

# Package: resultant (via r-universe)

September 30, 2024

**Title** Utilities for Multivariate Polynomials with Rational Coefficients

**Version** 1.0.0

**Description** Computation of resultant, subresultants, greatest common divisor, integral division (aka division without remainder) of two multivariate polynomials with rational coefficients, Sturm-Habicht sequence and square-free factorization of a multivariate polynomial with rational coefficients. The computations are performed by the 'C++' library 'CGAL' (<<https://www.cgal.org/>>). Resultants have applications in polynomial systems solving, number theory, and algebraic geometry. The package also contains some functions computing the number of real roots of a univariate polynomial with rational coefficients, and a function computing the division with remainder of two univariate polynomials with rational coefficients.

**License** GPL-3

**URL** <https://github.com/stla/resultant>

**BugReports** <https://github.com/stla/resultant/issues>

**Depends** qspray (>= 3.1.0)

**Imports** Rcpp, gmp, utils

**Suggests** testthat (>= 3.0.0)

**LinkingTo** BH, Rcpp, RcppCGAL

**Config/testthat/edition** 3

**Encoding** UTF-8

**RoxygenNote** 7.3.1

**SystemRequirements** C++17, gmp, mpfr

**NeedsCompilation** yes

**Author** Stéphane Laurent [aut, cre]

**Maintainer** Stéphane Laurent <laurent\_step@outlook.fr>

**Repository** CRAN

**Date/Publication** 2024-07-31 10:50:02 UTC

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gcd	<i>Greatest common divisor of two polynomials</i>
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### Description

Greatest common divisor of two polynomials with rational coefficients.

### Usage

```
gcd(qspray1, qspray2, utcf = FALSE)
```

### Arguments

qspray1, qspray2	two qspray polynomials with at more nine variables
utcf	Boolean, whether to get the greatest common divisor up to a constant factor (this can be faster)

### Value

A qspray polynomial.

### Examples

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
g <- x^2 + 2*x*y + 1
p <- g * (y^2 + x^2)
q <- g * (y + x^3 + 2)
gcd(p, q)
```

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integralDivision	<i>Integral division of two polynomials</i>
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**Description**

Integral division (division without remainder) of two polynomials with rational coefficients.

**Usage**

```
integralDivision(qspray1, qspray2, check = TRUE)
```

**Arguments**

qspray1, qspray2	two qspray polynomials having at most nine variables
check	Boolean, whether to check that qspray2 divides qspray1

**Value**

If check=TRUE, this returns NULL if qspray2 does not divide qspray1, otherwise this returns a qspray polynomial, the quotient of qspray1 by qspray2. If check=FALSE, this always returns a qspray polynomial, which is the quotient of qspray1 by qspray2 if qspray2 divides qspray1, otherwise it is an undefined polynomial. So you can use check=FALSE only when you are sure that qspray2 divides qspray1.

**See Also**

[univariateDivision](#), [qsprayDivision](#).

**Examples**

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
q <- x^2 + 2*x*y + 1
qspray1 <- q * (x^4 + y^2 + 2)
qspray2 <- x^4 + y^2 + 2
integralDivision(qspray1, qspray2) == q # should be TRUE
```

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numberOfRealRoots	<i>Number of real roots</i>
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**Description**

Number of distinct real roots of a univariate polynomial.

**Usage**

```
numberOfRealRoots(qspray)
```

**Arguments**

qspray            a univariate qspray polynomial

**Value**

An integer, the number of real roots of the polynomial.

**Note**

The roots are not counted with their multiplicity.

**Examples**

```
library(resultant)
x <- qlone(1)
P <- 2*x^4 + x^3 - 3*x^2 - x + 1
numberOfRealRoots(P)
```

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numberOfRealRootsInInterval	<i>Number of real roots in an interval</i>
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**Description**

Number of distinct real roots of a univariate polynomial in a given interval.

**Usage**

```
numberOfRealRootsInInterval(qspray, lower, upper, closed = TRUE)
```

**Arguments**

qspray            a univariate qspray polynomial  
lower, upper      the bounds of the interval, bigq numbers or objects coercible to bigq numbers,  
and it is also possible to set lower = -Inf and upper = Inf  
closed            Boolean, whether to consider the interval is closed or open

**Value**

An integer, the number of real roots of the polynomial in the interval.

**Note**

The roots are not counted with their multiplicity.

**Examples**

```
library(resultant)
x <- qlone(1)
P <- 2*x^4 + x^3 - 3*x^2 - x + 1
numberOfRealRootsInInterval(P, 0, 1)
```

---

principalSturmHabicht *Principal Sturm-Habicht sequence of a polynomial*

---

**Description**

Principal Sturm-Habicht sequence of a polynomial with rational coefficients.

**Usage**

```
principalSturmHabicht(qspray, var = 1)
```

**Arguments**

qspray	a qspray polynomial with at most nine variables
var	integer indicating with respect to which variable the resultant is desired (e.g. 1 for x and 2 for y)

**Value**

For a univariate polynomial, this returns a vector of bigq rational numbers. For a multivariate polynomial, this returns a list of qspray polynomials that do not involve the var-th variable.

**Examples**

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
qspray <- x^3*y^2 - 5*x*y^2 + 7*x - 2
principalSturmHabicht(qspray, var = 1)
principalSturmHabicht(qspray, var = 2)
```

---

 principalSubresultants

*Principal subresultants of two polynomials*


---

### Description

Principal subresultants of two polynomials with rational coefficients.

### Usage

```
principalSubresultants(qspray1, qspray2, var = 1)
```

### Arguments

qspray1, qspray2

two qspray polynomials with at most nine variables

var

integer indicating with respect to which variable the subresultants are desired (e.g. 1 for x and 2 for y)

### Value

If both qspray1 and qspray2 are univariate polynomials, the function returns a vector of bigq rational numbers. Otherwise, it returns a list of qspray polynomials that do not involve the var-th variable.

### Examples

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
p <- x^2 * y * (y^2 - 5*x + 6)
q <- x^2 * y * (3*y + 2)
principalSubresultants(p, q, var = 1) # should be 0, 0, non-zero, ...
principalSubresultants(p, q, var = 2) # should be 0, non-zero, ...
```

---

 resultant

*Resultant of two polynomials*


---

### Description

Resultant of two polynomials with rational coefficients.

### Usage

```
resultant(qspray1, qspray2, var = 1)
```

**Arguments**

qspray1, qspray2      two qspray polynomials with at most nine variables

var                      integer indicating with respect to which variable the resultant is desired (e.g. 1 for x and 2 for y)

**Value**

If both qspray1 and qspray2 are univariate polynomials, the function returns a `bigq` rational number. Otherwise, it returns a qspray polynomial that does not involve the var-th variable.

**Examples**

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
f <- x^4 - x^3 + x^2 - 2*x*y^2 + y^4
g <- x - 2*y^2
resultant(f, g, var = 1)
resultant(f, g, var = 2)
```

---

squareFreeFactorization

*Square-free factorization of a polynomial*

---

**Description**

Square-free factorization of a polynomial with rational coefficients.

**Usage**

```
squareFreeFactorization(qspray)
```

**Arguments**

qspray                  a qspray polynomial having at most nine variables

**Value**

A list with two fields `constantFactor` and `nonConstantFactors`. In the field `constantFactor`, there is a `bigq` rational number, the constant factor of the factorization. In the field `nonConstantFactors`, there is a list providing the square-free and pairwise coprime qspray polynomials of the factorization with their multiplicity.

**Examples**

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
p <- x^8*y^7 + 2*x^7*y^5 + x^6*y^4 + 2*x^5*y^2
squareFreeFactorization(p)
```

---

SturmHabicht

*Sturm-Habicht sequence of a polynomial*

---

**Description**

Sturm-Habicht sequence of a polynomial with rational coefficients.

**Usage**

```
SturmHabicht(qspray, var = 1)
```

**Arguments**

qspray	a qspray polynomial having at most nine variables
var	index of the variable with respect to which the Sturm-Habicht sequence will be computed

**Value**

A list of qspray polynomials, the Sturm-Habicht sequence of qspray, starting with the 0-th Sturm-Habicht polynomial.

**Examples**

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
SturmHabicht(x^3*y^2 + 2*x*y + 1)
```



---

subresultants      *Subresultants of two polynomials*

---

**Description**

Subresultants of two polynomials with rational coefficients.

**Usage**

```
subresultants(qspray1, qspray2, var = 1)
```

**Arguments**

qspray1, qspray2      two qspray polynomials having at most nine variables

var      integer indicating with respect to which variable the subresultants will be computed (e.g. 1 for x and 2 for y)

**Value**

A list of qspray polynomials.

**Examples**

```
library(resultant)
x <- qlone(1)
y <- qlone(2)
p <- x^2 * y * (y^2 - 5*x + 6)
q <- x^2 * y * (3*y + 2)
subresultants(p, q, var = 1)
subresultants(p, q, var = 2)
```

---

univariateDivision      *Division of univariate polynomials*

---

**Description**

Division with remainder of univariate polynomials with rational coefficients.

**Usage**

```
univariateDivision(qspray1, qspray2)
```

**Arguments**

qspray1, qspray2      two univariate qspray polynomials

**Value**

A list of two univariate qspray polynomials, the quotient of the division in the field  $Q$  of the list, and the remainder in the field  $R$ .

**See Also**

[integralDivision](#), [qsprayDivision](#).

**Examples**

```
library(resultant)
x <- qlone(1)
qspray1 <- 2*x^4 + x^3 - 3*x^2 - x + 1
qspray2 <- x^2 - 5*x + 10
division <- univariateDivision(qspray1, qspray2)
Q <- division[["Q"]]; R <- division[["R"]]
qspray1 == Q*qspray2 + R # should be TRUE
```

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