculate the expected improvement over XX, as well as to rank all of the points in XX in the order that they should be sampled to minimize the expected multivariate improvement (refer to Schonlau et al, 1998). Alternatively, improv can be set to any positive integer 'g', in which case the ranking is performed with respect to the expectation for improvement raised to the power 'g'. Increasing 'g' leads to rankings that are more oriented towards a global optimization. The option FALSE (default) skips these computations, resulting in a faster implementation. Optionally, a two-vector can be supplied where improv[2] is interpreted as the (maximum) number of points to rank by improvement. See the note below. If not specified, the entire XX matrix is ranked.

sens.p

Either NULL or a vector of parameters for sensitivity analysis, built by the function sens. Refer there for details

nu

"beta" functionality: fixed smoothness parameter for the Matern correlation function; nu + 0.5 times differentiable predictive surfaces result

trace

TRUE results in a saving of samples from the posterior distribution for most of the parameters in the model. The default is FALSE for speed/storage reasons. See note below

verb

Level of verbosity of R-console print statements: from 0 (none); 1 (default) which shows the "progress meter"; 2 includes an echo of initialization parameters; up to 3 and 4 (max) with more info about successful tree operations

. . .

These ellipses arguments are interpreted as augmentations to the prior specification generated by

params <- tgp.default.params(ncol(X)+1).</pre>

You may use these to specify a custom setting of any of default parameters in the output list params except those for which a specific argument is already provided (e.g., params\$corr or params\$bprior) or those which contradict the type of b\* function being called (e.g., params\$tree or params\$gamma); these redundant or possibly conflicting specifications will be ignored. Refer to tgp.default.params for details on the prior specification

### **Details**

The functions and their arguments can be categorized by whether or not they use treed partitioning (T), GP models, and jumps to the LLM (or LM)

blm	LM	Linear Model
btlm	T, LM	Treed Linear Model
bcart	T	Treed Constant Model
bgp	GP	GP Regression
bgpllm	GP, LLM	GP with jumps to the LLM
btgp	T, GP	treed GP Regression
btgpllm	T, GP, LLM	treed GP with jumps to the LLM

Each function implements a special case of the generic function tgp which is an interface to C/C++ code for treed Gaussian process modeling of varying parameterization. Documentation for tgp has

been declared redundant, and has subsequently been removed. To see how the b\* functions use tgp simply examine the function. In the latest version, with the addition of the ellipses "..." argument, there is nothing that can be done with the direct tgp function that cannot also be done with a b\* function

Only functions in the T (tree) category take the tree argument; GP category functions take the corr argument; and LLM category functions take the gamma argument. Non-tree class functions omit the parts output, see below

bcart is the same as btlm except that only the intercept term in the LM is estimated; the others are zero, thereby implementing a Bayesian version of the original CART model

The sens.p argument contains a vector of parameters for sensitivity analysis. It should be NULL unless created by the sens function. Refer to help(sens) for details.

If itemps =! NULL then importance tempering (IT) is performed to get better mixing. After each restart (when R > 1) the observation counts are used to update the pseudo-prior. Stochastic approximation is performed in the first burn-in rounds (for B-T rounds, not B) when c0 and n0 are positive. Every subsequent burn-in after the first restart is for B rounds in order to settle-in after using the observation counts. See default.itemps for more details and an example

Please see vignette("tgp") for a detailed illustration

#### Value

bgp returns an object of class "tgp". The function plot.tgp can be used to help visualize results.

An object of class "tgp" is a list containing at least the following components... The parts output is unique to the T (tree) category functions. Tree viewing is supported by tgp. trees

Χ	Input argument: data.frame of inputs X
n	Number of rows in X, i.e., nrow(X)
d	Number of cols in X, i.e., ncol(X)
Z	Vector of output responses Z
XX	Input argument: data.frame of predictive locations XX
nn	Number of rows in XX, i.e., nrow(XX)
BTE	Input argument: Monte-carlo parameters
R	Input argument: restarts
linburn	Input argument: initialize MCMC with linear CART
params	list of model parameters generated by tgp.default.params and subsequently modified according to the calling b* function and its arguments
dparams	Double-representation of model input parameters used by the C-code
itemps	data.frame containing the importance tempering ladders and pseudo-prior: $k$ has inverse inverse temperatures (from the input argument), $k$ has an <i>updated</i> pseudo-prior based on observation counts and (possibly) stochastic approximation during burn-in and (input) stochastic approximation parameters $c_0$ and $n_0$ . See default.itemps for more info
Zp.mean	Vector of mean predictive estimates at X locations
Zp.q1	Vector of 5% predictive quantiles at X locations

Zp.q2	Vector of 95% predictive quantiles at X locations
Zp.q	Vector of quantile norms Zp.q2-Zp.q1
Zp.s2	If input zcov = TRUE, then this is a predictive covariance matrix for the inputs at locations X; otherwise then this is a vector of predictive variances at the X locations (diagonal of the predictive covariance matrix). Only appears when input pred.n = TRUE
Zp.km	Vector of (expected) kriging means at X locations
Zp.vark	Vector of posterior variance for kriging surface (no additive noise) at X locations
Zp.ks2	Vector of (expected) predictive kriging variances at X locations
ZZ.mean	Vector of mean predictive estimates at XX locations
ZZ.q1	Vector of 5% predictive quantiles at XX locations
ZZ.q2	Vector of 95% predictive quantiles at XX locations
ZZ.q	Vector of quantile norms ZZ.q2-ZZ.q1, used by the ALM adaptive sampling algorithm
ZZ.s2	If input zcov = TRUE, then this is a predictive covariance matrix for predictive locations XX; otherwise then this is a vector of predictive variances at the XX locations (diagonal of the predictive covariance matrix). Only appears when input XX != NULL
ZpZZ.s2	If input zcov = TRUE, then this is a predictive $n * nn$ covariance matrix between locations in X and XX; Only appears when zcov = TRUE and both pred. $n = TRUE$ and XX! = NULL
ZZ.km	Vector of (expected) kriging means at XX locations
ZZ.vark	Vector of posterior variance for kriging surface (no additive noise) at XX locations
ZZ.ks2	Vector of (expected) predictive kriging variances at XX locations
Ds2x	If argument Ds2x=TRUE, this vector contains ALC statistics for XX locations
improv	If argument improv is TRUE or a positive integer, this is a 'matrix' with first column set to the expected improvement statistics for XX locations, and the second column set to a ranking in the order that they should be sampled to minimize the expected multivariate improvement raised to a power determined by the argument improv
response	Name of response Z if supplied by data.frame in argument, or "z" if none provided $$
parts	Internal representation of the regions depicted by partitions of the maximum a' posteriori (MAP) tree
trees	list of trees ( <b>maptree</b> representation) which were MAP as a function of each tree height sampled between MCMC rounds B and T
trace	If trace==TRUE, this list contains traces of most of the model parameters and posterior predictive distributions at input locations XX. Otherwise the entry is FALSE. See note below

ess Importance tempering effective sample size (ESS). If itemps==NULL this corre-

sponds to the total number of samples collected, i.e..

R\*(BTE[2]-BTE[1])/BTE[3].

Otherwise the ESS will be lower due to a non-zero coefficient of variation of the

calculated importance tempering weights

sens See sens documentation for more details

#### Note

Inputs X, XX, Z containing NaN, NA, or Inf are discarded with non-fatal warnings

Upon execution, MCMC reports are made every 1,000 rounds to indicate progress

Stationary (non-treed) processes on larger inputs (e.g., X,Z) of size greater than 500, \*might\* be slow in execution, especially on older machines. Once the C code starts executing, it can be interrupted in the usual way: either via Ctrl-C (Unix-alikes) or pressing the Stop button in the R-GUI. When this happens, interrupt messages will indicate which required cleanup measures completed before returning control to R.

Whereas most of the **tgp** models will work reasonably well with little or no change to the default prior specification, GP's with the "mrexpsep" correlation imply a very specific relationship between fine and coarse data, and a careful prior specification is usually required.

The ranks provided in the second column of the improv field of a tgp object are based on the expectation of a multivariate improvement that may or may not be raised to a positive integer power. They can thus differ significantly from a simple ranking of the first column of expected univariate improvement values.

Regarding trace=TRUE: Samples from the posterior will be collected for all parameters in the model. GP parameters are collected with reference to the locations in XX, resulting nn=nrow{XX} traces of d,g,s2,tau2, etc. Therefore, it is recommended that nn is chosen to be a small, representative, set of input locations. Besides GP parameters, traces are saved for the tree partitions, areas under the LLM, log posterior (as a function of tree height), and samples from the posterior predictive distributions. Note that since some traces are stored in files, multiple tgp/R sessions in the same working directory can clobber the trace files of other sessions

#### Author(s)

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```
https://bobby.gramacy.com/r_packages/tgp/
```

### See Also

```
plot.tgp, tgp.trees, predict.tgp, sens, default.itemps
```

### **Examples**

```
##
## Many of the examples below illustrate the above
## function(s) on random data. Thus it can be fun
## (and informative) to run them several times.
##

# simple linear response
#

# input and predictive data
X <- seq(0,1,length=50)
XX <- seq(0,1,length=99)
Z <- 1 + 2*X + rnorm(length(X),sd=0.25)

out <- blm(X=X, Z=Z, XX=XX) # try Linear Model
plot(out) # plot the surface

#
# 1-d Example
#
# construct some 1-d nonstationary data</pre>
```

```
X < - seq(0,20, length=100)
XX \leftarrow seq(0,20,length=99)
Z \leftarrow (\sin(pi*X/5) + 0.2*\cos(4*pi*X/5)) * (X <= 9.6)
lin <- X>9.6;
Z[lin] <- -1 + X[lin]/10
Z \leftarrow Z + rnorm(length(Z), sd=0.1)
out <- btlm(X=X, Z=Z, XX=XX) # try Linear CART
plot(out) # plot the surface
                  # plot the MAP trees
tgp.trees(out)
out <- btgp(X=X, Z=Z, XX=XX) # use a treed GP
plot(out) # plot the surface
tgp.trees(out) # plot the MAP trees
#
# 2-d example
# (using the isotropic correlation function)
# construct some 2-d nonstationary data
exp2d.data <- exp2d.rand()</pre>
X <- exp2d.data$X; Z <- exp2d.data$Z</pre>
XX <- exp2d.data$XX
# try a GP
out <- bgp(X=X, Z=Z, XX=XX, corr="exp")</pre>
plot(out)
          # plot the surface
# try a treed GP LLM
out <- btgpllm(X=X, Z=Z, XX=XX, corr="exp")</pre>
plot(out) # plot the surface
tgp.trees(out) # plot the MAP trees
# Motorcycle Accident Data
# get the data
require(MASS)
# try a GP
out <- bgp(X=mcycle[,1], Z=mcycle[,2])</pre>
plot(out) # plot the surface
# try a treed GP LLM
# best to use the "b0" beta linear prior to capture common
# common linear process throughout all regions (using the
# ellipses "...")
out <- btgpllm(X=mcycle[,1], Z=mcycle[,2], bprior="b0")</pre>
plot(out) # plot the surface
tgp.trees(out) # plot the MAP trees
```

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default.itemps

Default Sigmoidal, Harmonic and Geometric Temperature Ladders

### **Description**

Parameterized by the minimum desired *inverse* temperature, this function generates a ladder of inverse temperatures k[1:m] starting at k[1] = 1, with m steps down to the final temperature k[m] = 1k. min progressing sigmoidally, harmonically or geometrically. The output is in a format convenient for the b\* functions in the tgp package (e.g. btgp), including stochastic approximation parameters  $c_0$  and  $n_0$  for tuning the uniform pseudo-prior output by this function

### Usage

```
default.itemps(m = 40, type = c("geometric", "harmonic", "sigmoidal"),
               k.min = 0.1, c0n0 = c(100, 1000), lambda = c("opt",
               "naive", "st"))
```

#### **Arguments**

m Number of temperatures in the ladder; m=1 corresponds to *importance sampling* 

at the temperature specified by k.min (in this case all other arguments are ig-

nored)

Choose from amongst two common defaults for simulated tempering and Metropolistype

coupled MCMC, i.e., geometric (default) or harmonic, or a sigmoidal ladder

(default) that concentrates more inverse temperatures near 1

Minimum inverse temperature desired k.min

c0n0 Stochastic approximation parameters used to tune the simulated tempering pseudo-

> prior (\$pk) to get a uniform posterior over the inverse temperatures; must be a 2-vector of positive integers c(c0, n0); see the Geyer & Thompson reference

below

Method for combining the importance samplers at each temperature. Optimal

combination ("opt") is the default, weighting the IS at each temperature k by

$$\lambda_k \propto (\sum_i w_{ki})^2 / \sum_i w_{ki}^2.$$

Setting lambda = "naive" allows each temperature to contribute equally ( $\lambda_k \propto$ 1, or equivalently ignores delineations due to temperature when using importance weights. Setting lambda = "st" allows only the first (cold) temperature to contribute to the estimator, thereby implementing simulated tempering

lambda

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### **Details**

The geometric and harmonic inverse temperature ladders are usually defined by an index  $i=1,\ldots,m$  and a parameter  $\Delta_k>0$ . The geometric ladder is defined by

$$k_i = (1 + \Delta_k)^{1-i},$$

and the harmonic ladder by

$$k_i = (1 + \Delta_k(i-1))^{-1}.$$

Alternatively, specifying the minimum temperature  $k_{\min}$  in the ladder can be used to uniquely determine  $\Delta_k$ . E.g., for the geometric ladder

$$\Delta_k = k_{\min}^{1/(1-m)} - 1,$$

and for the harmonic

$$\Delta_k = \frac{k_{\min}^{-1} - 1}{m - 1}.$$

In a similar spirit, the sigmoidal ladder is specified by first situating m indices  $j_i \in \Re$  so that  $k_1 = k(j_1) = 1$  and  $k_m = k(j_m) = k_{\min}$  under

$$k(j_i) = 1.01 - \frac{1}{1 + e^{j_i}}.$$

The remaining  $j_i, i = 2, ..., (m-1)$  are spaced evenly between  $j_1$  and  $j_m$  to fill out the ladder  $k_i = k(j_i), i = 1, ..., (m-1)$ .

For more details, see the *Importance tempering* paper cited below and a full demonstration in vignette("tgp2")

#### Value

The return value is a list which is compatible with the input argument itemps to the b\* functions (e.g. btgp), containing the following entries:

c0n0	A copy of the cono input argument
k	The generated inverse temperature ladder; a vector with length(k) = $m$ containing a decreasing sequence from 1 down to k.min
pk	A vector with length(pk) = $m$ containing an initial pseudo-prior for the temperature ladder of $1/m$ for each inverse temperature
lambda	IT method, as specified by the input argument

#### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

14 default.itemps

#### References

Gramacy, R.B., Samworth, R.J., and King, R. (2010) *Importance Tempering*. ArXiV article 0707.4242 Statistics and Computing, 20(1), pp. 1-7; https://arxiv.org/abs/0707.4242.

For stochastic approximation and simulated tempering (ST):

Geyer, C.~and Thompson, E.~(1995). *Annealing Markov chain Monte Carlo with applications to ancestral inference*. Journal of the American Statistical Association, **90**, 909–920.

For the geometric temperature ladder:

Neal, R.M.~(2001) Annealed importance sampling. Statistics and Computing, 11, 125–129

Justifying geometric and harmonic defaults:

Liu, J.S.~(1002) Monte Carlo Strategies in Scientific Computing. New York: Springer. Chapter 10 (pages 213 & 233)

https://bobby.gramacy.com/r\_packages/tgp/

#### See Also

btgp

# **Examples**

```
## comparing the different ladders
geo <- default.itemps(type="geometric")</pre>
har <- default.itemps(type="harmonic")</pre>
sig <- default.itemps(type="sigmoidal")</pre>
par(mfrow=c(2,1))
matplot(cbind(geo$k, har$k, sig$k), pch=21:23,
        main="inv-temp ladders", xlab="indx",
        ylab="itemp")
legend("topright", pch=21:23,
       c("geometric", "harmonic", "sigmoidal"), col=1:3)
matplot(log(cbind(sig$k, geo$k, har$k)), pch=21:23,
        main="log(inv-temp) ladders", xlab="indx",
        ylab="itemp")
## Not run:
## using Importance Tempering (IT) to improve mixing
## on the motorcycle accident dataset
library(MASS)
out.it <- btgpllm(X=mcycle[,1], Z=mcycle[,2], BTE=c(2000,22000,2),
        R=3, itemps=default.itemps(), bprior="b0", trace=TRUE,
        pred.n=FALSE)
## compare to regular tgp w/o IT
out.reg <- btgpllm(X=mcycle[,1], Z=mcycle[,2], BTE=c(2000,22000,2),
        R=3, bprior="b0", trace=TRUE, pred.n=FALSE)
## compare the heights explored by the three chains:
## REG, combining all temperatures, and IT
p <- out.it$trace$post</pre>
L <- length(p$height)
```

dopt.gp

dopt.gp

Sequential D-Optimal Design for a Stationary Gaussian Process

# **Description**

Create sequential D-Optimal design for a stationary Gaussian process model of fixed parameterization by subsampling from a list of candidates

# Usage

```
dopt.gp(nn, X=NULL, Xcand, iter=5000, verb=0)
```

# Arguments

nn	Number of new points in the design. Must be less than or equal to the number of candidates contained in Xcand, i.e., nn <= nrow(Xcand)
X	${\tt data.frame}, {\tt matrix}$ or vector of input locations which are forced into (already in) the design
Xcand	<pre>data.frame, matrix or vector of candidates from which new design points are subsampled. Must have the same dimension as X, i.e., ncol(X) == ncol(Xcand)</pre>
iter	number of iterations of stochastic accent algorithm, default 5000
verb	positive integer indicating after how many rounds of stochastic approximation to print each progress statement; default verb=0 results in no printing

# **Details**

Design is based on a stationary Gaussian process model with stationary isotropic exponential correlation function with parameterization fixed as a function of the dimension of the inputs. The algorithm implemented is a simple stochastic ascent which maximizes det(K)—the covariance matrix constructed with locations X and a subset of Xcand of size nn. The selected design is *locally* optimal

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#### Value

The output is a list which contains the inputs to, and outputs of, the C code used to find the optimal design. The chosen design locations can be accessed as list members XX or equivalently Xcand[fi,].

X Input argument: data.frame of inputs X, can be NULL nn Input argument: number new points in the design

Xcand Input argument: data.frame of candidate locations Xcand ncand Number of rows in Xcand, i.e., nncand = dim(Xcand)[1]

fi Vector of length nn describing the selected new design locations as indices into

Xcand

XX data.frame of selected new design locations, i.e., XX = Xcand[fi,]

#### Note

Inputs X, Xcand containing NaN, NA, Inf are discarded with non-fatal warnings. If nn > dim(Xcand)[1] then a non-fatal warning is displayed and execution commences with nn = dim(Xcand)[1]

In the current version there is no progress indicator. You will have to be patient. Creating D-optimal designs is no speedy task

### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. (See Chapter 6.) https://bobby.gramacy.com/surrogates/

Chaloner, K. and Verdinelli, I. (1995). *Bayesian experimental design: A review*. Statist. Sci., 10, (pp. 273–304).

### See Also

```
tgp.design, lhs
```

# **Examples**

```
#
# 2-d Exponential data
# (This example is based on random data.
# It might be fun to run it a few times)
#
# get the data
exp2d.data <- exp2d.rand()
X <- exp2d.data$X; Z <- exp2d.data$Z
Xcand <- exp2d.data$XX</pre>
```

exp2d

```
# find a treed sequential D-Optimal design
# with 10 more points
dgp <- dopt.gp(10, X, Xcand)

# plot the d-optimally chosen locations
# Contrast with locations chosen via
# the tgp.design function
plot(X, pch=19, xlim=c(-2,6), ylim=c(-2,6))
points(dgp$XX)</pre>
```

exp2d

2-d Exponential Data

# **Description**

A 2-dimensional data set that can be used to validate non-stationary models.

### Usage

data(exp2d)

#### **Format**

A data frame with 441 observations on the following 4 variables.

- X1 Numeric vector describing the first dimension of X inputs
- X2 Numeric vector describing the second dimension of X inputs
- Z Numeric vector describing the response Z(X)+N(0, sd=0.001)

Ztrue Numeric vector describing the true response Z(X), without noise

### **Details**

The true response is evaluated as

$$Z(X) = x_1 * \exp(x_1^2 - x_2^2).$$

Zero-mean normal noise with sd=0.001 has been added to the true response

# Note

This data is used in the examples of the functions listed below in the "See Also" section via the exp2d.rand function

### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

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### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. https://bobby.gramacy.com/surrogates/

Gramacy, R. B. (2007). **tgp**: An R Package for Bayesian Nonstationary, Semiparametric Nonlinear Regression and Design by Treed Gaussian Process Models. Journal of Statistical Software, **19**(9). https://www.jstatsoft.org/v19/i09 doi:10.18637/jss.v019.i09

Robert B. Gramacy, Matthew Taddy (2010). Categorical Inputs, Sensitivity Analysis, Optimization and Importance Tempering with tgp Version 2, an R Package for Treed Gaussian Process Models. Journal of Statistical Software, 33(6), 1–48. https://www.jstatsoft.org/v33/i06/.doi:10.18637/jss.v033.i06

Gramacy, R. B., Lee, H. K. H. (2008). *Bayesian treed Gaussian process models with an application to computer modeling*. Journal of the American Statistical Association, 103(483), pp. 1119-1130. Also available as ArXiv article 0710.4536 https://arxiv.org/abs/0710.4536

https://bobby.gramacy.com/r\_packages/tgp/

### See Also

exp2d.rand, exp2d.Z, btgp, and other b\* functions

exp2d.rand

Random 2-d Exponential Data

### **Description**

A Random subsample of data(exp2d), or Latin Hypercube sampled data evaluated with exp2d.Z

### Usage

```
exp2d.rand(n1 = 50, n2 = 30, lh = NULL, dopt = 1)
```

# Arguments

n1	Number of samples from the first, interesting, quadrant
n2	Number of samples from the other three, uninteresting, quadrants
lh	If !is.null(1h) then Latin Hypercube (LH) sampling (1hs) is used instead of subsampling from data(exp2d); 1h should be a single nonnegative integer specifying the desired number of predictive locations, XX; or, it should be a vector of length 4, specifying the number of predictive locations desired from each of the four quadrants (interesting quadrant first, then counter-clockwise)
dopt	If dopt >= 2 then d-optimal subsampling from LH candidates of the multiple indicated by the value of dopt will be used. This argument only makes sense when !is.null(lh)

exp2d.rand

#### **Details**

When is.null(1h), data is subsampled without replacement from data(exp2d). Of the n1 + n2 <= 441 input/response pairs X, Z, there are n1 are taken from the first quadrant, i.e., where the response is interesting, and the remaining n2 are taken from the other three quadrants. The remaining 441 - (n1 + n2) are treated as predictive locations

Otherwise, when !is.null(lh), Latin Hypercube Sampling (lhs) is used

If dopt >= 2 then n1\*dopt LH candidates are used for to get a D-optimal subsample of size n1 from the first (interesting) quadrant. Similarly n2\*dopt in the rest of the un-interesting region. A total of 1h\*dopt candidates will be used for sequential D-optimal subsampling for predictive locations XX in all four quadrants assuming the already-sampled X locations will be in the design.

In all three cases, the response is evaluated as

$$Z(X) = x_1 * \exp(x_1^2 - x_2^2).$$

thus creating the outputs Ztrue and ZZtrue. Zero-mean normal noise with sd=0.001 is added to the responses Z and ZZ

#### Value

Output is a list with entries:

Χ	2-d data.frame with n1 + n2 input locations
Z	Numeric vector describing the responses (with noise) at the X input locations
Ztrue	Numeric vector describing the true responses (without noise) at the X input locations
XX	2-d data.frame containing the remaining 441 - (n1 + n2) input locations
ZZ	Numeric vector describing the responses (with noise) at the XX predictive locations
ZZtrue	Numeric vector describing the responses (without noise) at the XX predictive locations

#### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

# References

Gramacy, R. B. (2007). **tgp**: An R Package for Bayesian Nonstationary, Semiparametric Nonlinear Regression and Design by Treed Gaussian Process Models. Journal of Statistical Software, **19**(9). https://www.jstatsoft.org/v19/i09 doi:10.18637/jss.v019.i09

Robert B. Gramacy, Matthew Taddy (2010). Categorical Inputs, Sensitivity Analysis, Optimization and Importance Tempering with tgp Version 2, an R Package for Treed Gaussian Process Models. Journal of Statistical Software, 33(6), 1–48. https://www.jstatsoft.org/v33/i06/doi:10.18637/jss.v033.i06

Gramacy, R. B., Lee, H. K. H. (2008). *Bayesian treed Gaussian process models with an application to computer modeling*. Journal of the American Statistical Association, 103(483), pp. 1119-1130. Also available as ArXiv article 0710.4536 https://arxiv.org/abs/0710.4536

https://bobby.gramacy.com/r\_packages/tgp/

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### See Also

1hs, exp2d, exp2d. Z, btgp, and other b\* functions

# **Examples**

```
## randomly subsampled data
## -----
eds <- exp2d.rand()
# higher span = 0.5 required because the data is sparse
# and was generated randomly
eds.g <- interp.loess(eds$X[,1], eds$X[,2], eds$Z, span=0.5)</pre>
# perspective plot, and plot of the input (X & XX) locations
par(mfrow=c(1,2), bty="n")
persp(eds.g, main="loess surface", theta=-30, phi=20,
      xlab="X[,1]", ylab="X[,2]", zlab="Z")
plot(eds$X, main="Randomly Subsampled Inputs")
points(eds$XX, pch=19, cex=0.5)
## Latin Hypercube sampled data
edlh \leftarrow exp2d.rand(lh=c(20, 15, 10, 5))
# higher span = 0.5 required because the data is sparse
# and was generated randomly
edlh.g <- interp.loess(edlh$X[,1], edlh$X[,2], edlh$Z, span=0.5)
# perspective plot, and plot of the input (X & XX) locations
par(mfrow=c(1,2), bty="n")
persp(edlh.g, main="loess surface", theta=-30, phi=20,
      xlab="X[,1]", ylab="X[,2]", zlab="Z")
plot(edlh$X, main="Latin Hypercube Sampled Inputs")
points(edlh$XX, pch=19, cex=0.5)
# show the quadrants
abline(h=2, col=2, lty=2, lwd=2)
abline(v=2, col=2, lty=2, lwd=2)
## Not run:
## D-optimal subsample with a factor of 10 (more) candidates
## -----
edlhd <- exp2d.rand(lh=c(20, 15, 10, 5), dopt=10)
# higher span = 0.5 required because the data is sparse
# and was generated randomly
edlhd.g <- interp.loess(edlhd$X[,1], edlhd$X[,2], edlhd$Z, span=0.5)
```

exp2d.Z

exp2d.Z

Random Z-values for 2-d Exponential Data

# **Description**

Evaluate the functional (mean) response for the 2-d exponential data (truth) at the X inputs, and randomly sample noisy Z-values having normal error with standard deviation provided.

# Usage

```
exp2d.Z(X, sd=0.001)
```

### **Arguments**

X Must be a matrix or a data. frame with two columns describing input locationssd Standard deviation of iid normal noise added to the responses

#### **Details**

The response is evaluated as

$$Z(X) = x_1 * \exp(x_1^2 - x_2^2).$$

thus creating the outputs Z and Z true. Zero-mean normal noise with sd=0.001 is added to the responses Z and Z

#### Value

Output is a data. frame with columns:

Numeric vector describing the responses (with noise) at the X input locations
 Numeric vector describing the true responses (without noise) at the X input locations

# Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

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### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. https://bobby.gramacy.com/surrogates/

Gramacy, R. B. (2007). **tgp**: An R Package for Bayesian Nonstationary, Semiparametric Nonlinear Regression and Design by Treed Gaussian Process Models. Journal of Statistical Software, **19**(9). https://www.jstatsoft.org/v19/i09 doi:10.18637/jss.v019.i09

Robert B. Gramacy, Matthew Taddy (2010). Categorical Inputs, Sensitivity Analysis, Optimization and Importance Tempering with tgp Version 2, an R Package for Treed Gaussian Process Models. Journal of Statistical Software, 33(6), 1–48. https://www.jstatsoft.org/v33/i06/doi:10.18637/jss.v033.i06

Gramacy, R. B., Lee, H. K. H. (2008). *Bayesian treed Gaussian process models with an application to computer modeling*. Journal of the American Statistical Association, 103(483), pp. 1119-1130. Also available as ArXiv article 0710.4536 https://arxiv.org/abs/0710.4536

```
https://bobby.gramacy.com/r_packages/tgp/
```

# See Also

```
exp2d, exp2d.rand
```

### **Examples**

friedman.1.data

First Friedman Dataset and a variation

### Description

Generate X and Y values from the 10-dim "first" Friedman data set used to validate the Multivariate Adaptive Regression Splines (MARS) model, and a variation involving boolean indicators. This test function has three non-linear and interacting variables, along with two linear, and five which are irrelevant. The version with indicators has parts of the response turned on based on the setting of the indicators

# Usage

```
friedman.1.data(n = 100)
fried.bool(n = 100)
```

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### Arguments

n Number of samples desired

#### **Details**

In the original formulation, as implemented by friedman. 1. data the function has 10-dim inputs X are drawn from Unif(0,1), and responses are N(m(X),1) where  $m(\mathbf{x})=E[f(\mathbf{x})]$  and

$$m(\mathbf{x}) = 10\sin(\pi x_1 x_2) + 20(x_3 - 0.5)^2 + 10x_4 + 5x_5$$

The variation fried.bool uses indicators  $I \in \{1,2,3,4\}$ . The function also has 10-dim inputs X with columns distributed as  $\mathrm{Unif}(0,1)$  and responses are  $N(m(\mathbf{x},I),1)$  where  $m(\mathbf{x},I)=E(f(\mathbf{x},I))$  and

$$m(\mathbf{x}, I) = f_1(\mathbf{x})_{[I=1]} + f_2(\mathbf{x})_{[I=2]} + f_3(\mathbf{x})_{[I=3]} + m([x_{10}, \dots, x_1])_{[I=4]}$$

where

$$f_1(\mathbf{x}) = 10\sin(\pi x_1 x_2), \ f_2(\mathbf{x}) = 20(x_3 - 0.5)^2, \ \text{and} \ f_3(\mathbf{x}) = 10x_4 + 5x_5.$$

The indicator I is coded in binary in the output data frame as: c(0,0,0) for I=1, c(0,0,1) for I=2, c(0,1,0) for I=3, and c(1,0,0) for I=4.

#### Value

Output is a data. frame with columns

X.1, ..., X.10 describing the 10-d randomly sampled inputs

I.1, ..., I.3 boolean version of the indicators provided only for fried.bool, as described

above

Y sample responses (with N(0,1) noise)

Ytrue true responses (without noise)

#### Note

An example using the original version of the data (friedman.1.data) is contained in the first package vignette: vignette("tgp"). The boolean version fried.bool is used in second vignette vignette("tgp2")

### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

Gramacy, R. B. (2007). **tgp**: An R Package for Bayesian Nonstationary, Semiparametric Nonlinear Regression and Design by Treed Gaussian Process Models. Journal of Statistical Software, **19**(9). https://www.jstatsoft.org/v19/i09 doi:10.18637/jss.v019.i09

Robert B. Gramacy, Matthew Taddy (2010). Categorical Inputs, Sensitivity Analysis, Optimization and Importance Tempering with tgp Version 2, an R Package for Treed Gaussian Process

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*Models.* Journal of Statistical Software, **33**(6), 1–48. https://www.jstatsoft.org/v33/i06/. doi:10.18637/jss.v033.i06

Friedman, J. H. (1991). *Multivariate adaptive regression splines*. "Annals of Statistics", **19**, No. 1, 1–67.

Gramacy, R. B., Lee, H. K. H. (2008). *Bayesian treed Gaussian process models with an application to computer modeling*. Journal of the American Statistical Association, 103(483), pp. 1119-1130. Also available as ArXiv article 0710.4536 https://arxiv.org/abs/0710.4536

Chipman, H., George, E., & McCulloch, R. (2002). *Bayesian treed models*. Machine Learning, 48, 303–324.

https://bobby.gramacy.com/r\_packages/tgp/

#### See Also

bgpllm, btlm, blm, bgp, btgpllm, bgp

interp.loess

Lowess 2-d interpolation onto a uniform grid

# **Description**

Use the loess function to interpolate the two-dimensional x, y, and z data onto a uniform grid. The output produced is an object directly usable by the plotting functions persp, image, and contour, etc.

This function is designed as an alternative to the interp functions from the akima library.

#### Usage

```
interp.loess(x, y, z, gridlen = c(40,40), span = 0.1, ...)
```

### Arguments

X	Vector of X spatial input locations
у	Vector of Y spatial input locations
z	Vector of Z responses interpreted as Z = f(X,Y)
gridlen	Size of the interpolated grid to be produced in x and y. The default of gridlen = $c(40, 40)$ causes a $40 * 40$ grid of X, Y, and Z values to be computed.
span	Kernel span argument to the loess function with default setting span = 0.1 set significantly lower than the loess default – see note below.
	Further arguments to be passed to the loess function

# **Details**

Uses expand.grid function to produce a uniform grid of size gridlen with domain equal to the rectangle implied by X and Y. Then, a loess a smoother is fit to the data Z = f(X, Y). Finally, predict.loess is used to predict onto the grid.

interp.loess 25

#### Value

The output is a list compatible with the 2-d plotting functions persp, image, and contour, etc.

The list contains...

```
v Vector of with length(x) == gridlen of increasing X grid locations
v Vector of with length(y) == gridlen of increasing Y grid locations

matrix of interpolated responses Z = f(X,Y) where z[i,j] contains an estimate of f(x[i],y[j])
```

#### Note

As mentioned above, the default span = 0.1 parameter is significantly smaller that the default loess setting. This asserts a tacit assumption that the input is densely packed and that the noise in z's is small. Such should be the case when the data are output from a **tgp** regression – this function was designed specifically for this situation. For data that is random or sparse, simply choose higher setting, e.g., the default loess setting of span = 0.75, or a more intermediate setting of span = 0.5 as in the example below

#### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

```
https://bobby.gramacy.com/r_packages/tgp/
```

#### See Also

```
interp, loess, persp, image, contour
```

#### **Examples**

```
# random data
ed <- exp2d.rand()

# higher span = 0.5 required because the data is sparse
# and was generated randomly
ed.g <- interp.loess(ed$X[,1], ed$X[,2], ed$Z, span=0.5)

# perspective plot
persp(ed.g)</pre>
```

26 itemps

itemps	Functions to plot summary information about the sampled inverse temperatures, tree heights, etc., stored in the traces of a "tgp"-class object

# **Description**

Functions for making barplots summarizing the progress of importance tempering. The itemps.barplot function can be used to make a histogram of the inverse temperatures visited in the trans-temporal Markov chain. The hist2bar function is useful for making a histogram of integer-valued samples (e.g., tree heights) encountered in one or several Markov chains

### Usage

# **Arguments**

obj	"tgp"-class object
main	Main plot label to be augmented by itemps.barplot
xlab	Label for the x-axis
ylab	Label for the y-axis
plot.it	whether to plot the barplot in addition to returning the data.frame for later use in a barplot call
	other arguments passed to barplot if plot.it = TRUE
X	matrix of integers whose columns are treated as different realizations of similar processes producing where each row represents a sample (e.g., tree height) under that process

# **Details**

itemps.barplot specifically works with the \$trace field of a "tgp"-class object. An error will be produced if this field is NULL, i.e., if the b\* function used the create the object was not run with the argument trace=TRUE

The hist2bar function can be used on any integer (or discrete) valued matrix. The columns are interpreted as different realizations of similar processes for comparison with one another via a histogram.

The histogram is obtained with the barplot command used with the argument beside=TRUE. See the examples section of default.itemps

# Value

Both functions return a data.frame that can be used within the barplot function with argument beside=TRUE

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### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

Gramacy, R.B., Samworth, R.J., and King, R. (2007) *Importance Tempering*. ArXiv article 0707.4242 https://arxiv.org/abs/0707.4242 https://bobby.gramacy.com/r\_packages/tgp/

### See Also

```
default.itemps, vignette(tgp2), barplot
```

lhs

Latin Hypercube sampling

# **Description**

Draw a (random) Latin Hypercube (LH) sample of size n from in the region outlined by the provided rectangle

# Usage

```
lhs(n, rect, shape=NULL, mode=NULL)
```

# **Arguments**

n	Size of the LH sample
rect	Rectangle describing the domain from which the LH sample is to be taken. The rectangle should be a matrix or data.frame with ncol(rect) = 2, and number of rows equal to the dimension of the domain. For 1-d data, a vector of length 2 is allowed
shape	Optional vector of shape parameters for the Beta distribution. Vector of length equal to the dimension of the domain, with elements > 1. If this is specified, the LH sample is proportional to a joint pdf formed by independent Beta distributions in each dimension of the domain, scaled and shifted to have support defined by rect. Only concave Beta distributions with shape > 1 are supported.
mode	Optional vector of mode values for the Beta distribution. Vector of length equal to the dimension of the domain, with elements within the support defined by rect. If shape is specified, but this is not, then the scaled Beta distributions will be symmetric

# Value

The output is a matrix with n rows and nrow(rect) columns. Each of the n rows represents a sample point.

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#### Note

The domain bounds specified by the rows of rect can be specified backwards with no change in effect

#### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. (See Chapter 4.) https://bobby.gramacy.com/surrogates/

McKay, M. D., W. J. Conover and R. J. Beckman. (1979). A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code, Technometrics 21: (pp. 239–245).

### See Also

```
tgp.design, dopt.gp, sens
```

### **Examples**

```
# get and plot a 2-d LH design
s1 <- lhs(10, rbind(c(-2,3), c(0.5, 0.8)))
plot(s1)

# plot a grid to show that there is one sample
# in each grid location
abline(v=seq(-2,3,length=11), lty=2, col=3)
abline(h=seq(0.5,0.8,length=11), lty=2, col=3)</pre>
```

mapT

Plot the MAP partition, or add one to an existing plot

# Description

Plot the maximum a' posteriori (MAP) tree from a "tgp"-class object, or add one on top of an existing plot. Like plot.tgp, projections and slices of trees can be plotted as specified

# Usage

```
mapT(out, proj = NULL, slice = NULL, add = FALSE, lwd = 2, ...)
```

mapT 29

out	"tgp"-class object which is the output of one the model functions with tree support (e.g. btgpllm)
proj	1-or-2-Vector describing the dimensions to be shown in a projection. The argument is ignored for 1-d data, i.e., if $x$d == 1$ . For 2-d data, no projection needs to be specified— the default argument (proj = NULL) will result in a 2-d plot. 1-d projections of 2-d or higher trees are are supported, e.g., proj = c(2) would show the second variable projection. For 3-d data or higher, proj=NULL defaults to proj = c(1,2) which plots a 2-d projection of the trees for the first two variables. Slices have priority over projections— see next argument (slice)—when non-null arguments are provided for both.
slice	list object with x and z fields, which are vectors of equal length describing the slice to be plotted, i.e., which z-values of the treed partitions in the x\$d - 2 inputs x\$X and x\$XX should be fixed to in order to obtain a 2-d visualization. For example, for 4-d data, slice = list(x=(2,4), z=c( $\emptyset$ .2, 1.5) will result in a 2-d plot of the first and third dimensions which have the second and fourth slice fixed at 0.5 and 1.5. The default is NULL, yielding to the proj argument. Argument is ignored for 1-d data, i.e., if x\$d == 1
add	Specify whether the to add partitions to an existing plot (add = TRUE) or to make a new plot showing the data out\$X along with the partitions (default add = FALSE)
lwd	Plotting argument specifying the width of the lines used to depict the partitions
	Additional arguments to plot used when add = FALSE

# Value

The only output of this function is a beautiful region-representation of the MAP tree.

# Note

For examples, see vignette("tgp") and the examples provided in the documentation for the tgp.design function

# Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

# References

```
https://bobby.gramacy.com/r_packages/tgp/
```

# See Also

```
plot.tgp, tgp.trees, tgp.design, vignette("tgp")
```

optim.tgp

optim.tgp

Surrogate-based optimization of noisy black-box function

### **Description**

Optimize (minimize) a noisy black-box function (i.e., a function which may not be differentiable, and may not execute deterministically). A b\* **tgp** model is used as a surrogate for adaptive sampling via improvement (and other) statistics. Note that this function is intended as a skeleton to be tailored as required for a particular application

# Usage

```
optim.step.tgp(f, rect, model = btgp, prev = NULL, X = NULL,
    Z = NULL, NN = 20 * length(rect), improv = c(1, 5), cands = c("lhs", "tdopt"),
    method = c("L-BFGS-B", "Nelder-Mead", "BFGS", "CG", "SANN", "optimize"), ...)
optim.ptgpf(start, rect, tgp.obj,
    method=c("L-BFGS-B", "Nelder-Mead", "BFGS", "CG", "SANN", "optimize"))
```

### **Arguments**

f	A function to be optimized, having only one free argument
rect	matrix indicating the domain of the argument of f over which an optimal should be searched; must have ncol(rect) = 2 and nrow agreeing with the argument of f indicating the dimension of the data. For 1-d data, a vector of length 2 is allowed
model	The b* regression model used as a surrogate for optimization; see btgp, and others, for more detail
prev	The output from a previous call to optim.step.tgp; this should be a list with entries as described the "Value" section below
Χ	data.frame, matrix, or vector of current inputs X, to be augmented
Z	Vector of current output responses Z of length equal to the leading dimension (rows) of X, i.e., $length(Z) == nrow(X)$ , to be augmented
NN	Number of candidate locations (XX) at which to sample from the improvement statistic
improv	Indicates the improv argument provided to a b* model function for sampling from the improvement statistic at the NN candidate locations (XX); see btgp, and others, for more detail
cands	The type of candidates (XX) at which samples from the improvement statistics are gathered. The default setting of "lhs" is recommended. However, a sequential treed D-optimal design can be used with "tdopt" for a more global exploration; see tgp.design for more details
method	A method from optim, or "optimize" which uses optimize as appropriate (when the input-space is $1\text{-d}$ )
	Further arguments to the b* model function

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A starting value for optimization of the MAP predictive (kriging) surface of a "tgp"-class object. A good starting value is the X or XX location found to be a minimum in the mean predictive surface contained in "tgp"-class object

A "tgp"-class object that is the output of one of the b\* functions: blm, btlm bgp, bgpllm, btgp, or btgpllm, as can be used by predict.tgp for optimizing on the MAP predictive (surrogate) kriging surface

#### **Details**

tgp.obj

optim.step.tgp executes one step in a search for the global optimum (minimum) of a noisy function (Z~f(X)) in a bounded rectangle (rect). The procedure essentially fits a tgp model and samples from the posterior distribution of improvement statistics at NN+1 candidates locations. NN of the candidates come from cands, i.e., "lhs" or "tdopt", plus one which is the location of the minima found in a previous run via prev by using optim (with a particular method or optimize instead) on the MAP model predictive surface using the "tgp"-class object contained therein. The improv[2] with the the highest expected improvement are recommended for adding into the design on output.

optim.ptgpf is the subroutine used by optim.step.tgp to find optimize on the MAP (surrogate) predictive surface for the "tgp"-class object contained in prev.

Please see vignette("tgp2") for a detailed illustration

#### Value

The list return has the following components.

X A matrix with nrow(rect) columns whose rows contain recommendations for

input locations to add into the design

progress A one-row data. frame indicating the the X-location and objective value of the

current best guess of the solution to the (kriging) surrogate optimization along

with the maximum values of the improvement statistic

obj the "tgp"-class object output from the model function

#### Note

The ellipses (...) argument is used differently here, as compared to optim, and optimize. It allows further arguments to be passed to the b\* model function, whereas for optimit would describe further (static) arguments to the function f to be optimized. If static arguments need to be set for f, then we recommend setting defaults via the formals of f

### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. (See Chapter 7.) https://bobby.gramacy.com/surrogates/

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Matthew Taddy, Herbert K.H. Lee, Genetha A. Gray, and Joshua D. Griffin. (2009) *Bayesian guided pattern search for robust local optimization*. Technometrics, 51(4), pp. 389-401

```
https://bobby.gramacy.com/r_packages/tgp/
```

#### See Also

```
btgp, etc., optim, optimize, tgp.design, predict.tgp, dopt.gp
```

# **Examples**

```
## optimize the simple exponential function
f <- function(x) { exp2d.Z(x)$Z }</pre>
## create the initial design with D-optimal candidates
rect <- rbind(c(-2,6), c(-2,6))
Xcand <- lhs(500, rect)</pre>
X <- dopt.gp(50, X=NULL, Xcand)$XX</pre>
Z \leftarrow f(X)
## do 10 rounds of adaptive sampling
out <- progress <- NULL
for(i in 1:10) {
  ## get recommendations for the next point to sample
  out <- optim.step.tgp(f, X=X, Z=Z, rect=rect, prev=out)</pre>
  ## add in the inputs, and newly sampled outputs
  X <- rbind(X, out$X)</pre>
  Z \leftarrow c(Z, f(out$X))
  ## keep track of progress and best optimum
  progress <- rbind(progress, out$progress)</pre>
  print(progress[i,])
}
## plot the progress so far
par(mfrow=c(2,2))
plot(out$obj, layout="surf")
plot(out$obj, layout="as", as="improv")
matplot(progress[,1:nrow(rect)], main="optim results",
        xlab="rounds", ylab="x[,1:2]", type="1", lwd=2)
plot(log(progress$improv), type="l", main="max log improv",
     xlab="rounds", ylab="max log(improv)")
```

partition

Partition data according to the MAP tree

### **Description**

Partition data according to the maximum a' posteriori (MAP) tree contained in a "tgp"-class object.

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### Usage

```
partition(X, out)
```

# **Arguments**

#### Value

Output is a list of data. frames populated with the inputs X contained in each region of the partition of the MAP tree in the "tgp"-class object out

### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

```
https://bobby.gramacy.com/r_packages/tgp/
```

#### See Also

```
tgp.design, tgp.trees
```

# **Examples**

```
# # 2-d Exponential data
# (This example is based on random data.
# It might be fun to run it a few times)
#
# get the data
exp2d.data <- exp2d.rand()
X <- exp2d.data$X; Z <- exp2d.data$Z
Xcand <- exp2d.data$XX
# fit treed GP LLM model to data w/o prediction
# basically just to get MAP tree (and plot it)
out <- btgpllm(X=X, Z=Z, pred.n=FALSE, BTE=c(2000,3000,2))
tgp.trees(out)
# find a treed sequential D-Optimal design
# with 10 more points
Xcand.parts <- partition(Xcand, out)</pre>
```

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plot.tgp

Plotting for Treed Gaussian Process Models

#### Description

A generic function for plotting of "tgp"-class objects. 1-d posterior mean and error plots, 2-d posterior mean and error image and perspective plots, and 3+-dimensional mean and error image and perspective plots are supported via projection and slicing.

### Usage

# Arguments

x "tgp"-class object that is the output of one of the b\* functions: blm, btlm bgp, bgpllm, btgp, or btgpllm

pparts If TRUE, partition-regions are plotted (default), otherwise they are not

1-or-2-Vector describing the dimensions to be shown in a projection. The argument is ignored for 1-d data, i.e., if x\$d == 1. For 2-d data, no projection needs be specified— the default argument (proj = NULL) will result in a 2-d perspective or image plot. 1-d projections of 2-d or higher data are are supported, e.g., proj = c(2) would show the second variable projection. For 3-d data or higher, proj=NULL defaults to proj = c(1,2) which plots a 2-d projection for the first two variables. Slices have priority over the projections— see next argument (slice)— when non-null arguments are provided for both.

list object with x and z fields, which are vectors of equal length describing the slice to be plotted, i.e., which z-values of the x\$d-2 inputs x\$X and x\$XX should be fixed to in order to obtain a 2-d visualization. For example, for 4-d data, slice = list(x=(2,4), z=c(0.2,1.5) will result in a 2-d plot of the first and third dimensions which have the second and fourth slice fixed at 0.5 and 1.5. The default is NULL, yielding to the proj argument. Argument is ignored for 1-d data, i.e., if x\$d==1

Optional 2-d map (longitude and latitude) from **maps** to be shown on top of image plots

Default center = "mean" causes the posterior predictive mean to be plotted as the centering statistic. Otherwise the median can be used with center = "med", or the kriging mean with center = "km"

proj

slice

map

center

plot.tgp 35

as Optional string indicator for plotting of adaptive sampling statistics: specify-

ing as = "alm" for ALM, as = "s2" for predictive variance, as = "ks2" for expected kriging variance, as = "alc" for ALC, and as = "improv" for expected improvement (about the minimum, see the rankmax argument below). The default as = NULL plots error-bars (1d-plots) or error magnitudes (2d-plots), which

is essentially the same as as = "alm"

layout Specify whether to plot the mean predictive surface (layout = "surf"), the er-

ror or adaptive sampling statistics (layout = "as"), or default (layout = "both") which shows both. If layout = "sens", plot the results of a sensitivity analysis

(see sens) in a format determined by the argument maineff below.

main Optional character string to add to the main title of the plot valab Optional character string to add to the x label of the plots optional character string to add to the y label of the plots

zlab Optional character string to add to the z label of the plots; ignored unless pc =

"p"

pc Selects perspective-posterior mean and image-error plots (pc = "pc", the de-

fault) or a double-image plot (pc = "c")

(only valid for 2-d plots)

gridlen Number of regular grid points for 2-d slices and projections in x and y. The

default of gridlen = c(40,40) causes a 40 \* 40 grid of X, Y, and Z values to be

computed. Ignored for 1-d plots and projections

span Span for loess kernel. The tgp package default (span = 0.1) is set lower than

the loess default. Smaller spans can lead to warnings from loess when the data or predictive locations are sparse and ugly plots may result. In this case, try

increasing the span

pXX scalar logical indicating if XX locations should be plotted

legendloc Location of the legend included in the plots of sensitivity analyses produced

with layout = "sens", or 1-d plots of multi-resolution models (with corr = "mrexpsep") and option mrlayout = "both"; otherwise the argument is ignored

maineff Format for the plots of sensitivity analyses produced with layout = "sens";

otherwise the argument is ignored. If maineff=TRUE main effect plots are produced alongside boxplots for posterior samples of the sensitivity indices, and if FALSE only the boxplots are produced. Alternatively, maineff can be a matrix containing input dimensions in the configuration that the corresponding main effects are to be plotted; that is, mfrow=dim(maineff). In this case, a 90 percent interval is plotted with each main effect and the sensitivity index boxplots

are not plotted.

mrlayout The plot layout for double resolution tgp objects with params\$corr == "mrexpsep".

For the default mrlayout="both", the coarse and fine fidelity are plotted together, either on the same plot for 1D inputs or through side-by-side image plots of the predicted center with axis determined by proj for inputs of greater dimension. Note that many of the standard arguments – such as slice, pc, and map – are either non-applicable or unsupported for mrlayout="both". If mrlayout="coarse" or mrlayout="fine", prediction for the respective fidelity

is plotted as usual and all of the standard options apply.

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rankmax

When as = "improv" is provided, the posterior expected improvements are plotted according the the first column of the improv field of the "tgp"-class object. Text is added to the plot near the XX positions of the first 1:rankmax predictive locations with the highest ranks in the second column of the improv field.

Extra arguments to 1-d (plot) and 2-d plotting functions persp and image

#### Value

The only output of this function is beautiful plots

### Note

This plotting function is provided with the intention that it will be used as an aid in the visualization of "tgp"-class objects. Users are encouraged to use the source code for this function in order to develop custom plotting functions.

1-d projections for 3-d or higher data are also available by specifying a 1-d projection argument (e.g. projec(1) for projecting onto the first input variable).

For examples, see vignette("tgp") and the help files of those functions in "See Also", below

### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

```
https://bobby.gramacy.com/r_packages/tgp/
```

#### See Also

```
plot, bgpllm, btlm, blm, bgp, btgpllm, predict.tgp, tgp.trees, mapT, loess, sens
```

predict.tgp

Predict method for Treed Gaussian process models

# Description

This generic prediction method was designed to obtain samples from the posterior predictive distribution after the b\* functions have finished. Samples, or kriging mean and variance estimates, can be obtained from the MAP model encoded in the "tgp"-class object, or this parameterization can be used as a jumping-off point in obtaining further samples from the joint posterior and posterior predictive distributions

# Usage

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#### **Arguments**

MAP

object "tgp"-class object that is the output of one of the b\* functions: blm, btlm bgp,

bgpllm, btgp, or btgpllm

XX Optional data. frame, matrix, or vector of predictive input locations with ncol(XX)

== ncol(object\$X)

BTE 3-vector of Monte-carlo parameters (B)urn in, (T)otal, and (E)very. Predictive

samples are saved every E MCMC rounds starting at round B, stopping at T. The default BTE=c(0,1,1) is specified to give the kriging means and variances

as outputs, plus one sample from the posterior predictive distribution

R Number of repeats or restarts of BTE MCMC rounds, default R=1 is no restarts

When TRUE (default) predictive data (i.e., kriging mean and variance estimates, and samples from the posterior predictive distribution) are obtained for the *fixed* MAP model encoded in object. Otherwise, when MAP=FALSE sampling from the joint posterior of the model parameters (i.e., tree and GPs) and the posterior predictive distribution are obtained starting from the MAP model and proceed-

ing just as the b\* functions

pred.n TRUE (default) value results in prediction at the inputs X; FALSE skips prediction

at X resulting in a faster implementation

krige TRUE (default) value results in collection of kriging means and variances at pre-

dictive (and/or data) locations; FALSE skips the gathering of kriging statistics

giving a savings in storage

zcov If TRUE then the predictive covariance matrix is calculated—can be computationally (and memory) intensive if X or XX is large. Otherwise only the variances

(diagonal of covariance matrices) are calculated (default). See outputs Zp. s2,

ZZ.s2, etc., below

Ds2x TRUE results in ALC (Active Learning–Cohn) computation of expected reduction in uncertainty calculations at the X locations, which can be used for adaptive

sampling; FALSE (default) skips this computation, resulting in a faster imple-

mentation

improv TRUE results in samples from the improvement at locations XX with respect to the observed data minimum. These samples are used to calculate the expected im-

provement over XX, as well as to rank all of the points in XX in the order that they should be sampled to minimize the expected multivariate improvement (refer to Schonlau et al, 1998). Alternatively, improv can be set to any positive integer 'g', in which case the ranking is performed with respect to the expectation for improvement raised to the power 'g'. Increasing 'g' leads to rankings that are more oriented towards a global optimization. The option FALSE (default) skips these computations, resulting in a faster implementation. Optionally, a two-vector can be supplied where improv[2] is interpreted as the (maximum) number of points to rank by improvement. See the note in btgp documentation.

If not specified, then the larger of 10% of nn = nrow(XX) and min(10, nn) is taken by default

sens.p Either NULL or a vector of parameters for sensitivity analysis, built by the function sens. Refer there for details 38 predict.tgp

trace	TRUE results in a saving of samples from the posterior distribution for most of the parameters in the model. The default is FALSE for speed/storage reasons. See note below
verb	Level of verbosity of R-console print statements: from 0 (default: none); 1 which shows the "progress meter"; 2 includes an echo of initialization parameters; up to 3 and 4 (max) with more info about successful tree operations
	Ellipses are not used in the current version of predict.tgp. They are are only included in order to maintain S3 generic/method consistency

#### **Details**

While this function was designed with prediction in mind, it is actually far more general. It allows a continuation of MCMC sampling where the b\* function left off (when MAP=FALSE) with a possibly new set of predictive locations XX. The intended use of this function is to obtain quick kriging-style predictions for a previously-fit MAP estimate (contained in a "tgp"-class object) on a new set of predictive locations XX. However, it can also be used simply to extend the search for an MAP model when MAP=FALSE, pred.n=FALSE, and XX=NULL

#### Value

The output is the same, or a subset of, the output produced by the b\* functions, for example see btgp

#### Note

It is important to note that this function is not a replacement for supplying XX to the b\* functions, which is the only way to get fully Bayesian samples from the posterior prediction at new inputs. It is only intended as a post-analysis (diagnostic) tool.

Inputs XX containing NaN, NA, or Inf are discarded with non-fatal warnings. Upon execution, MCMC reports are made every 1,000 rounds to indicate progress.

If XXs are provided which fall outside the range of X inputs provided to the original b\* function, then those will not be extrapolated properly, due to the way that bounding rectangles are defined in the original run. For a workaround, supply out\$Xsplit <- rbind(X, XX) before running predict on out.

See note for btgp or another b\* function regarding the handling and appropriate specification of traces.

The "tgp" class output produced by predict.tgp can also be used as input to predict.tgp, as well as others (e.g., plot.tgp.

## Author(s)

Robert B. Gramacy, com></p

#### References

https://bobby.gramacy.com/r\_packages/tgp/

## See Also

```
predict, blm, btlm, bgp, btgp, bgpllm, btgpllm, plot.tgp
```

## Examples

```
## revisit the Motorcycle data
require(MASS)
## fit a btgpllm without predictive sampling (for speed)
out <- btgpllm(X=mcycle[,1], Z=mcycle[,2], bprior="b0",</pre>
        pred.n=FALSE)
## nothing to plot here because there is no predictive data
## save the "tgp" class output object for use later and
save(out, file="out.Rsave")
## then remove it (for illustrative purposes)
out <- NULL
## (now imagine emailing the out.Rsave file to a friend who
## then performs the following in order to use your fitted
## tgp model on his/her own predictive locations)
## load in the "tgp" class object we just saved
load("out.Rsave")
## new predictive locations
XX < - seq(2.4, 56.7, length=200)
## now obtain kriging estimates from the MAP model
out.kp <- predict(out, XX=XX, pred.n=FALSE)</pre>
plot(out.kp, center="km", as="ks2")
## actually obtain predictive samples from the MAP
out.p <- predict(out, XX=XX, pred.n=FALSE, BTE=c(0,1000,1))
plot(out.p)
## use the MAP as a jumping-off point for more sampling
out2 <- predict(out, XX, pred.n=FALSE, BTE=c(0,2000,2),</pre>
                MAP=FALSE, verb=1)
plot(out2)
## (generally you would not want to remove the file)
unlink("out.Rsave")
```

## Description

Fully Bayesian Monte Carlo sensitivity analysis scheme, based upon any of the regression models contained in the **tgp** package. Random Latin hypercube samples are drawn at each MCMC iteration in order to estimate main effects as well as 1st order and total sensitivity indices.

## Usage

## Arguments

i guillelles	
Χ	data.frame, matrix, or vector of inputs X
Z	Vector of output responses Z of length equal to the leading dimension (rows) of $X$ , i.e., length(Z) == nrow(X)
nn.lhs	Size of each Latin hypercube drawn for use in the Monte Carlo integration scheme. Total number of locations for prediction is $nn.lhs*(ncol(X)+2)$
model	Either the regression model used for prediction, or NULL. If model=NULL, then the function just returns the sens.p vector of parameters to be passed with a regression model call. This can be used to perform sensitivity analysis through the predict.tgp framework
ngrid	The number of grid points in each input dimension upon which main effects will be estimated.
span	Smoothing parameter for main effects integration: the fraction of nn.1hs points that will be included in a moving average window that is used to estimate main effects at the ngrid locations in each input dimension.
ВТЕ	3-vector of Monte-Carlo parameters (B)urn in, (T)otal, and (E)very. Predictive samples are saved every E MCMC rounds starting at round B, stopping at T
rect	Rectangle describing the domain of the uncertainty distribution with respect to which the sensitivity is to be determined. This defines the domain from which the LH sample is to be taken. The rectangle should be a matrix or data.frame with ncol(rect) = 2, and number of rows equal to the dimension of the domain. For 1-d data, a vector of length 2 is allowed. Defaults to the input data range (X).
shape	Optional vector of shape parameters for the Beta distribution. Vector of length equal to the dimension of the domain, with elements > 1. If specified, the uncertainty distribution (i.e. the LH sample) is proportional to a joint pdf formed by independent Beta distributions in each dimension of the domain, scaled and shifted to have support defined by rect. Only concave Beta distributions with shape > 1 are supported. If unspecified, the uncertainty distribution is uniform over rect. The specification shape[i]=0 instructs sens to treat the i'th dimension as a binary variable. In this case, mode[i] is the probability parameter for a bernoulli uncertainty distribution, and we must also have rect[i,]=c(0,1).
mode	Optional vector of mode values for the Beta uncertainty distribution. Vector of length equal to the dimension of the domain, with elements within the support defined by rect. If shape is specified, but this is not, then the scaled Beta

distributions will be symmetric.

Extra arguments to the **tgp** model.

#### **Details**

Saltelli (2002) describes a Latin Hypercube sampling based method for estimation of the 'Sobal' sensitivity indices:

1st Order for input i,

$$S(i) = Var(E[f|x_i])/Var(f),$$

where  $x_i$  is the *i*-th input.

Total Effect for input i,

$$T(i) = E[\operatorname{Var}(f|x_{-i})]/\operatorname{Var}(f),$$

where  $x_{-i}$  is all inputs except for the *i*-th.

All moments are with respect to the appropriate marginals of the uncertainty distribution U – that is, the probability distribution on the inputs with respect to which sensitivity is being investigated. Under this approach, the integrals involved are approximated through averages over properly chosen samples based on two LH samples proportional to U. If nn.lhs is the sample size for the Monte Carlo estimate, this scheme requires nn.lhs\*(ncol(X)+2) function evaluations.

The sens function implements the method for unknown functions f, through prediction via one of the **tgp** regression models conditional on an observed set of X locations. At each MCMC iteration of the **tgp** model fitting, the nn.lhs\*(ncol(X)+2) locations are drawn randomly from the LHS scheme and realizations of the sensitivity indices are calculated. Thus we obtain a posterior sample of the indices, incorporating variability from both the Monte Carlo estimation and uncertainty about the function output. Since a subset of the predictive locations are actually an LHS proportional to the uncertainty distribution, we can also estimate the main effects through simple non-parametric regression (a moving average).

Please see vignette("tgp2") for a detailed illustration

## Value

The output is a "tgp"-class object. The details for btgp contain a complete description of this output. The list entry that is relevance to sensitivity analysis is sens, which itself has entries:

par	This contains a list of the input parameters used in the sensitivity analysis, as outlined above.
Xgrid	A matrix containing a grid in each input dimension (by column) over which the main effects are estimated.
ZZ.mean	A matrix, where each column contains the mean main effects over the corresponding column of sens.Xgrid.
ZZ.q1	A matrix, where each column contains the 5th percentile main effects over the corresponding column of sens.Xgrid.
ZZ.q2	A matrix, where each column contains the 5th percentile main effects over the corresponding column of sens.Xgrid.
S	A matrix, where each column contains the posterior sample of 1st order sensitivity indices for the corresponding input dimension.
T	A matrix, where each column contains the posterior sample of total sensitivity indices for the corresponding input dimension.

#### Note

The quality of sensitivity analysis is dependent on the size of the LH samples used for integral approximation; as with any Monte Carlo integration scheme, the sample size (nn. lhs) must increase with the dimensionality of the problem. The total sensitivity indices T are forced non-negative, and if negative values occur it is necessary to increase nn.lhs. The plot.tgp function replaces negative values with zero for illustration.

## Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. (See Chapter 8.) https://bobby.gramacy.com/surrogates/

R.D. Morris, A. Kottas, M. Taddy, R. Furfaro, and B. Ganapol. (2009) A statistical framework for the sensitivity analysis of radiative transfer models. IEEE Transactions on Geoscience and Remote Sensing, to appear.

Saltelli, A. (2002) *Making best use of model evaluations to compute sensitivity indices*. Computer Physics Communications, 145, 280-297.

#### See Also

```
btgp, plot.tgp, predict.tgp, lhs
```

## **Examples**

```
# Take a look at the air quality in New York: Sensitivity of
# ozone levels with respect to solar radiation, wind, and
# temperature. See help(airquality) for details.
X <- airquality[,2:4]</pre>
Z <- airquality$Ozone
# Uncertainty distribution is the default: uniform over range(X)
# There is missing data, which is removed automatically by tgp
# range(X).
s <- suppressWarnings(sens(X=X, Z=Z, nn.lhs=300, model=btgp,</pre>
          ngrid=100, span=0.3, BTE=c(5000,10000,10)))
# plot the results
plot(s, layout="sens", ylab="Ozone", main="main effects")
# plot only the sensitivity indices
plot(s, layout="sens", ylab="Ozone", maineff=FALSE)
# plot only the main effects, side by side
plot(s, layout="sens", ylab="Ozone", main="", maineff=t(1:3))
```

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```
# build a 'sens.p' parameter vector for a data-dependent
# informative uncertainty distribution. For each variable,
# the input distribution will be a scaled Beta with shape=2,
# and mode equal to the data mean
rect <- t(apply(X, 2, range, na.rm=TRUE))</pre>
mode <- apply(X , 2, mean, na.rm=TRUE)</pre>
shape \leftarrow rep(2,3)
# plot a sample from the marginal uncertainty distribution.
Udraw <- lhs(300, rect=rect, mode=mode, shape=shape)
par(mfrow=c(1,3))
for(i in 1:3) hist(Udraw[,i], breaks=15,xlab=names(X)[i])
# build sens.p with the 'sens' function.
sens.p <- suppressWarnings(sens(X=X, Z=Z, nn.lhs=300, model=NULL,</pre>
               ngrid=100, rect=rect, shape=shape, mode=mode))
# Use predict.tgp to quickly analyze with respect to this new
# uncertainty distribution without re-running the MCMC, then
# plot the results.
s.new <- predict(s, BTE=c(1,1000,1), sens.p=sens.p, verb=1)</pre>
plot(s.new, layout="sens", ylab="0zone", main="main effects")
```

tgp.default.params

Default Treed Gaussian Process Model Parameters

## **Description**

Construct a default list of parameters to the b\* functions- the interfaces to treed Gaussian process modeling

#### **Usage**

## **Arguments**

d number of input dimensions ncol(X)

meanfn A choice of mean function for the process. When meanfn = "linear" (default), then we have the process

$$Z = (\mathbf{1} \ \mathbf{X})\beta + W(\mathbf{X})$$

where  $W(\mathbf{X})$  represents the Gaussian process part of the model (if present). Otherwise, when meanfn = "constant", then

$$Z = \beta_0 + W(\mathbf{X})$$

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corr Gaussian process correlation model. Choose between the isotropic power ex-

ponential family ("exp") or the separable power exponential family ("expsep", default); the current version also supports the isotropic Matern ("matern") and single-index model ("sim") and "twovar" as "beta" functionality. The option "mrexpsep" uses a multi-resolution GP model, a depricated feature in the pack-

age (docs removed)

splitmin Indicates which column of the inputs X should be the first to allow splits via

treed partitioning. This is useful for excluding certain input directions from the

partitioning mechanism

basemax Indicates which column of the inputs X should be the last be fit under the base

model (e.g., LM or GP). This is useful for allowing some input directions (e.g., binary indicators) to only influence the tree partitioning mechanism, and not the

base model(s) at the leaves of the tree

These ellipses arguments are interpreted as augmentations to the prior specifica-

tion. You may use these to specify a custom setting of any of default parameters

in the output list detailed below

#### Value

The output is the following list of params...

col dimension of regression coefficients  $\beta$ : 1 for input meanfn = "constant", or

ncol(X)+1 for meanfn = "linear"

meanfn copied from the inputs corr copied from the inputs

bprior Linear (beta) prior, default is "bflat" which gives an "improper" prior which

can perform badly when the signal-to-noise ratio is low. In these cases the "proper" hierarchical specification "b0", "bmzt", or "bmznot" prior may per-

form better

beta rep(0,col) starting values for beta linear parameters

tree c(0.5, 2, max(c(10, col+1)), 1, d) indicating the tree prior process parameters

 $\alpha$ ,  $\beta$ , minpart, splitmin and basemax:

$$p_{ ext{ iny split}}(\eta, \mathcal{T}) = lpha * (1 + \eta)^{eta}$$

with zero probability given to trees with partitions containing less than nmin data points; *splitmin* indicates the first column of X which where treed partitioning is allowed; *basemax* gives the last column where the base model is used

s2.p c(5,10)  $\sigma^2$  inverse-gamma prior parameters c(a0, g0) where g0 is rate parameter

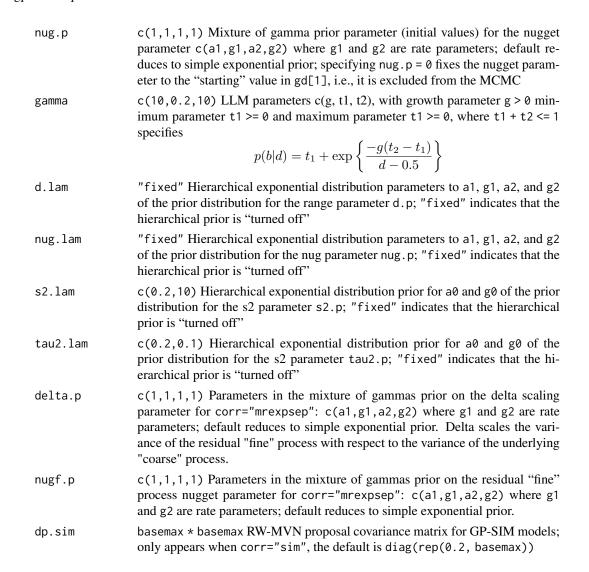
tau2.p  $c(5,10) \tau^2$  inverse-gamma prior parameters c(a0, g0) where g0 is rate parameters

d.p c(1.0,20.0,10.0,10.0) Mixture of gamma prior parameter (initial values) for the

range parameter(s) c(a1,g1,a2,g2) where g1 and g2 are rate parameters. If corr="mrexpsep", then this is a vector of length 8: The first four parameters remain the same and correspond to the "coarse" process, and the second set of four values, which default to c(1,10,1,10), are the equivalent prior parameters

for the range parameter(s) in the residual "fine" process.

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#### Note

Please refer to the examples for the functions in "See Also" below, vignette("tgp") and vignette(tgp2)

## Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

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Robert B. Gramacy, Heng Lian (2011). *Gaussian process single-index models as emulators for computer experiments*. Available as ArXiv article 1009.4241 https://arxiv.org/abs/1009.4241

https://bobby.gramacy.com/r\_packages/tgp/

#### See Also

blm, btlm, bgp, btgp, bgpllm, btgpllm

tgp.design	Sequential Treed D-Optimal Design for Treed Gaussian Process Models

## Description

Based on the maximum a' posteriori (MAP) treed partition extracted from a "tgp"-class object, calculate independent sequential treed D-Optimal designs in each of the regions.

## Usage

```
tgp.design(howmany, Xcand, out, iter = 5000, verb = 0)
```

## **Arguments**

howmany	Number of new points in the design. Must be less than the number of candidates contained in Xcand, i.e., howmany <= nrow(Xcand)
Xcand	<pre>data.frame, matrix or vector of candidates from which new design points are subsampled. Must have nrow(Xcand) == nrow(out\$X)</pre>
out	"tgp"-class object output from one of the model functions which has tree support, e.g., btgpllm, btgp, btlm
iter	number of iterations of stochastic accent algorithm, default 5000
verb	positive integer indicating after how many rounds of stochastic approximation in dopt.gp to print each progress statement; default verb=0 results in no printing

#### **Details**

This function partitions Xcand and out\$X based on the MAP tree (obtained on "tgp"-class out with partition) and calls dopt.gp in order to obtain a D-optimal design under independent stationary Gaussian processes models defined in each region. The aim is to obtain a design where new points from Xcand are spaced out relative to themselves, and relative to the existing locations (out\$X) in the region. The number of new points from each region of the partition is proportional to the number of candidates Xcand in the region.

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#### Value

Output is a list of data. frames containing XX design points for each region of the MAP tree in out

#### Note

Input Xcand containing NaN, NA, Inf are discarded with non-fatal warnings

D-Optimal computation in each region is preceded by a print statement indicated the number of new locations to be chosen and the number of candidates in the region. Other than that, there are no other indicators of progress. You will have to be patient. Creating treed sequential D-optimal designs is no speedy task. At least it faster than the non-treed version (see dopt.gp).

The example below is also part of vignette("tgp"). Please see vignette("tgp2") for a similar example based on optimization using the optim.step.tgp

#### Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

#### References

Gramacy, R. B. (2020) Surrogates: Gaussian Process Modeling, Design and Optimization for the Applied Sciences. Boca Raton, Florida: Chapman Hall/CRC. (See Chapter 9.) https://bobby.gramacy.com/surrogates/

Gramacy, R. B. (2007). **tgp**: An R Package for Bayesian Nonstationary, Semiparametric Nonlinear Regression and Design by Treed Gaussian Process Models. Journal of Statistical Software, **19**(9). https://www.jstatsoft.org/v19/i09 doi:10.18637/jss.v019.i09

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https://bobby.gramacy.com/r\_packages/tgp/

#### See Also

```
bgpllm, btlm, blm, bgp, btgpllm, plot.tgp, dopt.gp, lhs, partition, optim.step.tgp
```

## **Examples**

```
#
# 2-d Exponential data
# (This example is based on random data.
# It might be fun to run it a few times)
#
```

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```
# get the data
exp2d.data <- exp2d.rand()</pre>
X <- exp2d.data$X; Z <- exp2d.data$Z</pre>
Xcand <- exp2d.data$XX</pre>
# fit treed GP LLM model to data w/o prediction
# basically just to get MAP tree (and plot it)
out <- btgpllm(X=X, Z=Z, pred.n=FALSE, corr="exp")</pre>
tgp.trees(out)
# find a treed sequential D-Optimal design
# with 10 more points. It is interesting to
# contrast this design with one obtained via
# the dopt.gp function
XX <- tgp.design(10, Xcand, out)
# now fit the model again in order to assess
# the predictive surface at those new design points
dout <- btgpllm(X=X, Z=Z, XX=XX, corr="exp")</pre>
plot(dout)
```

tgp.trees

Plot the MAP Tree for each height encountered by the Markov Chain

## **Description**

Plot the maximum a' posteriori (MAP) tree as a function of tree height, and show the log posterior probabilities for comparison.

## Usage

```
tgp.trees(out, heights = NULL, main = NULL, ...)
```

## **Arguments**

out	"tgp"-class object which is the output of one the model functions with tree support (e.g. btgpllm)
heights	Index vector of length less than length(out\$trees) describing trees to plot by their height. Default (NULL) is to plot all trees, one for each height encountered when sampling from the Markov chain of the tree posterior. This is equivalent to heights = out\$posts\$height. Specifying heights = "map" causes (only) the maximum a' posteriori (MAP) height tree to be plotted
main	Optional character string to add to the main title of the plot
	Extra arguments to the draw. tree function from <b>maptree</b>

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## **Details**

The maximum a' posteriori (MAP) tree encountered at each height (in the MCMC chain) is plotted, and the log posterior probabilities are shown for comparison. The text at the branches in the tree show the splitting variable and value. The text at the leaves show the number of input data points (X and Z) that fall into the region(s) along with an estimate of the variability therein.

## Value

The only output of this function is beautiful tree diagrams.

## Note

Plotting trees that the **maptree** library is installed, which itself requires that the **combinat** library also be installed.

See vignette("tgp") and the examples sections of the functions under "See Also", below

## Author(s)

Robert B. Gramacy, <rbg@vt.edu>, and Matt Taddy, <mataddy@amazon.com>

## References

```
https://bobby.gramacy.com/r_packages/tgp/
```

## See Also

```
bgpllm, btlm, blm, bgp, btgpllm, plot.tgp, mapT, vignette("tgp")
```

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