

# Package: qfa (via r-universe)

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**Type** Package

**Title** Quantile-Frequency Analysis (QFA) of Time Series

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**Description** Quantile-frequency analysis (QFA) of univariate or multivariate time series based on trigonometric quantile regression. See Li, T.-H. (2012) ``Quantile periodograms'', Journal of the American Statistical Association, 107, 765–776, <doi:10.1080/01621459.2012.682815>; Li, T.-H. (2014) Time Series with Mixed Spectra, CRC Press, <doi:10.1201/b15154>; Li, T.-H. (2022) ``Quantile Fourier transform, quantile series, and nonparametric estimation of quantile spectra'', <doi:10.48550/arXiv.2211.05844>.

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**Imports** RhpBLASctl, doParallel, fields, foreach, mgcv, nlme, parallel, quantreg, splines, stats, graphics, colorRamps, MASS

**License** GPL (>= 2)

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ar2qspec	<i>Quantile Spectrum from AR Model of Quantile Series</i>
----------	---

---

### Description

This function computes quantile spectrum/cross-spectrum (QSPEC) from an AR model of quantile series (QSER).

### Usage

```
ar2qspec(fit, freq = NULL)
```

### Arguments

fit	object of AR model from qser2sar() or qser2ar()
freq	sequence of frequencies in [0,1) (default = NULL: all Fourier frequencies)

**Value**

a list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
freq	sequence of frequencies

---

qacf	<i>Quantile Autocovariance Function (QACF)</i>
------	--

---

**Description**

This function computes quantile autocovariance function (QACF) from time series or quantile discrete Fourier transform (QDFT).

**Usage**

```
qacf(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

**Arguments**

y	vector or matrix of time series (if matrix, <code>nrow(y)</code> = length of time series)
tau	sequence of quantile levels in (0,1)
y.qdft	matrix or array of pre-calculated QDFT (default = NULL: compute from y and tau); if y.qdft is supplied, y and tau can be left unspecified
n.cores	number of cores for parallel computing of QDFT if y.qdft = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

**Value**

matrix or array of quantile autocovariance function

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qacf <- qacf(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qacf <- qacf(y.qdft=y.qdft)
```

qdft

*Quantile Discrete Fourier Transform (QDFT)***Description**

This function computes quantile discrete Fourier transform (QDFT) for univariate or multivariate time series.

**Usage**

```
qdft(y, tau, n.cores = 1, cl = NULL)
```

**Arguments**

**y** vector or matrix of time series (if matrix, `nrow(y)` = length of time series)  
**tau** sequence of quantile levels in (0,1)  
**n.cores** number of cores for parallel computing (default = 1)  
**cl** pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

matrix or array of quantile discrete Fourier transform of `y`

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y,tau)
# Make a cluster for repeated use
n.cores <- 2
cl <- parallel::makeCluster(n.cores)
parallel::clusterExport(cl, c("tqr.fit"))
doParallel::registerDoParallel(cl)
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y1,tau,n.cores=n.cores,cl=cl)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y2,tau,n.cores=n.cores,cl=cl)
parallel::stopCluster(cl)
```

---

qdft2qacf

*Quantile Autocovariance Function (QACF)*


---

### Description

This function computes quantile autocovariance function (QACF) from QDFT.

### Usage

```
qdft2qacf(y.qdft, return.qser = FALSE)
```

### Arguments

`y.qdft` matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`  
`return.qser` if TRUE, return quantile series (QSER) along with QACF

### Value

matrix or array of quantile autocovariance function if `return.qser = FALSE` (default), else a list with the following elements:

`qacf` matrix or array of quantile autocovariance function  
`qser` matrix or array of quantile series

### Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qacf <- qdft2qacf(y.qdft)
plot(c(0:9),y.qacf[c(1:10),1],type='h',xlab="LAG",ylab="QACF")
y.qser <- qdft2qacf(y.qdft,return.qser=TRUE)$qser
plot(y.qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)
plot(c(0:9),y.qacf[1,2,c(1:10),1],type='h',xlab="LAG",ylab="QACF")
```

---

qdft2qper

*Quantile Periodogram and Cross-Periodogram (QPER)*


---

**Description**

This function computes quantile periodogram/cross-periodogram (QPER) from QDFT.

**Usage**

```
qdft2qper(y.qdft)
```

**Arguments**

`y.qdft`            matrix or array of QDFT from `qdft()`

**Value**

matrix or array of quantile periodogram/cross-periodogram

**Examples**

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qper <- qdft2qper(y.qdft)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(y.qper[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qper <- qdft2qper(y.qdft)
qfa.plot(ff[sel.f],tau,Re(y.qper[1,1,sel.f,]))
qfa.plot(ff[sel.f],tau,Re(y.qper[1,2,sel.f,]))
```

---

qdft2qser

*Quantile Series (QSER)*


---

**Description**

This function computes quantile series (QSER) from QDFT.

**Usage**

```
qdft2qser(y.qdft)
```

**Arguments**

`y.qdft` matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`

**Value**

matrix or array of quantile series

**Examples**

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qser <- qdft2qser(y.qdft)
plot(y.qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qser <- qdft2qser(y.qdft)
plot(y.qser[,1,1],type='l',xlab="TIME",ylab="QSER")
```

---

qfa.plot

*Quantile-Frequency Plot*


---

**Description**

This function creates an image plot of quantile spectrum.

**Usage**

```
qfa.plot(
  freq,
  tau,
  rqper,
  rg.qper = range(rqper),
  rg.tau = range(tau),
  rg.freq = c(0, 0.5),
  color = colorRamps::matlab.like2(1024),
  ylab = "QUANTILE LEVEL",
  xlab = "FREQUENCY",
  tlab = NULL,
  set.par = TRUE,
  legend.plot = TRUE
)
```

**Arguments**

freq	sequence of frequencies in (0,0.5) at which quantile spectrum is evaluated
tau	sequence of quantile levels in (0,1) at which quantile spectrum is evaluated
rqper	real-valued matrix of quantile spectrum evaluated on the freq x tau grid
rg.qper	zlim for qper (default = range(qper))
rg.tau	ylim for tau (default = range(tau))
rg.freq	xlim for freq (default = c(0, 0.5))
color	colors (default = colorRamps::matlab.like2(1024))
ylab	label of y-axis (default = "QUANTILE LEVEL")
xlab	label of x-axis (default = "FREQUENCY")
tlab	title of plot (default = NULL)
set.par	if TRUE, par() is set internally (single image)
legend.plot	if TRUE, legend plot is added

**Value**

no return value

---

qkl.divergence      *Kullback-Leibler Divergence of Quantile Spectral Estimate*

---

**Description**

This function computes Kullback-Leibler divergence (KLD) of quantile spectral estimate.

**Usage**

```
qkl.divergence(y.qper, qspec, sel.f = NULL, sel.tau = NULL)
```

**Arguments**

y.qper	matrix or array of quantile spectral estimate from, e.g., qspec.lw()
qspec	matrix of array of true quantile spectrum/cross-spectrum (same dimension as y.qper)
sel.f	index of selected frequencies for computation (default = NULL: all frequencies)
sel.tau	index of selected quantile levels for computation (default = NULL: all quantile levels)

**Value**

real number of Kullback-Leibler divergence



---

qper *Quantile Periodogram and Cross-Periodogram (QPER)*

---

**Description**

This function computes quantile periodogram/cross-periodogram (QPER) from time series or quantile discrete Fourier transform (QDFT).

**Usage**

```
qper(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

**Arguments**

y	vector or matrix of time series (if matrix, <code>nrow(y)</code> = length of time series)
tau	sequence of quantile levels in (0,1)
y.qdft	matrix or array of pre-calculated QDFT (default = NULL: compute from y and tau); if y.qdft is supplied, y and tau can be left unspecified
n.cores	number of cores for parallel computing of QDFT if y.qdft = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

**Value**

matrix or array of quantile periodogram/cross-periodogram

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qper <- qper(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qper <- qper(y.qdft=y.qdft)
```

---

qper2 *Quantile Periodogram Type II (QPER2)*

---

**Description**

This function computes type-II quantile periodogram for univariate time series.

**Usage**

```
qper2(y, freq, tau, weights = NULL, n.cores = 1, cl = NULL)
```

**Arguments**

y	univariate time series
freq	sequence of frequencies in [0,1)
tau	sequence of quantile levels in (0,1)
weights	sequence of weights in quantile regression (default = NULL: weights equal to 1)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

matrix of quantile periodogram evaluated on freq \* tau grid

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper2 <- qper2(y,ff,tau)
qfa.plot(ff[sel.f],tau,Re(y.qper2[sel.f,]))
```

---

qser

*Quantile Series (QSER)*


---

**Description**

This function computes quantile series (QSER) from time series or quantile discrete Fourier transform (QDFT).

**Usage**

```
qser(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

**Arguments**

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
y.qdft	matrix or array of pre-calculated QDFT (default = NULL: compute from y and tau); if y.qdft is supplied, y and tau can be left unspecified
n.cores	number of cores for parallel computing of QDFT if y.qdft = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

**Value**

matrix or array of quantile series

**Examples**

```

y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qser <- qser(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qser <- qser(y.qdft=y.qdft)

```

---

qser2ar

*Autoregression (AR) Model of Quantile Series*


---

**Description**

This function fits an autoregression (AR) model to quantile series (QSER) separately for each quantile level using `stats::ar()`.

**Usage**

```
qser2ar(y.qser, p = NULL, order.max = NULL)
```

**Arguments**

<code>y.qser</code>	matrix or array of pre-calculated QSER, e.g., using <code>qser()</code>
<code>p</code>	order of AR model (default = NULL: selected by AIC)
<code>order.max</code>	maximum order for AIC if <code>p = NULL</code> (default = NULL: determined by <code>stats::ar()</code> )

**Value**

a list with the following elements:

<code>A</code>	matrix or array of AR coefficients
<code>V</code>	vector or matrix of residual covariance
<code>p</code>	order of AR model
<code>n</code>	length of time series
<code>residuals</code>	matrix or array of residuals

qser2sar

*Spline Autoregression (SAR) Model of Quantile Series***Description**

This function fits spline autoregression (SAR) model to quantile series (QSER).

**Usage**

```
qser2sar(
  y.qser,
  tau,
  d = 1,
  p = NULL,
  order.max = NULL,
  spar = NULL,
  method = c("AIC", "BIC", "GCV"),
  weighted = FALSE
)
```

**Arguments**

y.qser	matrix or array of pre-calculated QSER, e.g., using qser()
tau	sequence of quantile levels where y.qser is calculated
d	subsampling rate of quantile levels (default = 1)
p	order of SAR model (default = NULL: automatically selected by AIC)
order.max	maximum order for AIC if p = NULL (default = NULL: determined by stats::ar())
spar	penalty parameter alla smooth.spline (default = NULL: automatically selected)
method	criterion for penalty parameter selection: "AIC" (default), "BIC", or "GCV"
weighted	if TRUE, penalty function is weighted (default = FALSE)

**Value**

a list with the following elements:

A	matrix or array of SAR coefficients
V	vector or matrix of SAR residual covariance
p	order of SAR model
spar	penalty parameter
tau	sequence of quantile levels
n	length of time series
d	subsampling rate of quantile levels
weighted	option for weighted penalty function
fit	object containing details of SAR fit

**Description**

This function computes quantile-smoothed version of quantile discrete Fourier transform (QDFT).

**Usage**

```
qsmooth.qdft(  
  y.qdft,  
  method = c("gamm", "sp"),  
  spar = "GCV",  
  n.cores = 1,  
  cl = NULL  
)
```

**Arguments**

y.qdft	matrix or array of QDFT from qdft()
method	smoothing method: "gamm" for mgcv::gamm() (default), "sp" for stats::smooth.spline()
spar	smoothing parameter in smooth.spline() if method = "sp" (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

matrix or array of quantile-smoothed QDFT

**Examples**

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)  
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)  
tau <- seq(0.1,0.9,0.05)  
n <- length(y1)  
ff <- c(0:(n-1))/n  
sel.f <- which(ff > 0 & ff < 0.5)  
y.qdft <- qdft(cbind(y1,y2),tau)  
y.qdft <- qsmooth.qdft(y.qdft,method="sp",spar=0.9)  
y.qacf <- qdft2qacf(y.qdft)  
y.qper.qslw <- qspec.lw(y.qacf,M=5)$spec  
qfa.plot(ff[sel.f],tau,Re(y.qper.qslw[1,1,sel.f,]))
```

---

qsmooth.qper

*Quantile Smoothing of Quantile Periodogram or Spectral Estimate*


---

## Description

This function computes quantile-smoothed version of quantile periodogram/cross-periodogram (QPER) or other quantile spectral estimate.

## Usage

```
qsmooth.qper(
  y.qper,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

## Arguments

y.qper	matrix or array of quantile periodogram/cross-periodogram or spectral estimate
method	smoothing method: "gamm" for <code>mgcv::gamm()</code> (default), "sp" for <code>stats::smooth.spline()</code>
spar	smoothing parameter in <code>smooth.spline()</code> if method = "sp" (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

## Value

matrix or array of quantile-smoothed quantile spectral estimate

## Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[1,1,sel.f,]))
y.qper.lwqs <- qsmooth.qper(y.qper.lw,method="sp",spar=0.9)
qfa.plot(ff[sel.f],tau,Re(y.qper.lwqs[1,1,sel.f,]))
```

---

qspec.ar

*Autoregression (AR) Estimator of Quantile Spectrum*


---

## Description

This function computes autoregression (AR) estimate of quantile spectrum/cross-spectrum from time series or quantile series (QSER).

## Usage

```
qspec.ar(
  y,
  tau,
  y.qser = NULL,
  p = NULL,
  order.max = NULL,
  freq = NULL,
  n.cores = 1,
  cl = NULL
)
```

## Arguments

y	vector or matrix of time series (if matrix, <code>nrow(y)</code> = length of time series)
tau	sequence of quantile levels in (0,1)
y.qser	matrix or array of pre-calculated QSER (default = NULL: compute from y and tau); if y.qser is supplied, y and tau can be left unspecified
p	order of AR model (default = NULL: automatically selected by AIC)
order.max	maximum order for AIC if p = NULL (default = NULL: determined by <code>stats::ar()</code> )
freq	sequence of frequencies in [0,1) (default = NULL: all Fourier frequencies)
n.cores	number of cores for parallel computing of QDFT if y.qser = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

## Value

a list with the following elements:

spec	matrix or array of AR quantile spectrum/cross-spectrum
freq	sequence of frequencies
fit	object of AR model
qser	matrix or array of quantile series if y.qser = NULL

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.ar <- qspec.ar(cbind(y1,y2),tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.ar$spec[1,1,sel.f,]))

```

qspec.lw

*Lag-Window (LW) Estimator of Quantile Spectrum***Description**

This function computes lag-window (LW) estimate of quantile spectrum/cross-spectrum from QACF.

**Usage**

```
qspec.lw(y.qacf, M = NULL)
```

**Arguments**

y.qacf	matrix or array of pre-calculated QACF from <code>qdft2qacf()</code>
M	bandwidth parameter of lag window (default = NULL: quantile periodogram)

**Value**

A list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
lw	lag-window sequence

**Examples**

```

# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qacf <- qdft2qacf(y.qdft)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)

```



```
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[1,2,sel.f,]))
```

---

qspec.lwqs	<i>Lag-Window-Quantile-Smoothing (LWQS) Estimator of Quantile Spectrum</i>
------------	--

---

### Description

This function computes lag-window-quantile-smoothing (LWQS) estimate of quantile spectrum/cross-spectrum from time series or quantile autocovariance function (QACF).

### Usage

```
qspec.lwqs(
  y,
  tau,
  y.qacf = NULL,
  M = NULL,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

### Arguments

y	vector or matrix of time series (if matrix, <code>nrow(y)</code> = length of time series)
tau	sequence of quantile levels in (0,1)
y.qacf	matrix or array of pre-calculated QACF (default = NULL: compute from y and tau); if y.qacf is supplied, y and tau can be left unspecified
M	bandwidth parameter of lag window (default = NULL: quantile periodogram)
method	smoothing method: "gamm" for <code>mgcv::gamm()</code> (default), "sp" for <code>stats::smooth.spline()</code>
spar	smoothing parameter in <code>smooth.spline()</code> if method = "sp" (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

### Value

A list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
spec.lw	matrix or array of quantile spectrum/cross-spectrum without quantile smoothing
lw	lag-window sequence
qacf	matrix or array of quantile autocovariance function if y.qacf = NULL

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper.lwqs <- qspec.lwqs(cbind(y1,y2), tau,M=5,method="sp",spar=0.9)$spec
qfa.plot(ff[sel.f], tau,Re(y.qper.lwqs[1,1,sel.f,]))

```

---

qspec.qslw	<i>Quantile-Smoothing-Lag-Window (QSLW) Estimator of Quantile Spectrum</i>
------------	--

---

**Description**

This function computes quantile-smoothing-lag-window (QSLW estimate of quantile spectrum/cross-spectrum from time series or quantile discrete Fourier transform (QDFT).

**Usage**

```

qspec.qslw(
  y,
  tau,
  y.qdft = NULL,
  M = NULL,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)

```

**Arguments**

y	vector or matrix of time series (if matrix, <code>nrow(y)</code> = length of time series)
tau	sequence of quantile levels in (0,1)
y.qdft	matrix or array of pre-calculated QDFT (default = NULL: compute from y and tau); if y.qdft is supplied, y and tau can be left unspecified
M	bandwidth parameter of lag window (default = NULL: quantile periodogram)
method	smoothing method: "gamm" for <code>mgcv::gamm()</code> (default), "sp" for <code>stats::smooth.spline()</code>
spar	smoothing parameter in <code>smooth.spline()</code> if method = 'sp' (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

A list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
lw	lag-window sequence
qdfc	matrix or array of quantile discrete Fourier transform if <code>y.qdfc = NULL</code>

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper.qslw <- qspec.qslw(cbind(y1,y2), tau, M=5, method="sp", spar=0.9)$spec
qfa.plot(ff[sel.f], tau, Re(y.qper.qslw[1,1,sel.f,]))

```

---

qspec.sar

*Spline Autoregression (SAR) Estimator of Quantile Spectrum*


---

**Description**

This function computes spline autoregression (SAR) estimate of quantile spectrum/cross-spectrum.

**Usage**

```

qspec.sar(
  y,
  y.qser = NULL,
  tau,
  d = 1,
  p = NULL,
  order.max = NULL,
  spar = NULL,
  method = c("AIC", "BIC", "GCV"),
  weighted = FALSE,
  freq = NULL,
  n.cores = 1,
  cl = NULL
)

```

**Arguments**

<code>y</code>	vector or matrix of time series (if matrix, <code>nrow(y) = length of time series</code> )
<code>y.qser</code>	matrix or array of pre-calculated QSER (default = <code>NULL</code> : compute from <code>y</code> and <code>tau</code> ); if <code>y.qser</code> is supplied, <code>y</code> can be left unspecified

tau	sequence of quantile levels in (0,1)
d	subsampling rate of quantile levels (default = 1)
p	order of SAR model (default = NULL: automatically selected by AIC)
order.max	maximum order for AIC if p = NULL (default = NULL: determined by stats::ar())
spar	penalty parameter alla smooth.spline (default = NULL: automatically selected)
method	criterion for penalty parameter selection: "AIC" (default), "BIC", or "GCV"
weighted	if TRUE, penalty function is weighted (default = FALSE)
freq	sequence of frequencies in [0,1) (default = NULL: all Fourier frequencies)
n.cores	number of cores for parallel computing of QDFT if y.qser = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

### Value

a list with the following elements:

spec	matrix or array of SAR quantile spectrum
freq	sequence of frequencies
fit	object of SAR model
qser	matrix or array of quantile series if y.qser = NULL

### Examples

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
# compute from time series
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.sar$spec[1,1,sel.f,]))
# compute from quantile series
y.qser <- qser(cbind(y1,y2),tau)
y.sar <- qspec.sar(y.qser=y.qser,tau=tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.sar$spec[1,1,sel.f,]))

```

---

qspec.sqr1w

*Spline-Quantile-Regression-Lag-Window (SQRLW) Estimator of  
Quantile Spectrum*

---

### Description

This function computes spline-quantile-regression-lag-window (SQRLW) estimate of quantile spectrum/cross-spectrum from time series or spline quantile discrete Fourier transform (SQDFT).

**Usage**

```

qspec.sqr1w(
  y,
  tau,
  y.sqdf1 = NULL,
  M = NULL,
  c0 = 0.02,
  d = 4,
  weighted = FALSE,
  n.cores = 1,
  cl = NULL
)

```

**Arguments**

y	vector or matrix of time series (if matrix, <code>nrow(y)</code> = length of time series)
tau	sequence of quantile levels in (0,1)
y.sqdf1	matrix or array of pre-calculated SQDFT (default = NULL: compute from y and tau); if y.sqdf1 is supplied, y and tau can be left unspecified
M	bandwidth parameter of lag window (default = NULL: quantile periodogram)
c0	penalty parameter for SQDFT
d	subsampling rate of quantile levels for SQDFT (default = 1)
weighted	if TRUE, SQR penalty function is weighted (default = FALSE)
n.cores	number of cores for parallel computing of SQDFT (default = 1)
cl	pre-existing cluster for repeated parallel computing of SQDFT (default = NULL)

**Value**

A list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
lw	lag-window sequence
sqdf1	matrix or array of spline quantile discrete Fourier transform if y.sqdf1 = NULL

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper.sqr1w <- qspec.sqr1w(cbind(y1,y2), tau,M=5,c0=0.02,d=4)$spec
qfa.plot(ff[sel.f], tau, Re(y.qper.sqr1w[1,1,sel.f]))

```

---

 qspec2qcoh

*Quantile Coherence Spectrum*


---

### Description

This function computes quantile coherence spectrum (QCOH) from quantile spectrum and cross-spectrum of multiple time series.

### Usage

```
qspec2qcoh(qspec, k = 1, kk = 2)
```

### Arguments

qspec	array of quantile spectrum/cross-spectrum
k	index of first series (default = 1)
kk	index of second series (default = 2)

### Value

matrix of quantile coherence evaluated at Fourier frequencies in (0,0.5)

### Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qacf <- qacf(cbind(y1,y2),tau)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
y.qcoh <- qspec2qcoh(y.qper.lw,k=1,kk=2)
qfa.plot(ff[sel.f],tau,y.qcoh)
```

---

 sar.eq.bootstrap

*Bootstrap Simulation of SAR Coefficients for Testing Equality of  
Granger-Causality in Two Samples*


---

### Description

This function simulates bootstrap samples of selected spline autoregression (SAR) coefficients for testing equality of Granger-causality in two samples based on their SAR models under H0: effect in each sample equals the average effect.

**Usage**

```

sar.eq.bootstrap(
  y.qser,
  fit,
  fit2,
  index = c(1, 2),
  nsim = 1000,
  method = c("ar", "sar"),
  n.cores = 1,
  mthreads = FALSE,
  seed = 1234567
)

```

**Arguments**

y.qser	matrix or array of QSER from <code>qser()</code> or <code>qspec.sar()\$qser</code>
fit	object of SAR model from <code>qser2sar()</code> or <code>qspec.sar()\$fit</code>
fit2	object of SAR model for the other sample
index	a pair of component indices for multiple time series or a sequence of lags for single time series (default = <code>c(1, 2)</code> )
nsim	number of bootstrap samples (default = 1000)
method	method of residual calculation: "ar" (default) or "sar"
n.cores	number of cores for parallel computing (default = 1)
mthreads	if TRUE, multithread BLAS is enabled when available (default = FALSE, required for parallel computing)
seed	seed for random sampling (default = 1234567)

**Value**

array of simulated bootstrap samples of selected SAR coefficients

**Examples**

```

y11 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y21 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y12 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y22 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y1.sar <- qspec.sar(cbind(y11,y21), tau=tau,p=1)
y2.sar <- qspec.sar(cbind(y12,y22), tau=tau,p=1)
A1.sim <- sar.eq.bootstrap(y1.sar$qser,y1.sar$fit,y2.sar$fit,index=c(1,2),nsim=5)
A2.sim <- sar.eq.bootstrap(y2.sar$qser,y2.sar$fit,y1.sar$fit,index=c(1,2),nsim=5)

```

---

 sar.eq.test

*Wald Test and Confidence Band for Equality of SAR-Based Granger-Causality in Two Samples*


---

### Description

This function computes Wald test and confidence band for equality of Granger-causality in two samples using bootstrap samples generated by `sar.eq.bootstrap()` based on the spline autoregression (SAR) models of quantile series (QSER).

### Usage

```
sar.eq.test(A1, A1.sim, A2, A2.sim, sel.lag = NULL, sel.tau = NULL)
```

### Arguments

A1	matrix of selected SAR coefficients for sample 1
A1.sim	simulated bootstrap samples from <code>sar.eq.bootstrap()</code> for sample 1
A2	matrix of selected SAR coefficients for sample 2
A2.sim	simulated bootstrap samples from <code>sar.eq.bootstrap()</code> for sample 2
sel.lag	indices of time lags for Wald test (default = NULL: all lags)
sel.tau	indices of quantile levels for Wald test (default = NULL: all quantiles)

### Value

a list with the following elements:

test	list of Wald test result containing wald and p.value
D.u	matrix of upper limits of 95% confidence band for A1 - A2
D.l	matrix of lower limits of 95% confidence band for A1 - A2

### Examples

```
y11 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y21 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y12 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y22 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y1.sar <- qspec.sar(cbind(y11,y21),tau=tau,p=1)
y2.sar <- qspec.sar(cbind(y12,y22),tau=tau,p=1)
A1.sim <- sar.eq.bootstrap(y1.sar$qser,y1.sar$fit,y2.sar$fit,index=c(1,2),nsim=5)
A2.sim <- sar.eq.bootstrap(y2.sar$qser,y2.sar$fit,y1.sar$fit,index=c(1,2),nsim=5)
A1 <- sar.gc.coef(y1.sar$fit,index=c(1,2))
A2 <- sar.gc.coef(y2.sar$fit,index=c(1,2))
test <- sar.eq.test(A1,A1.sim,A2,A2.sim,sel.lag=NULL,sel.tau=NULL)
```



---

sar.gc.bootstrap	<i>Bootstrap Simulation of SAR Coefficients for Granger-Causality Analysis</i>
------------------	--

---

## Description

This function simulates bootstrap samples of selected spline autoregression (SAR) coefficients for Granger-causality analysis based on the SAR model of quantile series (QSER) under H0: (a) for multiple time series, the second series specified in `index` is not causal for the first series specified in `index`; (b) for single time series, the series is not causal at the lags specified in `index`.

## Usage

```
sar.gc.bootstrap(
  y.qser,
  fit,
  index = c(1, 2),
  nsim = 1000,
  method = c("ar", "sar"),
  n.cores = 1,
  mthreads = FALSE,
  seed = 1234567
)
```

## Arguments

<code>y.qser</code>	matrix or array of QSER from <code>qser()</code> or <code>qspec.sar()\$qser</code>
<code>fit</code>	object of SAR model from <code>qser2sar()</code> or <code>qspec.sar()\$fit</code>
<code>index</code>	a pair of component indices for multiple time series or a sequence of lags for single time series (default = <code>c(1, 2)</code> )
<code>nsim</code>	number of bootstrap samples (default = 1000)
<code>method</code>	method of residual calculation: "ar" (default) or "sar"
<code>n.cores</code>	number of cores for parallel computing (default = 1)
<code>mthreads</code>	if TRUE, multithread BLAS is enabled when available (default = FALSE, required for parallel computing)
<code>seed</code>	seed for random sampling (default = 1234567)

## Value

array of simulated bootstrap samples of selected SAR coefficients

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A.sim <- sar.gc.bootstrap(y.sar$qser,y.sar$fit,index=c(1,2),nsim=5)

```

---

sar.gc.coef

---

*Extraction of SAR Coefficients for Granger-Causality Analysis*


---

**Description**

This function extracts the spline autoregression (SAR) coefficients from an SAR model for Granger-causality analysis. See `sar.gc.bootstrap` for more details regarding the use of `index`.

**Usage**

```
sar.gc.coef(fit, index = c(1, 2))
```

**Arguments**

<code>fit</code>	object of SAR model from <code>qser2sar()</code> or <code>qspec.sar()</code> \$fit
<code>index</code>	a pair of component indices for multiple time series or a sequence of lags for single time series (default = <code>c(1, 2)</code> )

**Value**

matrix of selected SAR coefficients (number of lags by number of quantiles)

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A <- sar.gc.coef(y.sar$fit,index=c(1,2))

```

---

sar.gc.test	<i>Wald Test and Confidence Band for SAR-Based Granger-Causality Analysis</i>
-------------	---

---

## Description

This function computes Wald test and confidence band for Granger-causality using bootstrap samples generated by `sar.gc.bootstrap()` based the spline autoregression (SAR) model of quantile series (QSER).

## Usage

```
sar.gc.test(A, A.sim, sel.lag = NULL, sel.tau = NULL)
```

## Arguments

A	matrix of selected SAR coefficients
A.sim	simulated bootstrap samples from <code>sar.gc.bootstrap()</code>
sel.lag	indices of time lags for Wald test (default = NULL: all lags)
sel.tau	indices of quantile levels for Wald test (default = NULL: all quantiles)

## Value

a list with the following elements:

test	list of Wald test result containing wald and p.value
A.u	matrix of upper limits of 95% confidence band of A
A.l	matrix of lower limits of 95% confidence band of A

## Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A <- sar.gc.coef(y.sar$fit,index=c(1,2))
A.sim <- sar.gc.bootstrap(y.sar$qser,y.sar$fit,index=c(1,2),nsim=5)
y.gc <- sar.gc.test(A,A.sim)
```

sqdft

*Spline Quantile Discrete Fourier Transform (SQDFT)***Description**

This function computes spline quantile discrete Fourier transform (SQDFT) for univariate or multivariate time series through trigonometric spline quantile regression.

**Usage**

```
sqdft(y, tau, c0 = 0.02, d = 4, weighted = FALSE, n.cores = 1, cl = NULL)
```

**Arguments**

<code>y</code>	vector or matrix of time series (if matrix, <code>nrow(y) = length of time series</code> )
<code>tau</code>	sequence of quantile levels in (0,1)
<code>c0</code>	penalty parameter
<code>d</code>	subsampling rate of quantile levels (default = 1)
<code>weighted</code>	if TRUE, penalty function is weighted (default = FALSE)
<code>n.cores</code>	number of cores for parallel computing (default = 1)
<code>cl</code>	pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

matrix or array of the spline quantile discrete Fourier transform of `y`

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sqdft <- sqdft(y,tau,c0=0.02,d=4)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qacf <- qdft2qacf(y.sqdft)
y.qper.sqr1w <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.sqr1w[sel.f,]))
```

---

sqr.fit                      *Spline Quantile Regression (SQR)*

---

### Description

This function computes spline quantile regression (SQR) solution from response vector and design matrix. It uses the FORTRAN code `rqfnb.f` in the "quantreg" package with the kind permission of Dr. R. Koenker.

### Usage

```
sqr.fit(y, X, tau, c0, d = 1, weighted = FALSE, mthreads = FALSE)
```

### Arguments

<code>y</code>	response vector
<code>X</code>	design matrix ( <code>nrow(X) = length(y)</code> )
<code>tau</code>	sequence of quantile levels in (0,1)
<code>c0</code>	penalty parameter
<code>d</code>	subsampling rate of quantile levels (default = 1)
<code>weighted</code>	if TRUE, penalty function is weighted (default = FALSE)
<code>mthreads</code>	if TRUE, multithread BLAS is enabled when available (default = FALSE, required for parallel computing)

### Value

A list with the following elements:

<code>coefficients</code>	matrix of regression coefficients
<code>nit</code>	number of iterations

---

tqr.fit                      *Trigonometric Quantile Regression (TQR)*

---

### Description

This function computes trigonometric quantile regression (TQR) for univariate time series at a single frequency.

### Usage

```
tqr.fit(y, f0, tau, prepared = TRUE)
```

**Arguments**

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

**Value**

object of `rq()` (coefficients in `$coef`)

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tsqr.fit(y,f0=0.1,tau=tau)
plot(tau,fit$coef[1,],type='o',pch=0.75,xlab='QUANTILE LEVEL',ylab='TQR COEF')
```

---

tsqr.fit

*Trigonometric Spline Quantile Regression (TSQR)*

---

**Description**

This function computes trigonometric spline quantile regression (TSQR) for univariate time series at a single frequency.

**Usage**

```
tsqr.fit(y, f0, tau, c0, d = 1, weighted = FALSE, prepared = TRUE)
```

**Arguments**

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

**Value**

object of `sqr.fit()` (coefficients in `$coef`)

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
fit.sqr <- tsqr.fit(y,f0=0.1,tau=tau,c0=0.02,d=4)
plot(tau,fit$coef[1,],type='p',xlab='QUANTILE LEVEL',ylab='TQR COEF')
lines(tau,fit.sqr$coef[1,],type='l')
```

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