

# Package: signal (via r-universe)

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**Suggests** pracma

**Enhances** matlab

**Description** A set of signal processing functions originally written for 'Matlab' and 'Octave'. Includes filter generation utilities, filtering functions, resampling routines, and visualization of filter models. It also includes interpolation functions.

**License** GPL-2

**URL** <https://signal.R-forge.R-project.org>

**NeedsCompilation** yes

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signal-package	<i>Signal processing</i>
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## Description

A set of generally Matlab/Octave-compatible signal processing functions. Includes filter generation utilities, filtering functions, resampling routines, and visualization of filter models. It also includes interpolation functions and some Matlab compatibility functions.

## Details

The main routines are:

*Filtering:* filter, fftfilt, filtfilt, medfilt1, sgolay, sgolayfilt

*Resampling:* interp, resample, decimate

*IIR filter design:* bilinear, butter, buttord, cheb1ord, cheb2ord, cheby1, cheby2, ellip, ellipord, sftrans

*FIR filter design:* fir1, fir2, remez, kaiserord, spencer

*Interpolation:* interp1, pchip

*Compatibility routines and utilities:* ifft, sinc, postpad, chirp, poly, polyval

*Windowing:* bartlett, blackman, boxcar, flattopwin, gausswin, hamming, hanning, triang

*Analysis and visualization:* freqs, freqz, impz, zplane, grpdelay, specgram

Most of the functions accept Matlab-compatible argument lists, but many are generic functions and can accept simpler argument lists.

For a complete list, use `library(help="signal")`.

## Author(s)

Most of these routines were translated from Octave Forge routines. The main credit goes to the original Octave authors:

Paul Kienzle, John W. Eaton, Kurt Hornik, Andreas Weingessel, Kai Habel, Julius O. Smith III, Bill Lash, André Carezia, Paulo Neis, David Billingham, Friedrich Leisch

Translations by Tom Short <tshort@eprisolutions.com> (who maintained the package until 2009).

Current maintainer is Uwe Ligges <ligges@statistik.tu-dortmund.de>.

## References

[https://en.wikipedia.org/wiki/Category:Signal\\_processing](https://en.wikipedia.org/wiki/Category:Signal_processing)

Octave Forge <https://octave.sourceforge.io/>

Package `matlab` by P. Roebuck

For Matlab/Octave conversion and compatibility, see <https://mathesaurus.sourceforge.net/octave-r.html> by Vidar Bronken Gundersen and <https://cran.r-project.org/doc/contrib/R-and-octave.txt> by Robin Hankin.

### Examples

```
## The R implementation of these routines can be called "matlab-style",
bf <- butter(5, 0.2)
freqz(bf$b, bf$a)
## or "R-style" as:
freqz(bf)

## make a Chebyshev type II filter:
ch <- cheby2(5, 20, 0.2)
freqz(ch, Fs = 100) # frequency plot for a sample rate = 100 Hz

zplane(ch) # look at the poles and zeros

## apply the filter to a signal
t <- seq(0, 1, by = 0.01) # 1 second sample, Fs = 100 Hz
x <- sin(2*pi*t*2.3) + 0.25*rnorm(length(t)) # 2.3 Hz sinusoid+noise
z <- filter(ch, x) # apply filter
plot(t, x, type = "l")
lines(t, z, col = "red")

# look at the group delay as a function of frequency
grpdelay(ch, Fs = 100)
```

---

an	<i>Complex unit phasor of the given angle in degrees.</i>
----	---

---

### Description

Complex unit phasor of the given angle in degrees.

### Usage

```
an(degrees)
```

### Arguments

degrees      Angle in degrees.

### Details

This is a utility function to make it easier to specify phasor values as a magnitude times an angle in degrees.

### Value

A complex value or array of  $\exp(1i \cdot \text{degrees} \cdot \pi / 180)$ .

### Examples

```
120*an(30) + 125*an(-160)
```

Arma

*Create an autoregressive moving average (ARMA) model.***Description**

Returns an ARMA model. The model could represent a filter or system model.

**Usage**

```
Arma(b, a)

## S3 method for class 'Zpg'
as.Arma(x, ...)

## S3 method for class 'Arma'
as.Arma(x, ...)

## S3 method for class 'Ma'
as.Arma(x, ...)
```

**Arguments**

b	moving average (MA) polynomial coefficients.
a	autoregressive (AR) polynomial coefficients.
x	model or filter to be converted to an ARMA representation.
...	additional arguments (ignored).

**Details**

The ARMA model is defined by:

$$a(L)y(t) = b(L)x(t)$$

The ARMA model can define an analog or digital model. The AR and MA polynomial coefficients follow the Matlab/Octave convention where the coefficients are in decreasing order of the polynomial (the opposite of the definitions for filter from the stats package and polyroot from the base package). For an analog model,

$$H(s) = \frac{b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_m}{a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}$$

For a z-plane digital model,

$$H(z) = \frac{b_1 + b_2 z^{-1} + \dots + b_m z^{-m+1}}{a_1 + a_2 z^{-1} + \dots + a_n z^{-n+1}}$$

as.Arma converts from other forms, including Zpg and Ma.

Value

A list of class Arma with the following list elements:

- b moving average (MA) polynomial coefficients
- a autoregressive (AR) polynomial coefficients

Author(s)

Tom Short, EPRI Solutions, Inc., (<tshort@eprisolutions.com>)

See Also

See also [as.Zpg](#), [Ma](#), [filter](#), and various filter-generation functions like [butter](#) and [cheby1](#) that return Arma models.

Examples

```
filt <- Arma(b = c(1, 2, 1)/3, a = c(1, 1))
zplane(filt)
```

---

bilinear	<i>Bilinear transformation</i>
----------	--------------------------------

---

Description

Transform a s-plane filter specification into a z-plane specification.

Usage

```
## Default S3 method:
bilinear(Sz, Sp, Sg, T, ...)

## S3 method for class 'Zpg'
bilinear(Sz, T, ...)

## S3 method for class 'Arma'
bilinear(Sz, T, ...)
```

Arguments

- Sz In the generic case, a model to be transformed. In the default case, a vector containing the zeros in a pole-zero-gain model.
- Sp a vector containing the poles in a pole-zero-gain model.
- Sg a vector containing the gain in a pole-zero-gain model.
- T the sampling frequency represented in the z plane.
- ... Arguments passed to the generic function.

## Details

Given a piecewise flat filter design, you can transform it from the s-plane to the z-plane while maintaining the band edges by means of the bilinear transform. This maps the left hand side of the s-plane into the interior of the unit circle. The mapping is highly non-linear, so you must design your filter with band edges in the s-plane positioned at  $2/T \tan(w * T/2)$  so that they will be positioned at  $w$  after the bilinear transform is complete.

The bilinear transform is:

$$z = \frac{1 + sT/2}{1 - sT/2}$$

$$s = \frac{T}{2} \frac{z - 1}{z + 1}$$

Please note that a pole and a zero at the same place exactly cancel. This is significant since the bilinear transform creates numerous extra poles and zeros, most of which cancel. Those which do not cancel have a “fill-in” effect, extending the shorter of the sets to have the same number of as the longer of the sets of poles and zeros (or at least split the difference in the case of the band pass filter). There may be other opportunistic cancellations, but it will not check for them.

Also note that any pole on the unit circle or beyond will result in an unstable filter. Because of cancellation, this will only happen if the number of poles is smaller than the number of zeros. The analytic design methods all yield more poles than zeros, so this will not be a problem.

## Value

For the default case or for `bilinear.Zpg`, an object of class “Zpg”, containing the list elements:

zero	complex vector of the zeros of the transformed model
pole	complex vector of the poles of the transformed model
gain	gain of the transformed model

For `bilinear.Arma`, an object of class “Arma”, containing the list elements:

b	moving average (MA) polynomial coefficients
a	autoregressive (AR) polynomial coefficients

## Author(s)

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

## References

Proakis & Manolakis (1992). *Digital Signal Processing*. New York: Macmillan Publishing Company.

[https://en.wikipedia.org/wiki/Bilinear\\_transform](https://en.wikipedia.org/wiki/Bilinear_transform)

Octave Forge <https://octave.sourceforge.io/>

See Also

[Zpg](#), [sftrans](#), [Arma](#)

---

butter	<i>Generate a Butterworth filter.</i>
--------	---------------------------------------

---

Description

Generate Butterworth filter polynomial coefficients.

Usage

```
## Default S3 method:
butter(n, W, type = c("low", "high", "stop", "pass"),
plane = c("z", "s"), ...)

## S3 method for class 'FilterOfOrder'
butter(n, ...)
```

Arguments

n	filter order or generic filter model
W	critical frequencies of the filter. W must be a scalar for low-pass and high-pass filters, and W must be a two-element vector c(low, high) specifying the lower and upper bands. For digital filters, W must be between 0 and 1 where 1 is the Nyquist frequency.
type	Filter type, one of "low" for a low-pass filter, "high" for a high-pass filter, "stop" for a stop-band (band-reject) filter, or "pass" for a pass-band filter.
plane	"z" for a digital filter or "s" for an analog filter.
...	additional arguments passed to butter, overriding those given by n of class FilterOfOrder.

Details

Because butter is generic, it can be extended to accept other inputs, using "buttord" to generate filter criteria for example.

Value

An Arma object with list elements:

b	moving average (MA) polynomial coefficients
a	autoregressive (AR) polynomial coefficients



**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Modified by Doug Stewart.  
Conversion to R by Tom Short.

**References**

Proakis & Manolakis (1992). *Digital Signal Processing*. New York: Macmillan Publishing Company.

[https://en.wikipedia.org/wiki/Butterworth\\_filter](https://en.wikipedia.org/wiki/Butterworth_filter)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[Arma](#), [filter](#), [cheby1](#), [ellip](#), and [buttord](#)

**Examples**

```
bf <- butter(4, 0.1)
freqz(bf)
zplane(bf)
```

---

 buttord

---

*Butterworth filter order and cutoff*


---

**Description**

Compute butterworth filter order and cutoff for the desired response characteristics.

**Usage**

```
buttord(Wp, Ws, Rp, Rs)
```

**Arguments**

Wp, Ws	pass-band and stop-band edges. For a low-pass or high-pass filter, Wp and Ws are scalars. For a band-pass or band-rejection filter, both are vectors of length 2. For a low-pass filter, $W_p < W_s$ . For a high-pass filter, $W_s > W_p$ . For a band-pass ( $W_s[1] < W_p[1] < W_p[2] < W_s[2]$ ) or band-reject ( $W_p[1] < W_s[1] < W_s[2] < W_p[2]$ ) filter design, Wp gives the edges of the pass band, and Ws gives the edges of the stop band. Frequencies are normalized to [0,1], corresponding to the range [0, $F_s/2$ ].
Rp	allowable decibels of ripple in the pass band.
Rs	minimum attenuation in the stop band in dB.

## Details

Deriving the order and cutoff is based on:

$$|H(W)|^2 = \frac{1}{1 + (W/W_c)^{2n}} = 10^{-R/10}$$

With some algebra, you can solve simultaneously for  $W_c$  and  $n$  given  $W_s$ ,  $R_s$  and  $W_p$ ,  $R_p$ . For high-pass filters, subtracting the band edges from  $F_s/2$ , performing the test, and swapping the resulting  $W_c$  back works beautifully. For bandpass- and bandstop-filters, this process significantly overdesigns. Artificially dividing  $n$  by 2 in this case helps a lot, but it still overdesigns.

## Value

An object of class `FilterOfOrder` with the following list elements:

<code>n</code>	filter order
<code>Wc</code>	cutoff frequency
<code>type</code>	filter type, one of “low”, “high”, “stop”, or “pass”

This object can be passed directly to `butter` to compute filter coefficients.

## Author(s)

Original Octave version by Paul Kienzle, <pkienzle@user.sf.net>. Conversion to R by Tom Short.

## References

Octave Forge <https://octave.sourceforge.io/>

## See Also

[butter](#), [FilterOfOrder](#), [cheb1ord](#)

## Examples

```
Fs <- 10000
btord <- buttord(1000/(Fs/2), 1200/(Fs/2), 0.5, 29)
plot(c(0, 1000, 1000, 0, 0), c(0, 0, -0.5, -0.5, 0),
     type = "l", xlab = "Frequency (Hz)", ylab = "Attenuation (dB)")
bt <- butter(btord)
plot(c(0, 1000, 1000, 0, 0), c(0, 0, -0.5, -0.5, 0),
     type = "l", xlab = "Frequency (Hz)", ylab = "Attenuation (dB)",
     col = "red", ylim = c(-10,0), xlim = c(0,2000))
hf <- freqz(bt, Fs = Fs)
lines(hf$f, 20*log10(abs(hf$h)))
```

---

cheb1ord	<i>Chebyshev type-I filter order and cutoff</i>
----------	---

---

**Description**

Compute discrete Chebyshev type-I filter order and cutoff for the desired response characteristics.

**Usage**

```
cheb1ord(Wp, Ws, Rp, Rs)
```

**Arguments**

Wp, Ws	pass-band and stop-band edges. For a low-pass or high-pass filter, Wp and Ws are scalars. For a band-pass or band-rejection filter, both are vectors of length 2. For a low-pass filter, $W_p < W_s$ . For a high-pass filter, $W_s > W_p$ . For a band-pass ( $W_s[1] < W_p[1] < W_p[2] < W_s[2]$ ) or band-reject ( $W_p[1] < W_s[1] < W_s[2] < W_p[2]$ ) filter design, Wp gives the edges of the pass band, and Ws gives the edges of the stop band. Frequencies are normalized to [0,1], corresponding to the range [0, Fs/2].
Rp	allowable decibels of ripple in the pass band.
Rs	minimum attenuation in the stop band in dB.

**Value**

An object of class `FilterOfOrder` with the following list elements:

n	filter order
Wc	cutoff frequency
Rp	allowable decibels of ripple in the pass band
type	filter type, one of “low”, “high”, “stop”, or “pass”

This object can be passed directly to `cheby1` to compute filter coefficients.

**Author(s)**

Original Octave version by Paul Kienzle, <pkienzle@user.sf.net> and by Laurent S. Mazet.  
Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[cheby1](#), [FilterOfOrder](#), [buttord](#)

**Examples**

```

Fs <- 10000
chord <- cheb1ord(1000/(Fs/2), 1200/(Fs/2), 0.5, 29)
plot(c(0, 1000, 1000, 0, 0), c(0, 0, -0.5, -0.5, 0),
     type = "l", xlab = "Frequency (Hz)", ylab = "Attenuation (dB)")
ch1 <- cheby1(chord)
plot(c(0, 1000, 1000, 0, 0), c(0, 0, -0.5, -0.5, 0),
     type = "l", xlab = "Frequency (Hz)", ylab = "Attenuation (dB)",
     col = "red", ylim = c(-10,0), xlim = c(0,2000))
hf <- freqz(ch1, Fs = Fs)
lines(hf$f, 20*log10(abs(hf$h)))

```

chebwin

*Dolph-Chebyshev window coefficients***Description**

Returns the filter coefficients of the n-point Dolph-Chebyshev window with a given attenuation.

**Usage**

```
chebwin(n, at)
```

**Arguments**

n	length of the filter; number of coefficients to generate.
at	dB of attenuation in the stop-band of the corresponding Fourier transform.

**Details**

The window is described in frequency domain by the expression:

$$W(k) = \frac{Cheb(n-1, \beta * \cos(\pi * k/n))}{Cheb(n-1, \beta)}$$

with

$$\beta = \cosh(1/(n-1) * \operatorname{acosh}(10^{at/20}))$$

and  $Cheb(m, x)$  denoting the  $m$ -th order Chebyshev polynomial calculated at the point  $x$ .

Note that the denominator in  $W(k)$  above is not computed, and after the inverse Fourier transform the window is scaled by making its maximum value unitary.

**Value**

An array of length n with the filter coefficients.

**Author(s)**

Original Octave version by André Carezia, <acarezia@uol.com.br>. Conversion to R by Tom Short.

**References**

Peter Lynch, “The Dolph-Chebyshev Window: A Simple Optimal Filter”, Monthly Weather Review, Vol. 125, pp. 655-660, April 1997. <http://mathsci.ucd.ie/~plynch/Publications/Dolph.pdf>

C. Dolph, “A current distribution for broadside arrays which optimizes the relationship between beam width and side-lobe level”, Proc. IEEE, 34, pp. 335-348.

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[kaiser](#)

**Examples**

```
plot(chebwin(50, 100))
```

---

cheby1

---

*Generate a Chebyshev filter.*


---

**Description**

Generate a Chebyshev type I or type II filter coefficients with specified dB of pass band ripple.

**Usage**

```
## Default S3 method:
cheby1(n, Rp, W, type = c("low", "high", "stop",
"pass"), plane = c("z", "s"), ...)

## S3 method for class 'FilterOfOrder'
cheby1(n, Rp = n$Rp, W = n$Wc, type = n$type, ...)

## Default S3 method:
cheby2(n, Rp, W, type = c("low", "high", "stop",
"pass"), plane = c("z", "s"), ...)

## S3 method for class 'FilterOfOrder'
cheby2(n, ...)
```

**Arguments**

<code>n</code>	filter order or generic filter model
<code>Rp</code>	dB of pass band ripple
<code>W</code>	critical frequencies of the filter. <code>W</code> must be a scalar for low-pass and high-pass filters, and <code>W</code> must be a two-element vector <code>c(low, high)</code> specifying the lower and upper bands. For digital filters, <code>W</code> must be between 0 and 1 where 1 is the Nyquist frequency.
<code>type</code>	Filter type, one of "low" for a low-pass filter, "high" for a high-pass filter, "stop" for a stop-band (band-reject) filter, or "pass" for a pass-band filter.
<code>plane</code>	"z" for a digital filter or "s" for an analog filter.
<code>...</code>	additional arguments passed to <code>cheby1</code> or <code>cheby2</code> , overriding those given by <code>n</code> of class <code>FilterOfOrder</code> .

**Details**

Because `cheby1` and `cheby2` are generic, they can be extended to accept other inputs, using "`cheb1ord`" to generate filter criteria for example.

**Value**

An Arma object with list elements:

<code>b</code>	moving average (MA) polynomial coefficients
<code>a</code>	autoregressive (AR) polynomial coefficients

For `cheby1`, the ARMA model specifies a type-I Chebyshev filter, and for `cheby2`, a type-II Chebyshev filter.

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Modified by Doug Stewart. Conversion to R by Tom Short.

**References**

Parks & Burrus (1987). *Digital Filter Design*. New York: John Wiley & Sons, Inc.

[https://en.wikipedia.org/wiki/Chebyshev\\_filter](https://en.wikipedia.org/wiki/Chebyshev_filter)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[Arma](#), [filter](#), [butter](#), [ellip](#), and [cheblord](#)

**Examples**

```
# compare the frequency responses of 5th-order Butterworth and Chebyshev filters.
bf <- butter(5, 0.1)
cf <- cheby1(5, 3, 0.1)
bfr <- freqz(bf)
cfr <- freqz(cf)
plot(bfr$f/pi, 20 * log10(abs(bfr$h)), type = "l", ylim = c(-40, 0),
      xlim = c(0, .5), xlab = "Frequency", ylab = c("dB"))
lines(cfr$f/pi, 20 * log10(abs(cfr$h)), col = "red")
# compare type I and type II Chebyshev filters.
c1fr <- freqz(cheby1(5, .5, 0.5))
c2fr <- freqz(cheby2(5, 20, 0.5))
plot(c1fr$f/pi, abs(c1fr$h), type = "l", ylim = c(0, 1),
      xlab = "Frequency", ylab = c("Magnitude"))
lines(c2fr$f/pi, abs(c2fr$h), col = "red")
```

chirp

*A chirp signal***Description**

Generate a chirp signal. A chirp signal is a frequency swept cosine wave.

**Usage**

```
chirp(t, f0 = 0, t1 = 1, f1 = 100,
      form = c("linear", "quadratic", "logarithmic"), phase = 0)
```

**Arguments**

t	array of times at which to evaluate the chirp signal.
f0	frequency at time t=0.
t1	time, s.
f1	frequency at time t=t1.
form	shape of frequency sweep, one of "linear", "quadratic", or "logarithmic".
phase	phase shift at t=0.

**Details**

'linear' is:

$$f(t) = (f1 - f0) * (t/t1) + f0$$

'quadratic' is:

$$f(t) = (f1 - f0) * (t/t1)^2 + f0$$

'logarithmic' is:

$$f(t) = (f1 - f0)^{t/t1} + f0$$

### Value

Chirp signal, an array the same length as t.

### Author(s)

Original Octave version by Paul Kienzle. Conversion to R by Tom Short.

### References

Octave Forge <https://octave.sourceforge.io/>

### See Also

[specgram](#)

### Examples

```
ch <- chirp(seq(0, 0.6, len=5000))
plot(ch, type = "l")

# Shows a quadratic chirp of 400 Hz at t=0 and 100 Hz at t=10
# Time goes from -2 to 15 seconds.
specgram(chirp(seq(-2, 15, by=0.001), 400, 10, 100, "quadratic"))

# Shows a logarithmic chirp of 200 Hz at t=0 and 500 Hz at t=2
# Time goes from 0 to 5 seconds at 8000 Hz.
specgram(chirp(seq(0, 5, by=1/8000), 200, 2, 500, "logarithmic"))
```

---

conv

*Convolution*

---

### Description

A Matlab/Octave compatible convolution function that uses the Fast Fourier Transform.

### Usage

```
conv(x, y)
```

### Arguments

x, y                      numeric sequences to be convolved.



**Details**

The inputs `x` and `y` are post padded with zeros as follows:

```
ifft(fft(postpad(x, n) * fft(postpad(y, n))))
```

where  $n = \text{length}(x) + \text{length}(y) - 1$

**Value**

An array of length equal to  $\text{length}(x) + \text{length}(y) - 1$ . If `x` and `y` are polynomial coefficient vectors, `conv` returns the coefficients of the product polynomial.

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[convolve](#), [fft](#), [ifft](#), [fftfilt](#), [poly](#)

**Examples**

```
conv(c(1,2,3), c(1,2))
conv(c(1,2), c(1,2,3))
conv(c(1,-2), c(1,2))
```

---

decimate

*Decimate or downsample a signal*

---

**Description**

Downsample a signal by a factor, using an FIR or IIR filter.

**Usage**

```
decimate(x, q, n = if (ftype == "iir") 8 else 30, ftype = "iir")
```

**Arguments**

<code>x</code>	signal to be decimated.
<code>q</code>	integer factor to downsample by.
<code>n</code>	filter order used in the downsampling.
<code>ftype</code>	filter type, "iir" or "fir"

**Details**

By default, an order 8 Chebyshev type I filter is used or a 30-point FIR filter if `ftype` is `'fir'`. Note that `q` must be an integer for this rate change method.

Makes use of the `filtfilt` function with all its limitations.

**Value**

The decimated signal, an array of length `ceiling(length(x) / q)`.

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

`filter`, `resample`, `interp`

**Examples**

```
# The signal to decimate starts away from zero, is slowly varying
# at the start and quickly varying at the end, decimate and plot.
# Since it starts away from zero, you will see the boundary
# effects of the antialiasing filter clearly. You will also see
# how it follows the curve nicely in the slowly varying early
# part of the signal, but averages the curve in the quickly
# varying late part of the signal.
t <- seq(0, 2, by = 0.01)
x <- chirp(t, 2, 0.5, 10, 'quadratic') + sin(2*pi*t*0.4)
y <- decimate(x, 4) # factor of 4 decimation
plot(t, x, type = "l")
lines(t[seq(1,length(t), by = 4)], y, col = "blue")
```

---

ellip

*Elliptic or Cauer filter*

---

**Description**

Generate an Elliptic or Cauer filter (discrete and continuous).

## Usage

```
## Default S3 method:
ellip(n, Rp, Rs, W, type = c("low", "high", "stop",
"pass"), plane = c("z", "s"), ...)

## S3 method for class 'FilterOfOrder'
ellip(n, Rp = n$Rp, Rs = n$Rs, W = n$Wc, type = n$type, ...)
```

## Arguments

n	filter order or generic filter model
Rp	dB of pass band ripple
Rs	dB of stop band ripple
W	critical frequencies of the filter. W must be a scalar for low-pass and high-pass filters, and W must be a two-element vector c(low, high) specifying the lower and upper bands. For digital filters, W must be between 0 and 1 where 1 is the Nyquist frequency.
type	Filter type, one of "low" for a low-pass filter, "high" for a high-pass filter, "stop" for a stop-band (band-reject) filter, or "pass" for a pass-band filter.
plane	"z" for a digital filter or "s" for an analog filter.
...	additional arguments passed to ellip, overriding those given by n of class FilterOfOrder.

## Details

Because ellip is generic, it can be extended to accept other inputs, using "ellipord" to generate filter criteria for example.

## Value

An Arma object with list elements:

b	moving average (MA) polynomial coefficients
a	autoregressive (AR) polynomial coefficients

## Author(s)

Original Octave version by Paulo Neis <p\_neis@yahoo.com.br>. Modified by Doug Stewart. Conversion to R by Tom Short.

## References

Oppenheim, Alan V., *Discrete Time Signal Processing*, Hardcover, 1999.

Parente Ribeiro, E., Notas de aula da disciplina TE498 - Processamento Digital de Sinais, UFPR, 2001/2002.

[https://en.wikipedia.org/wiki/Elliptic\\_filter](https://en.wikipedia.org/wiki/Elliptic_filter)

Octave Forge <https://octave.sourceforge.io/>

See Also

[Arma](#), [filter](#), [butter](#), [cheby1](#), and [ellipord](#)

Examples

```
# compare the frequency responses of 5th-order Butterworth and elliptic filters.
bf <- butter(5, 0.1)
ef <- ellip(5, 3, 40, 0.1)
bfr <- freqz(bf)
efr <- freqz(ef)
plot(bfr$f, 20 * log10(abs(bfr$h)), type = "l", ylim = c(-50, 0),
     xlab = "Frequency, radians", ylab = c("dB"))
lines(efr$f, 20 * log10(abs(efr$h)), col = "red")
```

---

ellipord	<i>Elliptic filter order and cutoff</i>
----------	---

---

Description

Compute discrete elliptic filter order and cutoff for the desired response characteristics.

Usage

```
ellipord(Wp, Ws, Rp, Rs)
```

Arguments

Wp, Ws	pass-band and stop-band edges. For a low-pass or high-pass filter, Wp and Ws are scalars. For a band-pass or band-rejection filter, both are vectors of length 2. For a low-pass filter, Wp < Ws. For a high-pass filter, Ws > Wp. For a band-pass (Ws[1] < Wp[1] < Wp[2] < Ws[2]) or band-reject (Wp[1] < Ws[1] < Ws[2] < Wp[2]) filter design, Wp gives the edges of the pass band, and Ws gives the edges of the stop band. Frequencies are normalized to [0,1], corresponding to the range [0, Fs/2].
Rp	allowable decibels of ripple in the pass band.
Rs	minimum attenuation in the stop band in dB.

Value

An object of class `FilterOfOrder` with the following list elements:

n	filter order
Wc	cutoff frequency
type	filter type, one of "low", "high", "stop", or "pass"
Rp	dB of pass band ripple
Rs	dB of stop band ripple

This object can be passed directly to `ellip` to compute discrete filter coefficients.

**Author(s)**

Original Octave version by Paulo Neis <p\_neis@yahoo.com.br>. Modified by Doug Stewart. Conversion to R by Tom Short.

**References**

Lamar, Marcus Vinicius, Notas de aula da disciplina TE 456 - Circuitos Analogicos II, UFPR, 2001/2002.

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[Arma](#), [filter](#), [butter](#), [cheby1](#), and [ellipord](#)

**Examples**

```
Fs <- 10000
elord <- ellipord(1000/(Fs/2), 1200/(Fs/2), 0.5, 29)
plot(c(0, 1000, 1000, 0, 0), c(0, 0, -0.5, -0.5, 0),
     type = "l", xlab = "Frequency (Hz)", ylab = "Attenuation (dB)")
el1 <- ellip(elord)
plot(c(0, 1000, 1000, 0, 0), c(0, 0, -0.5, -0.5, 0),
     type = "l", xlab = "Frequency (Hz)", ylab = "Attenuation (dB)",
     col = "red", ylim = c(-35,0), xlim = c(0,2000))
lines(c(5000, 1200, 1200, 5000, 5000), c(-1000, -1000, -29, -29, -1000),
      col = "red")
hf <- freqz(el1, Fs = Fs)
lines(hf$f, 20*log10(abs(hf$h)))
```

---

fftfilt

---

*Filters with an FIR filter using the FFT*


---

**Description**

Filters with an FIR filter using the FFT.

**Usage**

```
fftfilt(b, x, n = NULL)

FftFilter(b, n)

## S3 method for class 'FftFilter'
filter(filt, x, ...)
```

### Arguments

<code>b</code>	the moving-average (MA) coefficients of an FIR filter.
<code>x</code>	the input signal to be filtered.
<code>n</code>	if given, the length of the FFT window for the overlap-add method.
<code>filt</code>	filter to apply to the signal.
<code>...</code>	additional arguments (ignored).

### Details

If `n` is not specified explicitly, we do not use the overlap-add method at all because loops are really slow. Otherwise, we only ensure that the number of points in the FFT is the smallest power of two larger than `n` and `length(b)`.

### Value

For `fftfilt`, the filtered signal, the same length as the input signal `x`.

For `FftFilter`, a filter of class `FftFilter` that can be used with `filter`.

### Author(s)

Original Octave version by Kurt Hornik and John W. Eaton. Conversion to R by Tom Short.

### References

Octave Forge <https://octave.sourceforge.io/>

### See Also

[Ma](#), [filter](#), [fft](#), [filtfilt](#)

### Examples

```
t <- seq(0, 1, len = 100)           # 1 second sample
x <- sin(2*pi*t*2.3) + 0.25*rnorm(length(t)) # 2.3 Hz sinusoid+noise
z <- fftfilt(rep(1, 10)/10, x) # apply 10-point averaging filter
plot(t, x, type = "l")
lines(t, z, col = "red")
```

---

filter	<i>Filter a signal</i>
--------	------------------------

---

### Description

Generic filtering function. The default is to filter with an ARMA filter of given coefficients. The default filtering operation follows Matlab/Octave conventions.

### Usage

```
## Default S3 method:
filter(filt, a, x, init, init.x, init.y, ...)

## S3 method for class 'Arma'
filter(filt, x, ...)

## S3 method for class 'Ma'
filter(filt, x, ...)

## S3 method for class 'Zpg'
filter(filt, x, ...)
```

### Arguments

<code>filt</code>	For the default case, the moving-average coefficients of an ARMA filter (normally called ‘b’). Generically, <code>filt</code> specifies an arbitrary filter operation.
<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>x</code>	the input signal to be filtered.
<code>init, init.x, init.y</code>	
<code>init, init.x, init.y</code>	allows to supply initial data for the filter - this allows to filter very large time-series in pieces.
<code>...</code>	additional arguments (ignored).

### Details

The default filter is an ARMA filter defined as:

$$a_1 y_n + a_2 y_{n-1} + \cdots + a_n y_1 = b_1 x_n + b_2 x_{n-1} + \cdots + b_m x_1$$

The default filter calls `stats:::filter`, so it returns a time-series object.

Since `filter` is generic, it can be extended to call other filter types.

### Value

The filtered signal, normally of the same length of the input signal `x`.

Author(s)

Tom Short, EPRI Solutions, Inc., (<tshort@eprisolutions.com>)

References

[https://en.wikipedia.org/wiki/Digital\\_filter](https://en.wikipedia.org/wiki/Digital_filter)

Octave Forge <https://octave.sourceforge.io/>

See Also

[filter](#) in the **stats** package, [Arma](#), [fftfilt](#), [filtfilt](#), and [runmed](#).

Examples

```
bf <- butter(3, 0.1)                # 10 Hz low-pass filter
t <- seq(0, 1, len = 100)           # 1 second sample
x <- sin(2*pi*t*2.3) + 0.25*rnorm(length(t)) # 2.3 Hz sinusoid+noise
z <- filter(bf, x) # apply filter
plot(t, x, type = "l")
lines(t, z, col = "red")
```

---

FilterOfOrder	<i>Filter of given order and specifications.</i>
---------------	--

---

Description

IIR filter specifications, including order, frequency cutoff, type, and possibly others.

Usage

FilterOfOrder(n, Wc, type, ...)

Arguments

n	filter order
Wc	cutoff frequency
type	filter type, normally one of "low", "high", "stop", or "pass"
...	other filter description characteristics, possibly including Rp for dB of pass band ripple or Rs for dB of stop band ripple, depending on filter type (Chebyshev, etc.).

Details

The filter is



**Value**

A list of class `FilterOfOrder` with the following elements (repeats of the input arguments):

<code>n</code>	filter order
<code>Wc</code>	cutoff frequency
<code>type</code>	filter type, normally one of "low", "high", "stop", or "pass"
<code>...</code>	other filter description characteristics, possibly including <code>Rp</code> for dB of pass band ripple or <code>Rs</code> for dB of stop band ripple, depending on filter type (Chebyshev, etc.).

**Author(s)**

Tom Short

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [butter](#) and [buttord](#) [cheby1](#) and [cheb1ord](#), and [ellip](#) and [ellipord](#)

---

`filtfilt`*Forward and reverse filter a signal*

---

**Description**

Using two passes, forward and reverse filter a signal.

**Usage**

```
## Default S3 method:
filtfilt(filt, a, x, ...)

## S3 method for class 'Arma'
filtfilt(filt, x, ...)

## S3 method for class 'Ma'
filtfilt(filt, x, ...)

## S3 method for class 'Zpg'
filtfilt(filt, x, ...)
```

## Arguments

<code>filt</code>	For the default case, the moving-average coefficients of an ARMA filter (normally called ‘b’). Generically, <code>filt</code> specifies an arbitrary filter operation.
<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>x</code>	the input signal to be filtered.
<code>...</code>	additional arguments (ignored).

## Details

This corrects for phase distortion introduced by a one-pass filter, though it does square the magnitude response in the process. That’s the theory at least. In practice the phase correction is not perfect, and magnitude response is distorted, particularly in the stop band.

In this version, we zero-pad the end of the signal to give the reverse filter time to ramp up to the level at the end of the signal. Unfortunately, the degree of padding required is dependent on the nature of the filter and not just its order, so this function needs some work yet - and is in the state of the year 2000 version of the Octave code.

Since `filtfilt` is generic, it can be extended to call other filter types.

## Value

The filtered signal, normally the same length as the input signal `x`.

## Author(s)

Original Octave version by Paul Kienzle, <pkienzle@user.sf.net>. Conversion to R by Tom Short.

## References

Octave Forge <https://octave.sourceforge.io/>

## See Also

`filter`, `Arma`, `fftfilt`

## Examples

```
bf <- butter(3, 0.1)                # 10 Hz low-pass filter
t <- seq(0, 1, len = 100)           # 1 second sample
x <- sin(2*pi*t*2.3) + 0.25*rnorm(length(t)) # 2.3 Hz sinusoid+noise
y <- filtfilt(bf, x)
z <- filter(bf, x) # apply filter
plot(t, x)
points(t, y, col="red")
points(t, z, col="blue")
legend("bottomleft", legend = c("data", "filtfilt", "filter"),
      pch = 1, col = c("black", "red", "blue"), bty = "n")
```

---

fir1	<i>FIR filter generation</i>
------	------------------------------

---

**Description**

FIR filter coefficients for a filter with the given order and frequency cutoff.

**Usage**

```
fir1(n, w, type = c("low", "high", "stop", "pass", "DC-0", "DC-1"),
     window = hamming(n + 1), scale = TRUE)
```

**Arguments**

n	order of the filter (1 less than the length of the filter)
w	band edges, strictly increasing vector in the range [0, 1], where 1 is the Nyquist frequency. A scalar for highpass or lowpass filters, a vector pair for bandpass or bandstop, or a vector for an alternating pass/stop filter.
type	character specifying filter type, one of "low" for a low-pass filter, "high" for a high-pass filter, "stop" for a stop-band (band-reject) filter, "pass" for a pass-band filter, "DC-0" for a bandpass as the first band of a multiband filter, or "DC-1" for a bandstop as the first band of a multiband filter.
window	smoothing window. The returned filter is the same shape as the smoothing window.
scale	whether to normalize or not. Use TRUE or 'scale' to set the magnitude of the center of the first passband to 1, and FALSE or 'noscale' to not normalize.

**Value**

The FIR filter coefficients, an array of length(n+1), of class Ma.

**Author(s)**

Original Octave version by Paul Kienzle, <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

[https://en.wikipedia.org/wiki/Fir\\_filter](https://en.wikipedia.org/wiki/Fir_filter)  
 Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [Ma](#), [fftfilt](#), [fir2](#)

**Examples**

```

freqz(fir1(40, 0.3))
freqz(fir1(10, c(0.3, 0.5), "stop"))
freqz(fir1(10, c(0.3, 0.5), "pass"))

```

---

fir2

---

*FIR filter generation*


---

**Description**

FIR filter coefficients for a filter with the given order and frequency cutoffs.

**Usage**

```

fir2(n, f, m, grid_n = 512, ramp_n = grid_n/20, window = hamming(n + 1))

```

**Arguments**

n	order of the filter (1 less than the length of the filter)
f	band edges, strictly increasing vector in the range [0, 1] where 1 is the Nyquist frequency. The first element must be 0 and the last element must be 1. If elements are identical, it indicates a jump in frequency response.
m	magnitude at band edges, a vector of length(f).
grid_n	length of ideal frequency response function defaults to 512, should be a power of 2 bigger than n.
ramp_n	transition width for jumps in filter response defaults to grid_n/20. A wider ramp gives wider transitions but has better stopband characteristics.
window	smoothing window. The returned filter is the same shape as the smoothing window.

**Value**

The FIR filter coefficients, an array of length(n+1), of class Ma.

**Author(s)**

Original Octave version by Paul Kienzle, <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [Ma](#), [fftfilt](#), [fir1](#)

**Examples**

```
f <- c(0, 0.3, 0.3, 0.6, 0.6, 1)
m <- c(0, 0, 1, 1/2, 0, 0)
fh <- freqz(fir2(100, f, m))
op <- par(mfrow = c(1, 2))
plot(f, m, type = "b", ylab = "magnitude", xlab = "Frequency")
lines(fh$f / pi, abs(fh$h), col = "blue")
# plot in dB:
plot(f, 20*log10(m+1e-5), type = "b", ylab = "dB", xlab = "Frequency")
lines(fh$f / pi, 20*log10(abs(fh$h)), col = "blue")
par(op)
```

---

freqs	<i>s-plane frequency response</i>
-------	-----------------------------------

---

**Description**

Compute the *s*-plane frequency response of an ARMA model (IIR filter).

**Usage**

```
## Default S3 method:
freqs(filt = 1, a = 1, W, ...)

## S3 method for class 'Arma'
freqs(filt, ...)

## S3 method for class 'Ma'
freqs(filt, ...)

## S3 method for class 'freqs'
print(x, ...)

## S3 method for class 'freqs'
plot(x, ...)

## Default S3 method:
freqs_plot(w, h, ...)

## S3 method for class 'freqs'
freqs_plot(w, ...)
```

**Arguments**

<code>filt</code>	for the default case, the moving-average coefficients of an ARMA model or filter. Generically, <code>filt</code> specifies an arbitrary model or filter operation.
-------------------	--

<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>W</code>	the frequencies at which to evaluate the model.
<code>w</code>	for the default case, the array of frequencies. Generically, <code>w</code> specifies an object from which to plot a frequency response.
<code>h</code>	a complex array of frequency responses at the given frequencies.
<code>x</code>	object to be plotted.
<code>...</code>	additional arguments passed through to <code>plot</code> .

### Details

When results of `freqs` are printed, `freqs_plot` will be called to display frequency plots of magnitude and phase. As with `lattice` plots, automatic printing does not work inside loops and function calls, so explicit calls to `print` are needed there.

### Value

For `freqs` list of class `freqs` with items:

<code>H</code>	array of frequencies.
<code>W</code>	complex array of frequency responses at those frequencies.

### Author(s)

Original Octave version by Julius O. Smith III. Conversion to R by Tom Short.

### See Also

[filter](#), [Arma](#), [freqz](#)

### Examples

```
b <- c(1, 2)
a <- c(1, 1)
w <- seq(0, 4, length=128)
freqs(b, a, w)
```

---

freqz	<i>z-plane frequency response</i>
-------	-----------------------------------

---

### Description

Compute the *z*-plane frequency response of an ARMA model or IIR filter.

**Usage**

```
## Default S3 method:
freqz(filt = 1, a = 1, n = 512, region = NULL, Fs = 2 * pi, ...)

## S3 method for class 'Arma'
freqz(filt, ...)

## S3 method for class 'Ma'
freqz(filt, ...)

## S3 method for class 'freqz'
print(x, ...)

## S3 method for class 'freqz'
plot(x, ...)

## Default S3 method:
freqz_plot(w, h, ...)

## S3 method for class 'freqz'
freqz_plot(w, ...)
```

**Arguments**

<code>filt</code>	for the default case, the moving-average coefficients of an ARMA model or filter. Generically, <code>filt</code> specifies an arbitrary model or filter operation.
<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>n</code>	number of points at which to evaluate the frequency response.
<code>region</code>	'half' (the default) to evaluate around the upper half of the unit circle or 'whole' to evaluate around the entire unit circle.
<code>Fs</code>	sampling frequency in Hz. If not specified, the frequencies are in radians.
<code>w</code>	for the default case, the array of frequencies. Generically, <code>w</code> specifies an object from which to plot a frequency response.
<code>h</code>	a complex array of frequency responses at the given frequencies.
<code>x</code>	object to be plotted.
<code>...</code>	for methods of <code>freqz</code> , arguments are passed to the default method. For <code>freqz_plot</code> , additional arguments are passed through to <code>plot</code> .

**Details**

For fastest computation, `n` should factor into a small number of small primes.

When results of `freqz` are printed, `freqz_plot` will be called to display frequency plots of magnitude and phase. As with lattice plots, automatic printing does not work inside loops and function calls, so explicit calls to `print` or `plot` are needed there.

**Value**

For freqz list of class freqz with items:

h                      complex array of frequency responses at those frequencies.  
f                      array of frequencies.

**Author(s)**

Original Octave version by John W. Eaton. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [Arma](#), [freqs](#)

**Examples**

```
b <- c(1, 0, -1)
a <- c(1, 0, 0, 0, 0.25)
freqz(b, a)
```

---

grpdelay	<i>Group delay of a filter or model</i>
----------	---

---

**Description**

The group delay of a filter or model. The group delay is the time delay for a sinusoid at a given frequency.

**Usage**

```
## Default S3 method:
grpdelay(filt, a = 1, n = 512, whole = FALSE, Fs = NULL, ...)

## S3 method for class 'Arma'
grpdelay(filt, ...)

## S3 method for class 'Ma'
grpdelay(filt, ...)

## S3 method for class 'Zpg'
grpdelay(filt, ...)

## S3 method for class 'grpdelay'
plot(x, xlab = if(x$HzFlag) 'Hz' else 'radian/sample',
```



```

        ylab = 'Group delay (samples)', type = "l", ...)

## S3 method for class 'grpdelay'
print(x, ...)

```

### Arguments

<code>filt</code>	for the default case, the moving-average coefficients of an ARMA model or filter. Generically, <code>filt</code> specifies an arbitrary model or filter operation.
<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>n</code>	number of points at which to evaluate the frequency response.
<code>whole</code>	FALSE (the default) to evaluate around the upper half of the unit circle or TRUE to evaluate around the entire unit circle.
<code>Fs</code>	sampling frequency in Hz. If not specified, the frequencies are in radians.
<code>x</code>	object to be plotted.
<code>xlab, ylab, type</code>	as in <a href="#">plot</a> , but with more sensible defaults.
<code>...</code>	for methods of <code>grpdelay</code> , arguments are passed to the default method. For <code>plot.grpdelay</code> , additional arguments are passed through to <code>plot</code> .

### Details

For fastest computation, `n` should factor into a small number of small primes.

If the denominator of the computation becomes too small, the group delay is set to zero. (The group delay approaches infinity when there are poles or zeros very close to the unit circle in the  $z$  plane.)

When results of `grpdelay` are printed, the group delay will be plotted. As with `lattice` plots, automatic printing does not work inside loops and function calls, so explicit calls to `print` or `plot` are needed there.

### Value

A list of class `grpdelay` with items:

<code>gd</code>	the group delay, in units of samples. It can be converted to seconds by multiplying by the sampling period (or dividing by the sampling rate <code>Fs</code> ).
<code>w</code>	frequencies at which the group delay was calculated.
<code>ns</code>	number of points at which the group delay was calculated.
<code>HzFlag</code>	TRUE for frequencies in Hz, FALSE for frequencies in radians.

### Author(s)

Original Octave version by Julius O. Smith III and Paul Kienzle. Conversion to R by Tom Short.

### References

[https://ccrma.stanford.edu/~jos/filters/Numerical\\_Computation\\_Group\\_Delay.html](https://ccrma.stanford.edu/~jos/filters/Numerical_Computation_Group_Delay.html)

[https://en.wikipedia.org/wiki/Group\\_delay](https://en.wikipedia.org/wiki/Group_delay)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [Arma](#), [freqz](#)

**Examples**

```
# Two Zeros and Two Poles
b <- poly(c(1/0.9*exp(1i*pi*0.2), 0.9*exp(1i*pi*0.6)))
a <- poly(c(0.9*exp(-1i*pi*0.6), 1/0.9*exp(-1i*pi*0.2)))
gpd <- grpdelay(b, a, 512, whole = TRUE, Fs = 1)
print(gpd)
plot(gpd)
```

---

ifft

*Inverse FFT*


---

**Description**

Matlab/Octave-compatible inverse FFT.

**Usage**

```
ifft(x)
```

**Arguments**

x                      the input array.

**Details**

It uses `fft` from the `stats` package as follows:

```
fft(x, inverse = TRUE)/length(x)
```

Note that it does not attempt to make the results real.

**Value**

The inverse FFT of the input, the same length as x.

**Author(s)**

Tom Short

**See Also**

[fft](#)

**Examples**

```
ifft(fft(1:4))
```

impz

*Impulse-response characteristics***Description**

Impulse-response characteristics of a discrete filter.

**Usage**

```
## Default S3 method:
impz(filt, a = 1, n = NULL, Fs = 1, ...)

## S3 method for class 'Arma'
impz(filt, ...)

## S3 method for class 'Ma'
impz(filt, ...)

## S3 method for class 'impz'
plot(x, xlab = "Time, msec", ylab = "", type = "l",
     main = "Impulse response", ...)

## S3 method for class 'impz'
print(x, xlab = "Time, msec", ylab = "", type = "l",
     main = "Impulse response", ...)
```

**Arguments**

<code>filt</code>	for the default case, the moving-average coefficients of an ARMA model or filter. Generically, <code>filt</code> specifies an arbitrary model or filter operation.
<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>n</code>	number of points at which to evaluate the frequency response.
<code>Fs</code>	sampling frequency in Hz. If not specified, the frequencies are in per unit.
<code>...</code>	for methods of <code>impz</code> , arguments are passed to the default method. For <code>plot. impz</code> , additional arguments are passed through to <code>plot</code> .
<code>x</code>	object to be plotted.
<code>xlab, ylab, main</code>	axis labels and main title with sensible defaults.
<code>type</code>	as in <a href="#">plot</a> , uses lines to connect the points

**Details**

When results of `impz` are printed, the impulse response will be plotted. As with lattice plots, automatic printing does not work inside loops and function calls, so explicit calls to `print` or `plot` are needed there.

**Value**

For `impz`, a list of class `impz` with items:

<code>x</code>	impulse response signal.
<code>t</code>	time.

**Author(s)**

Original Octave version by Kurt Hornik and John W. Eaton. Conversion to R by Tom Short.

**References**

[https://en.wikipedia.org/wiki/Impulse\\_response](https://en.wikipedia.org/wiki/Impulse_response)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [freqz](#), [zplane](#)

**Examples**

```
bt <- butter(5, 0.3)
impz(bt)
impz(ellip(5, 0.5, 30, 0.3))
```

---

interp	<i>Interpolate / Increase the sample rate</i>
--------	---

---

**Description**

Upsample a signal by a constant factor by using an FIR filter to interpolate between points.

**Usage**

```
interp(x, q, n = 4, Wc = 0.5)
```

**Arguments**

<code>x</code>	the signal to be upsampled.
<code>q</code>	the integer factor to increase the sampling rate by.
<code>n</code>	the FIR filter length.
<code>Wc</code>	the FIR filter cutoff frequency.

**Details**

It uses an order  $2*q*n+1$  FIR filter to interpolate between samples.

**Value**

The upsampled signal, an array of length  $q * \text{length}(x)$ .

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

<https://en.wikipedia.org/wiki/Upsampling>

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[fir1](#), [resample](#), [interp1](#), [decimate](#)

**Examples**

```
# The graph shows interpolated signal following through the
# sample points of the original signal.
t <- seq(0, 2, by = 0.01)
x <- chirp(t, 2, 0.5, 10, 'quadratic') + sin(2*pi*t*0.4)
y <- interp(x[seq(1, length(x), by = 4)], 4, 4, 1) # interpolate a sub-sample
plot(t, x, type = "l")
idx <- seq(1,length(t),by = 4)
lines(t, y[1:length(t)], col = "blue")
points(t[idx], y[idx], col = "blue", pch = 19)
```

---

 interp1

*Interpolation*


---

**Description**

Interpolation methods, including linear, spline, and cubic interpolation.

**Usage**

```
interp1(x, y, xi, method = c("linear", "nearest", "pchip", "cubic", "spline"),
        extrap = NA, ...)
```

**Arguments**

x, y	vectors giving the coordinates of the points to be interpolated. x is assumed to be strictly monotonic.
xi	points at which to interpolate.
method	one of "linear", "nearest", "pchip", "cubic", "spline".

extrap            if TRUE or 'extrap', then extrapolate values beyond the endpoints. If extrap is a number, replace values beyond the endpoints with that number (defaults to NA).

...                for method='spline', additional arguments passed to splinefun.

### Details

The following methods of interpolation are available:

'nearest': return nearest neighbour

'linear': linear interpolation from nearest neighbours

'pchip': piecewise cubic hermite interpolating polynomial

'cubic': cubic interpolation from four nearest neighbours

'spline': cubic spline interpolation—smooth first and second derivatives throughout the curve

### Value

The interpolated signal, an array of length(xi).

### Author(s)

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

### References

Octave Forge <https://octave.sourceforge.io/>

### See Also

[approx](#), [filter](#), [resample](#), [interp](#), [spline](#)

### Examples

```
xf <- seq(0, 11, length=500)
yf <- sin(2*pi*xf/5)
#xp <- c(0:1,3:10)
#yp <- sin(2*pi*xp/5)
xp <- c(0:10)
yp <- sin(2*pi*xp/5)
extrap <- TRUE
lin <- interp1(xp, yp, xf, 'linear', extrap = extrap)
spl <- interp1(xp, yp, xf, 'spline', extrap = extrap)
pch <- interp1(xp, yp, xf, 'pchip', extrap = extrap)
cub <- interp1(xp, yp, xf, 'cubic', extrap = extrap)
near <- interp1(xp, yp, xf, 'nearest', extrap = extrap)
plot(xp, yp, xlim = c(0, 11))
lines(xf, lin, col = "red")
lines(xf, spl, col = "green")
lines(xf, pch, col = "orange")
```

```
lines(xf, cub, col = "blue")
lines(xf, near, col = "purple")
```

---

kaiser

*Kaiser window*

---

## Description

Returns the filter coefficients of the n-point Kaiser window with parameter beta.

## Usage

```
kaiser(n, beta)
```

## Arguments

n	filter order.
beta	bessel shape parameter; larger beta gives narrower windows.

## Value

An array of filter coefficients of length(n).

## Author(s)

Original Octave version by Kurt Hornik. Conversion to R by Tom Short.

## References

Oppenheim, A. V., Schafer, R. W., and Buck, J. R. (1999). *Discrete-time signal processing*. Upper Saddle River, N.J.: Prentice Hall.

[https://en.wikipedia.org/wiki/Kaiser\\_window](https://en.wikipedia.org/wiki/Kaiser_window)

Octave Forge <https://octave.sourceforge.io/>

## See Also

[hamming](#), [kaiserord](#)

## Examples

```
plot(kaiser(101, 2), type = "l", ylim = c(0,1))
lines(kaiser(101, 10), col = "blue")
lines(kaiser(101, 50), col = "green")
```

kaiserord

*Parameters for an FIR filter from a Kaiser window***Description**

Returns the parameters needed for `fir1` to produce a filter of the desired specification from a Kaiser window.

**Usage**

```
kaiserord(f, m, dev, Fs = 2)
```

**Arguments**

f	frequency bands, given as pairs, with the first half of the first pair assumed to start at 0 and the last half of the last pair assumed to end at 1. It is important to separate the band edges, since narrow transition regions require large order filters.
m	magnitude within each band. Should be non-zero for pass band and zero for stop band. All passbands must have the same magnitude, or you will get the error that pass and stop bands must be strictly alternating.
dev	deviation within each band. Since all bands in the resulting filter have the same deviation, only the minimum deviation is used. In this version, a single scalar will work just as well.
Fs	sampling rate. Used to convert the frequency specification into the [0, 1], where 1 corresponds to the Nyquist frequency, $F_s/2$ .

**Value**

An object of class `FilterOfOrder` with the following list elements:

n	filter order
Wc	cutoff frequency
type	filter type, one of "low", "high", "stop", "pass", "DC-0", or "DC-1"
beta	shape parameter

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@users.sf.net>. Conversion to R by Tom Short.

**References**

Oppenheim, A. V., Schafer, R. W., and Buck, J. R. (1999). *Discrete-time signal processing*. Upper Saddle River, N.J.: Prentice Hall.

[https://en.wikipedia.org/wiki/Kaiser\\_window](https://en.wikipedia.org/wiki/Kaiser_window)

Octave Forge <https://octave.sourceforge.io/>



**See Also**[hamming](#), [kaiser](#)**Examples**

```

Fs <- 11025
op <- par(mfrow = c(2, 2), mar = c(3, 3, 1, 1))
for (i in 1:4) {
  switch(i,
    "1" = {
      bands <- c(1200, 1500)
      mag <- c(1, 0)
      dev <- c(0.1, 0.1)
    },
    "2" = {
      bands <- c(1000, 1500)
      mag <- c(0, 1)
      dev <- c(0.1, 0.1)
    },
    "3" = {
      bands <- c(1000, 1200, 3000, 3500)
      mag <- c(0, 1, 0)
      dev <- 0.1
    },
    "4" = {
      bands <- 100 * c(10, 13, 15, 20, 30, 33, 35, 40)
      mag <- c(1, 0, 1, 0, 1)
      dev <- 0.05
    }
  })
}

kaisprm <- kaiserord(bands, mag, dev, Fs)
with(kaisprm, {
  d <- max(1, trunc(n/10))
  if (length(mag)==1 && (d %% 2) == 1)
    d <- d+1
  f1 <- freqz(fir1(n, Wc, type, kaiser(n+1, beta), 'noscale'),
    Fs = Fs)
  f2 <- freqz(fir1(n-d, Wc, type, kaiser(n-d+1, beta), 'noscale'),
    Fs = Fs)
})
plot(f1$f,abs(f1$h), col = "blue", type = "l",
  xlab = "", ylab = "")
lines(f2$f,abs(f2$h), col = "red")
legend("right", paste("order", c(kaisprm$n-d, kaisprm$n)),
  col = c("red", "blue"), lty = 1, bty = "n")
b <- c(0, bands, Fs/2)
for (i in seq(2, length(b), by=2)) {
  hi <- mag[i/2] + dev[1]
  lo <- max(mag[i/2] - dev[1], 0)
  lines(c(b[i-1], b[i], b[i], b[i-1], b[i-1]), c(hi, hi, lo, lo, hi))
}

```

```
par(op)
```

---

levinson

*Durbin-Levinson Recursion*


---

## Description

Perform Durbin-Levinson recursion on a vector or matrix.

## Usage

```
levinson(x, p = NULL)
```

## Arguments

x	Input signal.
p	Lag (defaults to length(x) or nrow(x)).

## Details

Use the Durbin-Levinson algorithm to solve:

```
toeplitz(acf(1:p)) * y = -acf(2:p+1).
```

The solution [1, y'] is the denominator of an all pole filter approximation to the signal x which generated the autocorrelation function acf.

acf is the autocorrelation function for lags 0 to p.

## Value

a	The denominator filter coefficients.
v	Variance of the white noise = square of the numerator constant.
ref	Reflection coefficients = coefficients of the lattice implementation of the filter.

## Author(s)

Original Octave version by Paul Kienzle <pkienzle@users.sf.net> based on yulewalker.m by Friedrich Leisch <Friedrich.Leisch@boku.ac.at>. Conversion to R by Sebastian Krey <krey@statistik.tu-dortmund>.

## References

Steven M. Kay and Stanley Lawrence Marple Jr.: *Spectrum analysis – a modern perspective*, Proceedings of the IEEE, Vol 69, pp 1380-1419, Nov., 1981

Octave <https://octave.sourceforge.io/>

---

Ma	<i>Create a moving average (MA) model</i>
----	---

---

**Description**

Returns a moving average MA model. The model could represent a filter or system model.

**Usage**

Ma(b)

**Arguments**

b                      moving average (MA) polynomial coefficients

**Value**

A list with the MA polynomial coefficients of class Ma.

**Author(s)**

Tom Short, EPRI Solutions, Inc., (<tshort@eprisolutions.com>)

**See Also**

See also [Arma](#)

---

medfilt1	<i>Median filter</i>
----------	----------------------

---

**Description**

Deprecated! Performs an n-point running median. For Matlab/Octave compatibility.

**Usage**

```
medfilt1(x, n = 3, ...)
```

```
MedianFilter(n = 3)
```

```
## S3 method for class 'MedianFilter'
filter(filt, x, ...)
```

**Arguments**

<code>x</code>	signal to be filtered.
<code>n</code>	size of window on which to perform the median.
<code>filt</code>	filter to apply to the signal.
<code>...</code>	additional arguments passed to <code>runmed</code> .

**Details**

`medfilt1` is a wrapper for `runmed`.

**Value**

For `medfilt1`, the filtered signal of `length(x)`.

For `MedianFilter`, a class of “`MedianFilter`” that can be used with `filter` to apply a median filter to a signal.

**Author(s)**

Tom Short.

**References**

[https://en.wikipedia.org/wiki/Median\\_filter](https://en.wikipedia.org/wiki/Median_filter)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[runmed](#), [median](#), [filter](#)

**Examples**

```
t <- seq(0, 1, len=100) # 1 second sample
x <- sin(2*pi*t*2.3) + 0.25*rlnorm(length(t), 0.5) # 2.3 Hz sinusoid+noise
plot(t, x, type = "l")
# 3-point filter
lines(t, medfilt1(x), col="red", lwd=2)
# 7-point filter
lines(t, filter(MedianFilter(7), x), col = "blue", lwd=2) # another way to call it
```

pchip

*Piecewise cubic hermite interpolation***Description**

Piecewise cubic hermite interpolation.

**Usage**

```
pchip(x, y, xi = NULL)
```

**Arguments**

x, y	vectors giving the coordinates of the points to be interpolated. x must be strictly monotonic (either increasing or decreasing).
xi	points at which to interpolate.

**Details**

In contrast to `spline`, `pchip` preserves the monotonicity of x and y.

**Value**

Normally, the interpolated signal, an array of `length(xi)`.

if `xi == NULL`, a list of class `pp`, a piecewise polynomial representation with the following elements:

x	breaks between intervals.
P	a matrix with n times d rows and k columns. The ith row of P, <code>P[i, ]</code> , contains the coefficients for the polynomial over the ith interval, ordered from highest to lowest. There must be one row for each interval in x.
n	number of intervals ( <code>length(x) - 1</code> ).
k	polynomial order.
d	number of polynomials.

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

Fritsch, F. N. and Carlson, R. E., "Monotone Piecewise Cubic Interpolation", *SIAM Journal on Numerical Analysis*, vol. 17, pp. 238-246, 1980.

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[approx](#), [spline](#), [interp1](#)

**Examples**

```
xf <- seq(0, 11, length=500)
yf <- sin(2*pi*xf/5)
xp <- c(0:10)
yp <- sin(2*pi*xp/5)
pch <- pchip(xp, yp, xf)
plot(xp, yp, xlim = c(0, 11))
lines(xf, pch, col = "orange")
```

---

poly

*Polynomial given roots*

---

**Description**

Coefficients of a polynomial when roots are given or the characteristic polynomial of a matrix.

**Usage**

`poly(x)`

**Arguments**

`x` a vector or matrix. For a vector, it specifies the roots of the polynomial. For a matrix, the characteristic polynomial is found.

**Value**

An array of the coefficients of the polynomial in order from highest to lowest polynomial power.

**Author(s)**

Original Octave version by Kurt Hornik. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[polyval](#), [roots](#), [conv](#)

**Examples**

```
poly(c(1, -1))
poly(roots(1:3))
poly(matrix(1:9, 3, 3))
```

---

polyval	<i>Evaluate a polynomial</i>
---------	------------------------------

---

**Description**

Evaluate a polynomial at given points.

**Usage**

```
polyval(coef, z)
```

**Arguments**

coef	coefficients of the polynomial, defined in decreasing power.
z	the points at which to evaluate the polynomial.

**Value**

An array of `length(z)`, the polynomial evaluated at each element of `z`.

**Author(s)**

Tom Short

**See Also**

[poly](#), [roots](#)

**Examples**

```
polyval(c(1, 0, -2), 1:3) # s^2 - 2
```

---

remez	<i>Parks-McClellan optimal FIR filter design</i>
-------	--

---

**Description**

Parks-McClellan optimal FIR filter design.

**Usage**

```
remez(n, f, a, w = rep(1.0, length(f) / 2),  
      ftype = c('bandpass', 'differentiator', 'hilbert'),  
      density = 16)
```

**Arguments**

n	order of the filter (1 less than the length of the filter)
f	frequency at the band edges in the range (0, 1), with 1 being the Nyquist frequency.
a	amplitude at the band edges.
w	weighting applied to each band.
ftype	options are: 'bandpass', 'differentiator', and 'hilbert'.
density	determines how accurately the filter will be constructed. The minimum value is 16, but higher numbers are slower to compute.

**Value**

The FIR filter coefficients, an array of length(n+1), of class Ma.

**Author(s)**

Original Octave version by Paul Kienzle. Conversion to R by Tom Short. It uses C routines developed by Jake Janovetz.

**References**

Rabiner, L. R., McClellan, J. H., and Parks, T. W., "FIR Digital Filter Design Techniques Using Weighted Chebyshev Approximations", IEEE Proceedings, vol. 63, pp. 595 - 610, 1975.

[https://en.wikipedia.org/wiki/Fir\\_filter](https://en.wikipedia.org/wiki/Fir_filter)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [Ma](#), [fftfilt](#), [fir1](#)

**Examples**

```
f1 <- remez(15, c(0, 0.3, 0.4, 1), c(1, 1, 0, 0))
freqz(f1)
```

---

resample

---

*Change the sampling rate of a signal*


---

**Description**

Resample using bandlimited interpolation.

**Usage**

```
resample(x, p, q = 1, d = 5)
```



**Arguments**

<code>x</code>	signal to be resampled.
<code>p, q</code>	$p/q$ specifies the factor to resample by.
<code>d</code>	distance.

**Details**

Note that `p` and `q` do not need to be integers since this routine does not use a polyphase rate change algorithm, but instead uses bandlimited interpolation, wherein the continuous time signal is estimated by summing the sinc functions of the nearest neighbouring points up to distance `d`.

Note that `resample` computes all samples up to but not including time `n+1`. If you are increasing the sample rate, this means that it will generate samples beyond the end of the time range of the original signal. That is why `xf` must go all the way to 10.95 in the example below.

Nowadays, the signal version in Matlab and Octave contain more modern code for `resample` that has not been ported to the signal R package (yet).

**Value**

The resampled signal, an array of length `ceiling(length(x) * p / q)`.

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@user.sf.net>. Conversion to R by Tom Short.

**References**

J. O. Smith and P. Gossett (1984). A flexible sampling-rate conversion method. In ICASSP-84, Volume II, pp. 19.4.1-19.4.2. New York: IEEE Press.

[doi:10.1109/ICASSP.1984.1172555](https://doi.org/10.1109/ICASSP.1984.1172555)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [decimate](#), [interp](#)

**Examples**

```
xf <- seq(0, 10.95, by=0.05)
yf <- sin(2*pi*xf/5)
xp <- 0:10
yp <- sin(2*pi*xp/5)
r <- resample(yp, xp[2], xf[2])
title("confirm that the resampled function matches the original")
plot(xf, yf, type = "l", col = "blue")
lines(xf, r[1:length(xf)], col = "red")
points(xp,yp, pch = 19, col = "blue")
legend("bottomleft", c("Original", "Resample", "Data"),
```

```
col = c("blue", "red", "blue"),
pch = c(NA, NA, 19),
lty = c(1, 1, NA), bty = "n")
```

---

roots	<i>Roots of a polynomial</i>
-------	------------------------------

---

## Description

Roots of a polynomial

## Usage

```
roots(x, method = c("polyroot", "eigen"))
```

## Arguments

x	Polynomial coefficients with coefficients given in order from highest to lowest polynomial power. This is the Matlab/Octave convention; it is opposite of the convention used by <code>polyroot</code> .
method	Either “polyroot” (default) which uses <code>polyroot</code> for its computations internally (and is typically more accurate) or “eigen” which uses eigenvalues of the companion matrix for its computation. The latter returns complex values in case of real valued solutions in less cases.

## Value

A complex array with the roots of the polynomial.

## Author(s)

Original Octave version by Kurt Hornik. Conversion to R by Tom Short.

## References

Octave Forge <https://octave.sourceforge.io/>

## See Also

`polyroot`, `polyval`, `poly`, `conv`

## Examples

```
roots(1:3)
polyroot(3:1) # should be the same
poly(roots(1:3))

roots(1:3, method="eigen") # using eigenvalues
```

---

sftrans	<i>Transform filter band edges</i>
---------	------------------------------------

---

**Description**

Transform band edges of a generic lowpass filter to a filter with different band edges and to other filter types (high pass, band pass, or band stop).

**Usage**

```
## Default S3 method:
sftrans(Sz, Sp, Sg, W, stop = FALSE, ...)

## S3 method for class 'Arma'
sftrans(Sz, W, stop = FALSE, ...)

## S3 method for class 'Zpg'
sftrans(Sz, W, stop = FALSE, ...)
```

**Arguments**

Sz	In the generic case, a model to be transformed. In the default case, a vector containing the zeros in a pole-zero-gain model.
Sp	a vector containing the poles in a pole-zero-gain model.
Sg	a vector containing the gain in a pole-zero-gain model.
W	critical frequencies of the target filter specified in radians. W must be a scalar for low-pass and high-pass filters, and W must be a two-element vector c(low, high) specifying the lower and upper bands.
stop	FALSE for a low-pass or band-pass filter, TRUE for a high-pass or band-stop filter.
...	additional arguments (ignored).

**Details**

Given a low pass filter represented by poles and zeros in the splane, you can convert it to a low pass, high pass, band pass or band stop by transforming each of the poles and zeros individually. The following summarizes the transformations:

Low-Pass Transform

$$S \rightarrow CS/F_c$$

Zero at x	Pole at x
zero: $F_c x / C$	$F_c x / C$
gain: $C / F_c$	$F_c / C$

## High-Pass Transform

$$S \rightarrow CF_c/S$$

Zero at x	Pole at x
zero: $F_c C/x$	$F_c C/x$
pole: 0	0
gain: $-x$	$-1/x$

## Band-Pass Transform

$$S \rightarrow C \frac{S^2 + F_h F_l}{S(F_h - F_l)}$$

Zero at x	Pole at x
zero: $b \pm \sqrt{(b^2 - F_h F_l)}$	$b \pm \sqrt{(b^2 - F_h F_l)}$
pole: 0	0
gain: $C/(F_h - F_l)$	$(F_h - F_l)/C$
$b = x/C(F_h - F_l)/2$	$b = x/C(F_h - F_l)/2$

## Band-Stop Transform

$$S \rightarrow C \frac{S(F_h - F_l)}{S^2 + F_h F_l}$$

Zero at x	Pole at x
zero: $b \pm \sqrt{(b^2 - F_h F_l)}$	$b \pm \sqrt{(b^2 - F_h F_l)}$
pole: $\pm \sqrt{(-F_h F_l)}$	$\pm \sqrt{(-F_h F_l)}$
gain: $-x$	$-1/x$
$b = C/x(F_h - F_l)/2$	$b = C/x(F_h - F_l)/2$

## Bilinear Transform

$$S \rightarrow \frac{2}{T} \frac{z - 1}{z + 1}$$

Zero at x	Pole at x
zero: $(2 + xT)/(2 - xT)$	$(2 + xT)/(2 - xT)$
pole: $-1$	$-1$
gain: $(2 - xT)/T$	$(2 - xT)/T$

where  $C$  is the cutoff frequency of the initial lowpass filter,  $F_c$  is the edge of the target low/high pass filter and  $[F_l, F_h]$  are the edges of the target band pass/stop filter. With abundant tedious algebra, you can derive the above formulae yourself by substituting the transform for  $S$  into  $H(S) = S - x$  for a zero at  $x$  or  $H(S) = 1/(S - x)$  for a pole at  $x$ , and converting the result into the form:

$$H(S) = g \text{prod}(S - Xi) / \text{prod}(S - Xj)$$

Please note that a pole and a zero at the same place exactly cancel. This is significant for High Pass, Band Pass and Band Stop filters which create numerous extra poles and zeros, most of which cancel. Those which do not cancel have a ‘fill-in’ effect, extending the shorter of the sets to have the same number of as the longer of the sets of poles and zeros (or at least split the difference in the case of the band pass filter). There may be other opportunistic cancellations, but it does not check for them.

Also note that any pole on the unit circle or beyond will result in an unstable filter. Because of cancellation, this will only happen if the number of poles is smaller than the number of zeros and the filter is high pass or band pass. The analytic design methods all yield more poles than zeros, so this will not be a problem.

### Value

For the default case or for `sftrans.Zpg`, an object of class “Zpg”, containing the list elements:

zero	complex vector of the zeros of the transformed model
pole	complex vector of the poles of the transformed model
gain	gain of the transformed model

For `sftrans.Arma`, an object of class “Arma”, containing the list elements:

b	moving average (MA) polynomial coefficients
a	autoregressive (AR) polynomial coefficients

### Author(s)

Original Octave version by Paul Kienzle <pkienzle@users.sf.net>. Conversion to R by Tom Short.

### References

Proakis & Manolakis (1992). *Digital Signal Processing*. New York: Macmillan Publishing Company.

Octave Forge <https://octave.sourceforge.io/>

### See Also

[Zpg](#), [bilinear](#), [Arma](#)

---

sgolay

Savitzky-Golay smoothing filters

---

## Description

Computes the filter coefficients for all Savitzky-Golay smoothing filters.

## Usage

```
sgolay(p, n, m = 0, ts = 1)
```

## Arguments

p	filter order.
n	filter length (must be odd).
m	return the m-th derivative of the filter coefficients.
ts	time scaling factor.

## Details

The early rows of the result  $F$  smooth based on future values and later rows smooth based on past values, with the middle row using half future and half past. In particular, you can use row  $i$  to estimate  $x[k]$  based on the  $i-1$  preceding values and the  $n-i$  following values of  $x$  values as  $y[k] = F[i, :] * x[(k-i+1):(k+n-i)]$ .

Normally, you would apply the first  $(n-1)/2$  rows to the first  $k$  points of the vector, the last  $k$  rows to the last  $k$  points of the vector and middle row to the remainder, but for example if you were running on a realtime system where you wanted to smooth based on the all the data collected up to the current time, with a lag of five samples, you could apply just the filter on row  $n-5$  to your window of length  $n$  each time you added a new sample.

## Value

An square matrix with dimensions  $\text{length}(n)$  that is of class 'sgolayFilter' (so it can be used with `filter`).

## Author(s)

Original Octave version by Paul Kienzle <pkienzle@users.sf.net>. Modified by Pascal Dupuis. Conversion to R by Tom Short.

## References

William H. Press, Saul A. Teukolsky, William T. Vetterling, Brian P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, 2nd edition, Cambridge Univ. Press, N.Y., 1992.

Octave Forge <https://octave.sourceforge.io/>

**See Also**[sgolayfilt](#), [filter](#)

---

sgolayfilt*Apply a Savitzky-Golay smoothing filter*

---

**Description**

Smooth data with a Savitzky-Golay smoothing filter.

**Usage**

```
sgolayfilt(x, p = 3, n = p + 3 - p%%2, m = 0, ts = 1)
```

```
## S3 method for class 'sgolayFilter'  
filter(filt, x, ...)
```

**Arguments**

x	signal to be filtered.
p	filter order.
n	filter length (must be odd).
m	return the m-th derivative of the filter coefficients.
ts	time scaling factor.
filt	filter characteristics (normally generated by <code>sgolay</code> ).
...	additional arguments (ignored).

**Details**

These filters are particularly good at preserving lineshape while removing high frequency squiggles.

**Value**

The filtered signal, of `length(x)`.

**Author(s)**

Original Octave version by Paul Kienzle <pkienzle@users.sf.net>. Modified by Pascal Dupuis.  
Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**[sgolay](#), [filter](#)**Examples**

```
# Compare a 5 sample averager, an order-5 butterworth lowpass
# filter (cutoff 1/3) and sgolayfilt(x, 3, 5), the best cubic
# estimated from 5 points.
bf <- butter(5,1/3)
x <- c(rep(0,15), rep(10, 10), rep(0, 15))
sg <- sgolayfilt(x)
plot(sg, type="l")
lines(filtfilt(rep(1, 5)/5,1,x), col = "red") # averaging filter
lines(filtfilt(bf,x), col = "blue")          # butterworth
points(x, pch = "x")                        # original data
```

---

 signal-internal

---

*Internal or uncommented functions*


---

**Description**

Internal or barely commented functions not exported from the Namespace.

**Details**

```
# MOSTLY MATLAB/OCTAVE COMPATIBLE UTILITIES
fractdiff(x, d) # Fractional differences
postpad(x, n)  # pad \code{x} with zeros at the end for a total length \code{n}
                # truncates if length(x) < n
sinc(x)        # sin(pi * x) / (pi * x)

# MATLAB-INCOMPATIBLE UTILITIES
logseq(from, to, n = 500) # like \code{linspace} but equally spaced logarithmically

# MAINLY INTERNAL, BUT MATLAB COMPATIBLE
mkpp(x, P, d = round(NROW(P)/pp$n)) # used by \code{pchip}
## Construct a piece-wise polynomial structure from sample points x and
## coefficients P.
ppval(pp, xi) # used by \code{pchip}
## Evaluate piece-wise polynomial pp and points xi.
ncauer(Rp, Rs, n) # used by \code{ellip}
ellipke(m, Nmax)  # used by \code{ellip}
cheb(n, x) # nth-order Chebyshev polynomial calculated at x
            # used by \code{chebwin}
```



specgram

*Spectrogram plot***Description**

Generate a spectrogram for the signal. This chops the signal into overlapping slices, windows each slice and applies a Fourier transform to determine the frequency components at that slice.

**Usage**

```
specgram(x, n = min(256, length(x)), Fs = 2, window = hanning(n),
        overlap = ceiling(length(window)/2))

## S3 method for class 'specgram'
plot(x, col = gray(0:512 / 512), xlab="time", ylab="frequency", ...)

## S3 method for class 'specgram'
print(x, col = gray(0:512 / 512), xlab="time", ylab="frequency", ...)
```

**Arguments**

x	the vector of samples.
n	the size of the Fourier transform window.
Fs	the sample rate, Hz.
window	shape of the fourier transform window, defaults to hanning(n). The window length for a hanning window can be specified instead.
overlap	overlap with previous window, defaults to half the window length.
col	color scale used for the underlying <a href="#">image</a> function.
xlab, ylab	axis labels with sensible defaults.
...	additional arguments passed to the underlying plot functions.

**Details**

When results of `specgram` are printed, a spectrogram will be plotted. As with `lattice` plots, automatic printing does not work inside loops and function calls, so explicit calls to `print` or `plot` are needed there.

The choice of window defines the time-frequency resolution. In speech for example, a wide window shows more harmonic detail while a narrow window averages over the harmonic detail and shows more formant structure. The shape of the window is not so critical so long as it goes gradually to zero on the ends.

Step size (which is window length minus overlap) controls the horizontal scale of the spectrogram. Decrease it to stretch, or increase it to compress. Increasing step size will reduce time resolution, but decreasing it will not improve it much beyond the limits imposed by the window size (you do gain a little bit, depending on the shape of your window, as the peak of the window slides over peaks in the signal energy). The range 1-5 msec is good for speech.

FFT length controls the vertical scale. Selecting an FFT length greater than the window length does not add any information to the spectrum, but it is a good way to interpolate between frequency points which can make for prettier spectrograms.

After you have generated the spectral slices, there are a number of decisions for displaying them. First the phase information is discarded and the energy normalized:

```
S = abs(S); S = S/max(S)
```

Then the dynamic range of the signal is chosen. Since information in speech is well above the noise floor, it makes sense to eliminate any dynamic range at the bottom end. This is done by taking the max of the magnitude and some minimum energy such as  $\text{minE}=-40\text{dB}$ . Similarly, there is not much information in the very top of the range, so clipping to a maximum energy such as  $\text{maxE}=-3\text{dB}$  makes sense:

```
S = max(S, 10^(minE/10)); S = min(S, 10^(maxE/10))
```

The frequency range of the FFT is from 0 to the Nyquist frequency of one half the sampling rate. If the signal of interest is band limited, you do not need to display the entire frequency range. In speech for example, most of the signal is below 4 kHz, so there is no reason to display up to the Nyquist frequency of 10 kHz for a 20 kHz sampling rate. In this case you will want to keep only the first 40% of the rows of the returned *S* and *f*. More generally, to display the frequency range  $[\text{minF}, \text{maxF}]$ , you could use the following row index:

```
idx = (f >= minF & f <= maxF)
```

Then there is the choice of colormap. A brightness varying colormap such as copper or bone gives good shape to the ridges and valleys. A hue varying colormap such as jet or hsv gives an indication of the steepness of the slopes. The final spectrogram is displayed in log energy scale and by convention has low frequencies on the bottom of the image.

## Value

For specgram list of class specgram with items:

<i>S</i>	complex output of the FFT, one row per slice.
<i>f</i>	the frequency indices corresponding to the rows of <i>S</i> .
<i>t</i>	the time indices corresponding to the columns of <i>S</i> .

## Author(s)

Original Octave version by Paul Kienzle <pkienzle@users.sf.net>. Conversion to R by Tom Short.

## References

Octave Forge <https://octave.sourceforge.io/>

## See Also

[fft](#), [image](#)

**Examples**

```

specgram(chirp(seq(-2, 15, by = 0.001), 400, 10, 100, 'quadratic'))
specgram(chirp(seq(0, 5, by = 1/8000), 200, 2, 500, "logarithmic"), Fs = 8000)

data(wav) # contains wav$rate, wav$sound
Fs <- wav$rate
step <- trunc(5*Fs/1000)           # one spectral slice every 5 ms
window <- trunc(40*Fs/1000)       # 40 ms data window
fftn <- 2^ceiling(log2(abs(window))) # next highest power of 2
spg <- specgram(wav$sound, fftn, Fs, window, window-step)
S <- abs(spg$S[2:(fftn*4000/Fs),]) # magnitude in range 0<f<=4000 Hz.
S <- S/max(S)                     # normalize magnitude so that max is 0 dB.
S[S < 10^(-40/10)] <- 10^(-40/10) # clip below -40 dB.
S[S > 10^(-3/10)] <- 10^(-3/10)  # clip above -3 dB.
image(t(20*log10(S)), axes = FALSE) #, col = gray(0:255 / 255))

```

spencer

*Spencer filter***Description**

Spencer's 15-point moving average filter.

**Usage**

```

spencer(x)

spencerFilter()

```

**Arguments**

x                      signal to be filtered.

**Value**

For `spencer`, the filtered signal. For `spencerFilter`, a vector of filter coefficients with class `Ma` that can be passed to `filter`.

**Author(s)**

Original Octave version by Friedrich Leisch. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**See Also**

[filter](#), [Ma](#)

---

unwrap	<i>Unwrap radian phases</i>
--------	-----------------------------

---

**Description**

Unwrap radian phases by adding multiples of  $2\pi$  as appropriate to remove jumps.

**Usage**

```
unwrap(a, tol = pi, dim = 1)
```

**Arguments**

a	vector of phase angles in radians.
tol	tolerance for removing phase jumps.
dim	dimension with which to apply the phase unwrapping.

**Value**

A vector with the unwrapped phase angles.

**Author(s)**

Original Octave version by Bill Lash. Conversion to R by Tom Short.

**References**

Octave Forge <https://octave.sourceforge.io/>

**Examples**

```
phase <- c(seq(0, 2*pi, length=500), seq(0, 2*pi, length=500))
plot(phase, type = "l", ylim = c(0, 4*pi))
lines(unwrap(phase), col = "blue")
```

---

wav	<i>Example wav file</i>
-----	-------------------------

---

**Description**

Example wav file audio waveshape from Octave.

**Usage**

```
data(wav)
```

**Format**

The format is: List of 3 \$ sound: num [1, 1:17380] -0.000275 -0.00061 -0.000397 -0.000793 -0.000305 ... \$ rate : int 22050 \$ bits : int 16 - attr(\*, "class")= chr "Sample"

**Details**

Sound samples are in Element “sound” while “rate” is the sampling rate (in Hz) and “bits” the resolution of the underlying Wave file.

**Source**

Octave

**Examples**

```
data(wav)
str(wav)
```

---

Windowing functions      *Windowing functions*

---

**Description**

A variety of generally Matlab/Octave compatible filter generation functions, including Bartlett, Blackman, Hamming, Hanning, and triangular.

**Usage**

```
bartlett(n)
blackman(n)
boxcar(n)
flattopwin(n, sym = c('symmetric', 'periodic'))
gausswin(n, w = 2.5)
hamming(n)
hanning(n)
triang(n)
```

**Arguments**

n	length of the filter; number of coefficients to generate.
w	the reciprocal of the standard deviation for gausswin. Use larger a for a narrower window.
sym	'symmetric' for a symmetric window, 'periodic' for a periodic window.

**Details**

`triang`, unlike the `bartlett` window, does not go to zero at the edges of the window. For odd `n`, `triang(n)` is equal to `bartlett(n+2)` except for the zeros at the edges of the window.

A main use of `flattopwin` is for calibration, due to its negligible amplitude errors. This window has low pass-band ripple, but high bandwidth.

**Value**

Filter coefficients.

**Author(s)**

Original Octave versions by Paul Kienzle (`boxcar`, `gausswin`, `triang`) and Andreas Weingessel (`bartlett`, `blackman`, `hamming`, `hanning`). Conversion to R by Tom Short.

**References**

Oppenheim, A.V., and Schafer, R.W., *Discrete-Time Signal Processing*, Upper Saddle River, NJ: Prentice-Hall, 1999.

Gade, S., Herlufsen, H. (1987) "Use of weighting functions in DFT/FFT analysis (Part I)", *Bruel & Kjaer Technical Review* No. 3.

[https://en.wikipedia.org/wiki/Windowed\\_frame](https://en.wikipedia.org/wiki/Windowed_frame)

Octave Forge <https://octave.sourceforge.io/>

**See Also**

`filter`, `fftfilt`, `filtfilt`, `fir1`

**Examples**

```
n <- 51
op <- par(mfrow = c(3,3))
plot(bartlett(n), type = "l", ylim = c(0,1))
plot(blackman(n), type = "l", ylim = c(0,1))
plot(boxcar(n), type = "l", ylim = c(0,1))
plot(flattopwin(n), type = "l", ylim = c(0,1))
plot(gausswin(n, 5), type = "l", ylim = c(0,1))
plot(hanning(n), type = "l", ylim = c(0,1))
plot(hamming(n), type = "l", ylim = c(0,1))
plot(triang(n), type = "l", ylim = c(0,1))
par(op)
```

---

Zpg	<i>Zero-pole-gain model</i>
-----	-----------------------------

---

**Description**

Zero-pole-gain model of an ARMA filter.

**Usage**

```
Zpg(zero, pole, gain)

## S3 method for class 'Arma'
as.Zpg(x, ...)

## S3 method for class 'Ma'
as.Zpg(x, ...)

## S3 method for class 'Zpg'
as.Zpg(x, ...)
```

**Arguments**

zero	complex vector of the zeros of the model.
pole	complex vector of the poles of the model.
gain	gain of the model.
x	model to be converted.
...	additional arguments (ignored).

**Details**

as.Zpg converts from other forms, including Arma and Ma.

**Value**

An object of class “Zpg”, containing the list elements:

zero	complex vector of the zeros of the model.
pole	complex vector of the poles of the model.
gain	gain of the model.

**Author(s)**

Tom Short

**See Also**

[Arma](#), [bilinear](#)

zplane

*Pole-zero plot*

---

**Description**

Plot the poles and zeros of a model or filter.

**Usage**

```
## Default S3 method:
zplane(filt, a, ...)

## S3 method for class 'Arma'
zplane(filt, ...)

## S3 method for class 'Ma'
zplane(filt, ...)

## S3 method for class 'Zpg'
zplane(filt, ...)
```

**Arguments**

<code>filt</code>	for the default case, the moving-average coefficients of an ARMA model or filter. Generically, <code>filt</code> specifies an arbitrary model or filter operation.
<code>a</code>	the autoregressive (recursive) coefficients of an ARMA filter.
<code>...</code>	Additional arguments passed to <code>plot</code> .

**Details**

Poles are marked with an 'x', and zeros are marked with an 'o'.

**Value**

No value is returned.

**Author(s)**

Tom Short

**References**

Octave Forge <https://octave.sourceforge.io/>  
[https://en.wikipedia.org/wiki/Pole-zero\\_plot](https://en.wikipedia.org/wiki/Pole-zero_plot)

**See Also**

[Arma](#), [freqz](#)



**Examples**

```
filt <- ellip(5, 0.5, 20, .2)
zplane(filt)
```

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