

Logarithmic Chebyshev Approximation

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For a $p \times n$ ($p > n$) matrix \mathbf{B} and $p \times 1$ vector \mathbf{f} , the Logarithmic Chebyshev Approximation problem is stated as the following optimization problem ([1])

$$\begin{aligned} & \underset{\mathbf{x}, t}{\text{minimize}} && t \\ & \text{subject to} && 1/t \leq (\mathbf{x}^\top \mathbf{B}_i) / \mathbf{f}_i \leq t, \quad i = 1, \dots, p \end{aligned}$$

where \mathbf{B}_i denotes the i^{th} row of the matrix \mathbf{B} . Note that we require each element of $\mathbf{B}_i / \mathbf{f}$ to be greater than or equal to 0 for all j .

The function `logcheby` takes as input a matrix \mathbf{B} and vector \mathbf{f} , and returns the solution to the Logarithmic Chebyshev Approximation problem using `sqp1p`.

```
R> out <- logcheby(B,f)
```

Numerical Example

As a numerical example, consider the following

```
R> data(Blogcheby)
```

	V1	V2	V3	V4	V5
[1,]	9.148	9.040	3.796	6.756	5.816
[2,]	9.371	1.387	4.358	9.828	1.579
[3,]	2.861	9.889	0.374	7.595	3.590
[4,]	8.304	9.467	9.735	5.665	6.456
[5,]	6.417	0.824	4.318	8.497	7.758
[6,]	5.191	5.142	9.576	1.895	5.636
[7,]	7.366	3.902	8.878	2.713	2.337
[8,]	1.347	9.057	6.400	8.282	0.900
[9,]	6.570	4.470	9.710	6.932	0.856
[10,]	7.051	8.360	6.188	2.405	3.052
[11,]	4.577	7.376	3.334	0.430	6.674
[12,]	7.191	8.111	3.467	1.405	0.002
[13,]	9.347	3.881	3.985	2.164	2.086
[14,]	2.554	6.852	7.847	4.794	9.330
[15,]	4.623	0.039	0.389	1.974	9.256
[16,]	9.400	8.329	7.488	7.194	7.341
[17,]	9.782	0.073	6.773	0.079	3.331
[18,]	1.175	2.077	1.713	3.755	5.151

```
[19,] 4.750 9.066 2.611 5.144 7.440
[20,] 5.603 6.118 5.144 0.016 6.192
```

```
R> data(flogcheby)
```

```
      V1
[1,] 0.626
[2,] 0.217
[3,] 0.217
[4,] 0.389
[5,] 0.942
[6,] 0.963
[7,] 0.740
[8,] 0.733
[9,] 0.536
[10,] 0.002
[11,] 0.609
[12,] 0.837
[13,] 0.752
[14,] 0.453
[15,] 0.536
[16,] 0.537
[17,] 0.001
[18,] 0.356
[19,] 0.612
[20,] 0.829
```

Note that it must be the case that each element of $\mathbf{B}_{.j}/\mathbf{f}$ must be greater than or equal to 0 for every column of \mathbf{B} .

```
R> out <- logcheby(Blogcheby, flogcheby)
```

Here, the outputs of interest are the optimal value of the objective function (which again we need to negate due to the negation of the objective function), and the vector \mathbf{X} , which is stored in the output variable `y`.

```
R> -out$pobj
```

```
[1] 23.08812
```

```
R> m <- ncol(Blogcheby)
```

```
R> x <- out$y[1:m]
```

```
      [,1]
[1,] 0.001106650
[2,] 0.002661286
[3,] 0.001050662
[4,] 0.002180275
[5,] 0.001435069
```

References

- [1] Lieven Vandenberghe, Stephen Boyd, and Shao-Po Wu. Determinant maximization with linear matrix inequality constraints. *SIAM journal on matrix analysis and applications*, 19(2):499–533, 1998.