# <span id="page-0-0"></span>Package: performance (via r-universe)

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Type Package

Title Assessment of Regression Models Performance

Version 0.12.4

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Description Utilities for computing measures to assess model quality, which are not directly provided by R's 'base' or 'stats' packages. These include e.g. measures like r-squared, intraclass correlation coefficient (Nakagawa, Johnson & Schielzeth (2017) [<doi:10.1098/rsif.2017.0213>](https://doi.org/10.1098/rsif.2017.0213)), root mean squared error or functions to check models for overdispersion, singularity or zero-inflation and more. Functions apply to a large variety of regression models, including generalized linear models, mixed effects models and Bayesian models. References: Lüdecke et al. (2021) [<doi:10.21105/joss.03139>](https://doi.org/10.21105/joss.03139).

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URL <https://easystats.github.io/performance/>

BugReports <https://github.com/easystats/performance/issues>

Depends  $R$  ( $>= 3.6$ )

- **Imports** bayestestR  $(>= 0.15.0)$ , insight  $(>= 0.20.5)$ , datawizard  $(>=$ 0.13.0), stats, utils
- Suggests AER, afex, BayesFactor, bayesplot, betareg, bigutilsr, blavaan, boot, brms, car, carData, CompQuadForm, correlation, cplm, dagitty, dbscan, DHARMa, estimatr, fixest, flextable, forecast, ftExtra, gamm4, ggdag, glmmTMB (>= 1.1.10), graphics, Hmisc, httr2, ICS, ICSOutlier, ISLR, ivreg, lavaan, lme4, lmtest, loo, MASS, Matrix, mclogit, mclust, metadat, metafor, mgcv, mlogit, multimode, nestedLogit, nlme, nonnest2, ordinal, parallel, parameters (>= 0.21.6), patchwork, pscl, psych, quantreg, qqplotr  $(>= 0.0.6)$ , randomForest, RcppEigen, rempsyc, rmarkdown, rstanarm, rstantools, sandwich, see (>= 0.9.0), survey, survival, test that  $(>= 3.2.1)$ , tweedie, VGAM, with  $(>=$ 3.0.0)

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<span id="page-3-1"></span>binned\_residuals *Binned residuals for binomial logistic regression*

## Description

Check model quality of binomial logistic regression models.

## Usage

```
binned_residuals(
 model,
 term = NULL,
 n_bins = NULL,
 show_dots = NULL,
 ci = 0.95,ci_type = c("exact", "gaussian", "boot"),
 residuals = c("deviance", "pearson", "response"),
 iterations = 1000,
 verbose = TRUE,
  ...
)
```




### Details

Binned residual plots are achieved by "dividing the data into categories (bins) based on their fitted values, and then plotting the average residual versus the average fitted value for each bin." *(Gelman, Hill 2007: 97)*. If the model were true, one would expect about 95% of the residuals to fall inside the error bounds.

If term is not NULL, one can compare the residuals in relation to a specific model predictor. This may be helpful to check if a term would fit better when transformed, e.g. a rising and falling pattern of residuals along the x-axis is a signal to consider taking the logarithm of the predictor (cf. Gelman and Hill 2007, pp. 97-98).

#### Value

A data frame representing the data that is mapped in the accompanying plot. In case all residuals are inside the error bounds, points are black. If some of the residuals are outside the error bounds (indicated by the grey-shaded area), blue points indicate residuals that are OK, while red points indicate model under- or over-fitting for the relevant range of estimated probabilities.

#### Note

binned\_residuals() returns a data frame, however, the print() method only returns a short summary of the result. The data frame itself is used for plotting. The plot() method, in turn, creates a ggplot-object.

#### References

Gelman, A., and Hill, J. (2007). Data analysis using regression and multilevel/hierarchical models. Cambridge; New York: Cambridge University Press.

#### Examples

```
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
result <- binned_residuals(model)
result
```

```
# look at the data frame
as.data.frame(result)
# plot
if (require("see")) {
 plot(result, show_dots = TRUE)
}
```
<span id="page-5-1"></span>check\_autocorrelation *Check model for independence of residuals.*

## Description

Check model for independence of residuals, i.e. for autocorrelation of error terms.

## Usage

```
check_autocorrelation(x, ...)
```

```
## Default S3 method:
check_autocorrelation(x, nsim = 1000, ...)
```
#### **Arguments**



## Details

Performs a Durbin-Watson-Test to check for autocorrelated residuals. In case of autocorrelation, robust standard errors return more accurate results for the estimates, or maybe a mixed model with error term for the cluster groups should be used.

#### Value

Invisibly returns the p-value of the test statistics. A p-value < 0.05 indicates autocorrelated residuals.

#### See Also

Other functions to check model assumptions and and assess model quality: [check\\_collinearity\(](#page-7-1)), [check\\_convergence\(](#page-10-1)), [check\\_heteroscedasticity\(](#page-20-1)), [check\\_homogeneity\(](#page-21-1)), [check\\_model\(](#page-24-1)), [check\\_outliers\(](#page-30-1)), [check\\_overdispersion\(](#page-37-1)), [check\\_predictions\(](#page-39-1)), [check\\_singularity\(](#page-43-1)), [check\\_zeroinflation\(](#page-47-1))

## <span id="page-6-0"></span>Examples

```
m \le - \ln(mpg \sim wt + cyl + gear + disp, data = mtcars)check_autocorrelation(m)
```
<span id="page-6-1"></span>check\_clusterstructure

*Check suitability of data for clustering*

#### Description

This checks whether the data is appropriate for clustering using the Hopkins' H statistic of given data. If the value of Hopkins statistic is close to 0 (below 0.5), then we can reject the null hypothesis and conclude that the dataset is significantly clusterable. A value for H lower than 0.25 indicates a clustering tendency at the 90% confidence level. The visual assessment of cluster tendency (VAT) approach (Bezdek and Hathaway, 2002) consists in investigating the heatmap of the ordered dissimilarity matrix. Following this, one can potentially detect the clustering tendency by counting the number of square shaped blocks along the diagonal.

#### Usage

```
check_{\text{cluster}}tusterstructure(x, standardize = TRUE, distance = "euclidean", ...)
```
#### Arguments



#### Value

The H statistic (numeric)

## References

- Lawson, R. G., & Jurs, P. C. (1990). New index for clustering tendency and its application to chemical problems. Journal of chemical information and computer sciences, 30(1), 36-41.
- Bezdek, J. C., & Hathaway, R. J. (2002, May). VAT: A tool for visual assessment of (cluster) tendency. In Proceedings of the 2002 International Joint Conference on Neural Networks. IJCNN02 (3), 2225-2230. IEEE.

## See Also

[check\\_kmo\(\)](#page-17-1), [check\\_sphericity\\_bartlett\(\)](#page-17-1) and [check\\_factorstructure\(\)](#page-17-2).

#### Examples

```
library(performance)
check_clusterstructure(iris[, 1:4])
plot(check_clusterstructure(iris[, 1:4]))
```
<span id="page-7-1"></span>check\_collinearity *Check for multicollinearity of model terms*

## Description

check\_collinearity() checks regression models for multicollinearity by calculating the variance inflation factor (VIF). multicollinearity() is an alias for check\_collinearity(). check\_concurvity() is a wrapper around mgcv::concurvity(), and can be considered as a collinearity check for smooth terms in GAMs. Confidence intervals for VIF and tolerance are based on Marcoulides et al. (2019, Appendix B).

#### Usage

```
check_collinearity(x, ...)
multicollinearity(x, ...)
## Default S3 method:
check_collinearity(x, ci = 0.95, verbose = TRUE, \ldots)
## S3 method for class 'glmmTMB'
check_collinearity(
  x,
  component = c("all", "conditional", "count", "zi", "zero_inflated"),
  ci = 0.95,verbose = TRUE,
  ...
\lambda
```
## check\_concurvity(x, ...)



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component For models with zero-inflation component, multicollinearity can be checked for the conditional model (count component, component = "conditional" or component = "count"), zero-inflation component (component = "zero\_inflated" or component = "zi") or both components (component = "all"). Following model-classes are currently supported: hurdle, zeroinfl, zerocount, MixMod and glmmTMB.

## Value

A data frame with information about name of the model term, the variance inflation factor and associated confidence intervals, the factor by which the standard error is increased due to possible correlation with other terms, and tolerance values (including confidence intervals), where tolerance  $= 1/v$ if.

#### Multicollinearity

Multicollinearity should not be confused with a raw strong correlation between predictors. What matters is the association between one or more predictor variables, *conditional on the other variables in the model*. In a nutshell, multicollinearity means that once you know the effect of one predictor, the value of knowing the other predictor is rather low. Thus, one of the predictors doesn't help much in terms of better understanding the model or predicting the outcome. As a consequence, if multicollinearity is a problem, the model seems to suggest that the predictors in question don't seems to be reliably associated with the outcome (low estimates, high standard errors), although these predictors actually are strongly associated with the outcome, i.e. indeed might have strong effect (*McElreath 2020, chapter 6.1*).

Multicollinearity might arise when a third, unobserved variable has a causal effect on each of the two predictors that are associated with the outcome. In such cases, the actual relationship that matters would be the association between the unobserved variable and the outcome.

Remember: "Pairwise correlations are not the problem. It is the conditional associations - not correlations - that matter." (*McElreath 2020, p. 169*)

#### Interpretation of the Variance Inflation Factor

The variance inflation factor is a measure to analyze the magnitude of multicollinearity of model terms. A VIF less than 5 indicates a low correlation of that predictor with other predictors. A value between 5 and 10 indicates a moderate correlation, while VIF values larger than 10 are a sign for high, not tolerable correlation of model predictors (*James et al. 2013*). The *Increased SE* column in the output indicates how much larger the standard error is due to the association with other predictors conditional on the remaining variables in the model. Note that these thresholds, although commonly used, are also criticized for being too high. *Zuur et al. (2010)* suggest using lower values, e.g. a VIF of 3 or larger may already no longer be considered as "low".

#### Multicollinearity and Interaction Terms

If interaction terms are included in a model, high VIF values are expected. This portion of multicollinearity among the component terms of an interaction is also called "inessential ill-conditioning", which leads to inflated VIF values that are typically seen for models with interaction terms *(Francoeur 2013)*.

#### Concurvity for Smooth Terms in Generalized Additive Models

check\_concurvity() is a wrapper around mgcv::concurvity(), and can be considered as a collinearity check for smooth terms in GAMs."Concurvity occurs when some smooth term in a model could be approximated by one or more of the other smooth terms in the model." (see ?mgcv::concurvity). check\_concurvity() returns a column named *VIF*, which is the "worst" measure. While mgcv::concurvity() range between 0 and 1, the *VIF* value is 1 / (1 - worst), to make interpretation comparable to classical VIF values, i.e. 1 indicates no problems, while higher values indicate increasing lack of identifiability. The *VIF proportion* column equals the "estimate" column from  $mgcv$ ::concurvity(), ranging from 0 (no problem) to 1 (total lack of identifiability).

#### **Note**

The code to compute the confidence intervals for the VIF and tolerance values was adapted from the Appendix B from the Marcoulides et al. paper. Thus, credits go to these authors the original algorithm. There is also a plot()[-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

#### References

- Francoeur, R. B. (2013). Could Sequential Residual Centering Resolve Low Sensitivity in Moderated Regression? Simulations and Cancer Symptom Clusters. Open Journal of Statistics, 03(06), 24-44.
- James, G., Witten, D., Hastie, T., and Tibshirani, R. (eds.). (2013). An introduction to statistical learning: with applications in R. New York: Springer.
- Marcoulides, K. M., and Raykov, T. (2019). Evaluation of Variance Inflation Factors in Regression Models Using Latent Variable Modeling Methods. Educational and Psychological Measurement, 79(5), 874–882.
- McElreath, R. (2020). Statistical rethinking: A Bayesian course with examples in R and Stan. 2nd edition. Chapman and Hall/CRC.
- Vanhove, J. (2019). Collinearity isn't a disease that needs curing. [webpage](https://janhove.github.io/posts/2019-09-11-collinearity/)
- Zuur AF, Ieno EN, Elphick CS. A protocol for data exploration to avoid common statistical problems: Data exploration. Methods in Ecology and Evolution (2010) 1:3–14.

#### See Also

[see::plot.see\\_check\\_collinearity\(\)](#page-0-0) for options to customize the plot.

```
Other functions to check model assumptions and and assess model quality: check_autocorrelation(),
check_convergence(), check_heteroscedasticity(), check_homogeneity(), check_model(),
check_outliers(), check_overdispersion(), check_predictions(), check_singularity(),
check_zeroinflation()
```
## Examples

```
m \le -1m(mpg ~ wt + cyl + gear + disp, data = mtcars)
check_collinearity(m)
```
# plot results

```
x <- check_collinearity(m)
plot(x)
```
<span id="page-10-1"></span>check\_convergence *Convergence test for mixed effects models*

#### Description

check\_convergence() provides an alternative convergence test for merMod-objects.

## Usage

```
check\_convergence(x, tolerance = 0.001, ...)
```
#### **Arguments**



#### Value

TRUE if convergence is fine and FALSE if convergence is suspicious. Additionally, the convergence value is returned as attribute.

#### Convergence and log-likelihood

Convergence problems typically arise when the model hasn't converged to a solution where the log-likelihood has a true maximum. This may result in unreliable and overly complex (or nonestimable) estimates and standard errors.

#### Inspect model convergence

lme4 performs a convergence-check (see ?lme4::convergence), however, as as discussed [here](https://github.com/lme4/lme4/issues/120) and suggested by one of the lme4-authors in [this comment,](https://github.com/lme4/lme4/issues/120#issuecomment-39920269) this check can be too strict. check\_convergence() thus provides an alternative convergence test for merMod-objects.

#### Resolving convergence issues

Convergence issues are not easy to diagnose. The help page on ?lme4::convergence provides most of the current advice about how to resolve convergence issues. Another clue might be large parameter values, e.g. estimates (on the scale of the linear predictor) larger than 10 in (non-identity link) generalized linear model *might* indicate [complete separation.](https://stats.oarc.ucla.edu/other/mult-pkg/faq/general/faqwhat-is-complete-or-quasi-complete-separation-in-logisticprobit-regression-and-how-do-we-deal-with-them/) Complete separation can be addressed by regularization, e.g. penalized regression or Bayesian regression with appropriate priors on the fixed effects.

#### <span id="page-11-0"></span>Convergence versus Singularity

Note the different meaning between singularity and convergence: singularity indicates an issue with the "true" best estimate, i.e. whether the maximum likelihood estimation for the variance-covariance matrix of the random effects is positive definite or only semi-definite. Convergence is a question of whether we can assume that the numerical optimization has worked correctly or not.

## See Also

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-1)), [check\\_collinearity\(](#page-7-1)), [check\\_heteroscedasticity\(](#page-20-1)), [check\\_homogeneity\(](#page-21-1)), [check\\_model\(](#page-24-1)), [check\\_outliers\(](#page-30-1)), [check\\_overdispersion\(](#page-37-1)), [check\\_predictions\(](#page-39-1)), [check\\_singularity\(](#page-43-1)), [check\\_zeroinflation\(](#page-47-1))

## Examples

```
data(cbpp, package = "lme4")set.seed(1)
cbpp$x <- rnorm(nrow(cbpp))
cbpp$x2 <- runif(nrow(cbpp))
model <- lme4::glmer(
 cbind(incidence, size - incidence) \sim period + x + x2 + (1 + x | herd),
 data = cbp,
 family = binomial())
check_convergence(model)
model <- suppressWarnings(glmmTMB::glmmTMB(
 Sepal.Length ~ poly(Petal.Width, 4) * poly(Petal.Length, 4) +
    (1 + poly(Petal.Width, 4) | Species),
 data = iris
))
check_convergence(model)
```
check\_dag *Check correct model adjustment for identifying causal effects*

#### Description

The purpose of check\_dag() is to build, check and visualize your model based on directed acyclic graphs (DAG). The function checks if a model is correctly adjusted for identifying specific relationships of variables, especially directed (maybe also "causal") effects for given exposures on an outcome. In case of incorrect adjustments, the function suggests the minimal required variables that should be adjusted for (sometimes also called "controlled for"), i.e. variables that *at least* need to

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be included in the model. Depending on the goal of the analysis, it is still possible to add more variables to the model than just the minimally required adjustment sets.

check\_dag() is a convenient wrapper around ggdag::dagify(), dagitty::adjustmentSets() and dagitty::adjustedNodes() to check correct adjustment sets. It returns a dagitty object that can be visualized with plot(). as.dag() is a small convenient function to return the dagitty-string, which can be used for the online-tool from the dagitty-website.

## Usage

```
check_dag(
  ...,
 outcome = NULL,
  exposure = NULL,
  adjusted = NULL,
  latent = NULL,
  effect = c("all", "total", "direct"),
  coords = NULL
\lambda
```
 $as.dag(x, \ldots)$ 



- a list with elements that correspond to the variables in the DAG. Each element is a numeric vector of length two with x- and y-coordinate.
- a data frame with three columns: x, y and name (which contains the variable names).

See 'Examples'.

x An object of class check\_dag, as returned by check\_dag().

#### Value

An object of class check\_dag, which can be visualized with plot(). The returned object also inherits from class dagitty and thus can be used with all functions from the **ggdag** and **dagitty** packages.

#### Specifying the DAG formulas

The formulas have following syntax:

- One-directed paths: On the *left-hand-side* is the name of the variables where causal effects point to (direction of the arrows, in dagitty-language). On the *right-hand-side* are all variables where causal effects are assumed to come from. For example, the formula  $Y \sim X1 + X2$ , paths directed from both X1 and X2 to Y are assumed.
- Bi-directed paths: Use  $\sim$  to indicate bi-directed paths. For example,  $Y \sim \chi$  indicates that the path between Y and X is bi-directed, and the arrow points in both directions. Bi-directed paths often indicate unmeasured cause, or unmeasured confounding, of the two involved variables.

## Minimally required adjustments

The function checks if the model is correctly adjusted for identifying the direct and total effects of the exposure on the outcome. If the model is correctly specified, no adjustment is needed to estimate the direct effect. If the model is not correctly specified, the function suggests the minimally required variables that should be adjusted for. The function distinguishes between direct and total effects, and checks if the model is correctly adjusted for both. If the model is cyclic, the function stops and suggests to remove cycles from the model.

Note that it sometimes could be necessary to try out different combinations of suggested adjustments, because check\_dag() can not always detect whether *at least* one of several variables is required, or whether adjustments should be done for *all* listed variables. It can be useful to copy the dagitty-code (using as.dag(), which prints the dagitty-string into the console) into the dagittywebsite and play around with different adjustments.

## Direct and total effects

The direct effect of an exposure on an outcome is the effect that is not mediated by any other variable in the model. The total effect is the sum of the direct and indirect effects. The function checks if the model is correctly adjusted for identifying the direct and total effects of the exposure on the outcome.

#### Why are DAGs important - the Table 2 fallacy

Correctly thinking about and identifying the relationships between variables is important when it comes to reporting coefficients from regression models that mutually adjust for "confounders" or include covariates. Different coefficients might have different interpretations, depending on their relationship to other variables in the model. Sometimes, a regression coefficient represents the direct effect of an exposure on an outcome, but sometimes it must be interpreted as total effect, due to the involvement of mediating effects. This problem is also called "Table 2 fallacy" (*Westreich and Greenland 2013*). DAG helps visualizing and thereby focusing the relationships of variables in a regression model to detect missing adjustments or over-adjustment.

#### References

- Rohrer, J. M. (2018). Thinking clearly about correlations and causation: Graphical causal models for observational data. Advances in Methods and Practices in Psychological Science, 1(1), 27–42. [doi:10.1177/2515245917745629](https://doi.org/10.1177/2515245917745629)
- Westreich, D., & Greenland, S. (2013). The Table 2 Fallacy: Presenting and Interpreting Confounder and Modifier Coefficients. American Journal of Epidemiology, 177(4), 292–298. [doi:10.1093/aje/kws412](https://doi.org/10.1093/aje/kws412)

#### Examples

```
# no adjustment needed
check_dag(
  y \sim x + b,
  outcome = "y"exposure = "x")
# incorrect adjustment
dag <- check_dag(
  y \sim x + b + c,
  x \sim b,
  outcome = "y",exposure = "x"\lambdadag
plot(dag)
# After adjusting for `b`, the model is correctly specified
dag <- check_dag(
  y \sim x + b + c,
  x \sim b,
  outcome = "y",
  exposure = "x"adjusted = "b")
dag
# using formula interface for arguments "outcome", "exposure" and "adjusted"
check_dag(
```
 $y \sim x + b + c$ ,

```
x \sim b,
  outcome = \simy,
  exposure = -x,
  adjusted = ~\sim b + c
\lambda# if not provided, "outcome" is taken from first formula, same for "exposure"
# thus, we can simplify the above expression to
check_dag(
  y \sim x + b + c,
  x \sim b,
  adjusted = ~\sim b + c
\lambda# use specific layout for the DAG
dag <- check_dag(
  score \sim exp + b + c,
  exp ~ 0 ~ 6,outcome = "score",
  exposure = "exp",
  coords = list(
   # x-coordinates for all nodes
   x = c(\text{score} = 5, \text{ exp} = 4, \text{ b} = 3, \text{ c} = 3),# y-coordinates for all nodes
    y = c(score = 3, exp = 3, b = 2, c = 4)
 )
\mathcal{L}plot(dag)
# alternative way of providing the coordinates
dag <- check_dag(
 score \sim exp + b + c,
  exp \sim b,
  outcome = "score",
  exposure = "exp",
  coords = list(
    # x/y coordinates for each node
    score = c(5, 3),
    exp = c(4, 3),
    b = c(3, 2),
    c = c(3, 4))
)
plot(dag)
# Objects returned by `check_dag()` can be used with "ggdag" or "dagitty"
ggdag::ggdag_status(dag)
# Using a model object to extract information about outcome,
# exposure and adjusted variables
data(mtcars)
m \le -1m(mpg ~ wt + gear + disp + cyl, data = mtcars)
dag <- check_dag(
```
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```
m,
  wt \sim disp + cyl,
  wt \sim am
)
dag
plot(dag)
```
check\_distribution *Classify the distribution of a model-family using machine learning*

#### Description

Choosing the right distributional family for regression models is essential to get more accurate estimates and standard errors. This function may help to check a models' distributional family and see if the model-family probably should be reconsidered. Since it is difficult to exactly predict the correct model family, consider this function as somewhat experimental.

#### Usage

```
check_distribution(model)
```
#### Arguments

model Typically, a model (that should response to residuals()). May also be a numeric vector.

## Details

This function uses an internal random forest model to classify the distribution from a model-family. Currently, following distributions are trained (i.e. results of check\_distribution() may be one of the following): "bernoulli", "beta", "beta-binomial", "binomial", "cauchy", "chi", "exponential", "F", "gamma", "half-cauchy", "inverse-gamma", "lognormal", "normal", "negative binomial", "negative binomial (zero-inflated)", "pareto", "poisson", "poisson (zero-inflated)", "tweedie", "uniform" and "weibull".

Note the similarity between certain distributions according to shape, skewness, etc. Thus, the predicted distribution may not be perfectly representing the distributional family of the underlying fitted model, or the response value.

There is a plot() method, which shows the probabilities of all predicted distributions, however, only if the probability is greater than zero.

## Note

This function is somewhat experimental and might be improved in future releases. The final decision on the model-family should also be based on theoretical aspects and other information about the data and the model.

There is also a  $plot()$ [-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

## Examples

```
data(sleepstudy, package = "lme4")model <<- lme4::lmer(Reaction ~ Days + (Days | Subject), sleepstudy)
check_distribution(model)
```

```
plot(check_distribution(model))
```
<span id="page-17-2"></span>check\_factorstructure *Check suitability of data for Factor Analysis (FA) with Bartlett's Test of Sphericity and KMO*

#### <span id="page-17-1"></span>**Description**

This checks whether the data is appropriate for Factor Analysis (FA) by running the Bartlett's Test of Sphericity and the Kaiser, Meyer, Olkin (KMO) Measure of Sampling Adequacy (MSA). See details below for more information about the interpretation and meaning of each test.

#### Usage

```
check_factorstructure(x, n = NULL, ...)
check_kmo(x, n = NULL, ...)check_sphericity_bartlet(x, n = NULL, ...)
```
#### Arguments



#### Details

#### Bartlett's Test of Sphericity:

Bartlett's (1951) test of sphericity tests whether a matrix (of correlations) is significantly different from an identity matrix (filled with 0). It tests whether the correlation coefficients are all 0. The test computes the probability that the correlation matrix has significant correlations among at least some of the variables in a dataset, a prerequisite for factor analysis to work.

While it is often suggested to check whether Bartlett's test of sphericity is significant before starting with factor analysis, one needs to remember that the test is testing a pretty extreme scenario (that all correlations are non-significant). As the sample size increases, this test tends to be always significant, which makes it not particularly useful or informative in well-powered studies.

<span id="page-17-0"></span>

#### Kaiser, Meyer, Olkin (KMO):

*(Measure of Sampling Adequacy (MSA) for Factor Analysis.)*

Kaiser (1970) introduced a Measure of Sampling Adequacy (MSA), later modified by Kaiser and Rice (1974). The Kaiser-Meyer-Olkin (KMO) statistic, which can vary from 0 to 1, indicates the degree to which each variable in a set is predicted without error by the other variables.

A value of 0 indicates that the sum of partial correlations is large relative to the sum correlations, indicating factor analysis is likely to be inappropriate. A KMO value close to 1 indicates that the sum of partial correlations is not large relative to the sum of correlations and so factor analysis should yield distinct and reliable factors. It means that patterns of correlations are relatively compact, and so factor analysis should yield distinct and reliable factors. Values smaller than 0.5 suggest that you should either collect more data or rethink which variables to include.

Kaiser (1974) suggested that  $KMO > .9$  were marvelous, in the .80s, meritorious, in the .70s, middling, in the .60s, mediocre, in the .50s, miserable, and less than .5, unacceptable. Hair et al. (2006) suggest accepting a value > 0.5. Values between 0.5 and 0.7 are mediocre, and values between 0.7 and 0.8 are good.

Variables with individual KMO values below 0.5 could be considered for exclusion them from the analysis (note that you would need to re-compute the KMO indices as they are dependent on the whole dataset).

#### Value

A list of lists of indices related to sphericity and KMO.

#### References

This function is a wrapper around the KMO and the cortest.bartlett() functions in the psych package (Revelle, 2016).

- Revelle, W. (2016). How To: Use the psych package for Factor Analysis and data reduction.
- Bartlett, M. S. (1951). The effect of standardization on a Chi-square approximation in factor analysis. Biometrika, 38(3/4), 337-344.
- Kaiser, H. F. (1970). A second generation little jiffy. Psychometrika, 35(4), 401-415.
- Kaiser, H. F., & Rice, J. (1974). Little jiffy, mark IV. Educational and psychological measurement, 34(1), 111-117.
- Kaiser, H. F. (1974). An index of factorial simplicity. Psychometrika, 39(1), 31-36.

#### See Also

[check\\_clusterstructure\(\)](#page-6-1).

#### Examples

library(performance)

check\_factorstructure(mtcars)

# One can also pass a correlation matrix

r <- cor(mtcars)

check\_factorstructure(r, n = nrow(mtcars))

check\_heterogeneity\_bias

*Check model predictor for heterogeneity bias*

## Description

check\_heterogeneity\_bias() checks if model predictors or variables may cause a heterogeneity bias, i.e. if variables have a within- and/or between-effect (*Bell and Jones, 2015*).

## Usage

```
check_heterogeneity_bias(
 x,
  select = NULL,
 by = NULL,
 nested = FALSE,
 group = NULL
)
```


<span id="page-19-0"></span>

#### <span id="page-20-0"></span>References

• Bell A, Jones K. 2015. Explaining Fixed Effects: Random Effects Modeling of Time-Series Cross-Sectional and Panel Data. Political Science Research and Methods, 3(1), 133–153.

#### See Also

For further details, read the vignette [https://easystats.github.io/parameters/articles/](https://easystats.github.io/parameters/articles/demean.html) [demean.html](https://easystats.github.io/parameters/articles/demean.html) and also see documentation for [datawizard::demean\(\)](#page-0-0).

#### Examples

```
data(iris)
iris$ID <- sample(1:4, nrow(iris), replace = TRUE) # fake-ID
check_heterogeneity_bias(iris, select = c("Sepal.Length", "Petal.Length"), by = "ID")
```
<span id="page-20-1"></span>check\_heteroscedasticity

*Check model for (non-)constant error variance*

#### Description

Significance testing for linear regression models assumes that the model errors (or residuals) have constant variance. If this assumption is violated the p-values from the model are no longer reliable.

#### Usage

```
check_heteroscedasticity(x, ...)
```

```
check_heteroskedasticity(x, ...)
```
#### Arguments



## Details

This test of the hypothesis of (non-)constant error is also called *Breusch-Pagan test* (1979).

## Value

The p-value of the test statistics. A p-value < 0.05 indicates a non-constant variance (heteroskedasticity).

#### Note

There is also a  $plot()$ [-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

#### <span id="page-21-0"></span>References

Breusch, T. S., and Pagan, A. R. (1979) A simple test for heteroscedasticity and random coefficient variation. Econometrica 47, 1287-1294.

## See Also

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-1)), [check\\_collinearity\(](#page-7-1)), [check\\_convergence\(](#page-10-1)), [check\\_homogeneity\(](#page-21-1)), [check\\_model\(](#page-24-1)), [check\\_outliers\(](#page-30-1)), [check\\_overdispersion\(](#page-37-1)), [check\\_predictions\(](#page-39-1)), [check\\_singularity\(](#page-43-1)), [check\\_zeroinflation\(](#page-47-1))

## Examples

```
m \ll - \ln(mpg \sim wt + cyl + gear + disp, data = mtcars)check_heteroscedasticity(m)
# plot results
if (require("see")) {
  x <- check_heteroscedasticity(m)
  plot(x)
}
```
<span id="page-21-1"></span>check\_homogeneity *Check model for homogeneity of variances*

#### Description

Check model for homogeneity of variances between groups described by independent variables in a model.

## Usage

```
check_homogeneity(x, method = c("bartlett", "fligner", "levene", "auto"), ...)
```
## S3 method for class 'afex\_aov' check\_homogeneity(x, method = "levene",  $\dots$ )



## <span id="page-22-0"></span>check\_itemscale 23

## Value

Invisibly returns the p-value of the test statistics. A p-value  $< 0.05$  indicates a significant difference in the variance between the groups.

## Note

There is also a plot ()[-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

## See Also

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-1)), [check\\_collinearity\(](#page-7-1)), [check\\_convergence\(](#page-10-1)), [check\\_heteroscedasticity\(](#page-20-1)), [check\\_model\(](#page-24-1)), [check\\_outliers\(](#page-30-1)), [check\\_overdispersion\(](#page-37-1)), [check\\_predictions\(](#page-39-1)), [check\\_singularity\(](#page-43-1)), [check\\_zeroinflation\(](#page-47-1))

#### Examples

```
model <<- lm(len ~ supp + dose, data = ToothGrowth)
check_homogeneity(model)
# plot results
if (require("see")) {
  result <- check_homogeneity(model)
  plot(result)
}
```
check\_itemscale *Describe Properties of Item Scales*

#### Description

Compute various measures of internal consistencies applied to (sub)scales, which items were extracted using parameters::principal\_components().

#### Usage

```
check_itemscale(x, factor_index = NULL)
```


#### Details

check\_itemscale() calculates various measures of internal consistencies, such as Cronbach's alpha, item difficulty or discrimination etc. on subscales which were built from several items. Subscales are retrieved from the results of [parameters::principal\\_components\(\)](#page-0-0), i.e. based on how many components were extracted from the PCA, check\_itemscale() retrieves those variables that belong to a component and calculates the above mentioned measures.

## Value

A list of data frames, with related measures of internal consistencies of each subscale.

#### Note

- *Item difficulty* should range between 0.2 and 0.8. Ideal value is p+(1-p)/2 (which mostly is between 0.5 and 0.8). See [item\\_difficulty\(\)](#page-58-1) for details.
- For *item discrimination*, acceptable values are 0.20 or higher; the closer to 1.00 the better. See [item\\_reliability\(\)](#page-61-1) for more details.
- In case the total *Cronbach's alpha* value is below the acceptable cut-off of 0.7 (mostly if an index has few items), the *mean inter-item-correlation* is an alternative measure to indicate acceptability. Satisfactory range lies between 0.2 and 0.4. See also [item\\_intercor\(\)](#page-60-1).

## References

• Briggs SR, Cheek JM (1986) The role of factor analysis in the development and evaluation of personality scales. Journal of Personality, 54(1), 106-148. doi: 10.1111/j.1467- 6494.1986.tb00391.x

#### Examples

```
# data generation from '?prcomp', slightly modified
C \leftarrow \text{chol}(S \leftarrow \text{toe}(0.9^{(0.15)}))set.seed(17)
X <- matrix(rnorm(1600), 100, 16)
Z <- X %*% C
pca <- parameters::principal_components(
  as.data.frame(Z),
  rotation = "varimax",
  n = 3)
pca
check_itemscale(pca)
# as data frame
check_itemscale(
  as.data.frame(Z),
  factor_index = parameters::closest_component(pca)
)
```
<span id="page-24-1"></span><span id="page-24-0"></span>

### Description

Visual check of various model assumptions (normality of residuals, normality of random effects, linear relationship, homogeneity of variance, multicollinearity).

## Usage

```
check_model(x, ...)
## Default S3 method:
check_model(
  x,
 panel = TRUE,
 check = "all",determined = TRUE,bandwidth = "nrd",
  type = "density",
  residual_type = NULL,
  show_dots = NULL,
  dot\_size = 2,
  line\_size = 0.8,
  title_size = 12,
  axis_title_size = base_size,
  base_size = 10,
  alpha = 0.2,
  dot\_alpha = 0.8,
  colors = c("#3aaf85", "#1b6ca8", "#cd201f"),
  theme = "see::theme_lucid",
  verbose = FALSE,
  ...
)
```




## Details

For Bayesian models from packages rstanarm or brms, models will be "converted" to their frequentist counterpart, using [bayestestR::bayesian\\_as\\_frequentist](https://easystats.github.io/bayestestR/reference/convert_bayesian_as_frequentist.html). A more advanced modelcheck for Bayesian models will be implemented at a later stage.

See also the related [vignette.](https://easystats.github.io/performance/articles/check_model.html)

#### check\_model 27

#### Value

The data frame that is used for plotting.

#### Posterior Predictive Checks

Posterior predictive checks can be used to look for systematic discrepancies between real and simulated data. It helps to see whether the type of model (distributional family) fits well to the data. See [check\\_predictions\(\)](#page-39-1) for further details.

#### Linearity Assumption

The plot Linearity checks the assumption of linear relationship. However, the spread of dots also indicate possible heteroscedasticity (i.e. non-constant variance, hence, the alias "ncv" for this plot), thus it shows if residuals have non-linear patterns. This plot helps to see whether predictors may have a non-linear relationship with the outcome, in which case the reference line may roughly indicate that relationship. A straight and horizontal line indicates that the model specification seems to be ok. But for instance, if the line would be U-shaped, some of the predictors probably should better be modeled as quadratic term. See [check\\_heteroscedasticity\(\)](#page-20-1) for further details.

Some caution is needed when interpreting these plots. Although these plots are helpful to check model assumptions, they do not necessarily indicate so-called "lack of fit", e.g. missed non-linear relationships or interactions. Thus, it is always recommended to also look at [effect plots, including](https://strengejacke.github.io/ggeffects/articles/introduction_partial_residuals.html) [partial residuals.](https://strengejacke.github.io/ggeffects/articles/introduction_partial_residuals.html)

#### Homogeneity of Variance

This plot checks the assumption of equal variance (homoscedasticity). The desired pattern would be that dots spread equally above and below a straight, horizontal line and show no apparent deviation.

## Influential Observations

This plot is used to identify influential observations. If any points in this plot fall outside of Cook's distance (the dashed lines) then it is considered an influential observation. See [check\\_outliers\(\)](#page-30-1) for further details.

## Multicollinearity

This plot checks for potential collinearity among predictors. In a nutshell, multicollinearity means that once you know the effect of one predictor, the value of knowing the other predictor is rather low. Multicollinearity might arise when a third, unobserved variable has a causal effect on each of the two predictors that are associated with the outcome. In such cases, the actual relationship that matters would be the association between the unobserved variable and the outcome. See [check\\_collinearity\(\)](#page-7-1) for further details.

#### Normality of Residuals

This plot is used to determine if the residuals of the regression model are normally distributed. Usually, dots should fall along the line. If there is some deviation (mostly at the tails), this indicates that the model doesn't predict the outcome well for that range that shows larger deviations from the line. For generalized linear models and when residual\_type = "normal", a half-normal Q-Q plot of the absolute value of the standardized deviance residuals is shown, however, the interpretation of the plot remains the same. See [check\\_normality\(\)](#page-29-1) for further details. Usually, for generalized linear (mixed) models, a test for uniformity of residuals based on simulated residuals is conducted (see next section).

#### Uniformity of Residuals

Fore non-Gaussian models, when residual\_type = "simulated" (the default for generalized linear (mixed) models), residuals are not expected to be normally distributed. In this case, the created Q-Q plot checks the uniformity of residuals. The interpretation of the plot is the same as for the normal Q-Q plot. See [simulate\\_residuals\(\)](#page-105-1) and [check\\_residuals\(\)](#page-42-1) for further details.

#### Overdispersion

For count models, an *overdispersion plot* is shown. Overdispersion occurs when the observed variance is higher than the variance of a theoretical model. For Poisson models, variance increases with the mean and, therefore, variance usually (roughly) equals the mean value. If the variance is much higher, the data are "overdispersed". See [check\\_overdispersion\(\)](#page-37-1) for further details.

## Binned Residuals

For models from binomial families, a *binned residuals plot* is shown. Binned residual plots are achieved by cutting the the data into bins and then plotting the average residual versus the average fitted value for each bin. If the model were true, one would expect about 95% of the residuals to fall inside the error bounds. See [binned\\_residuals\(\)](#page-3-1) for further details.

#### Residuals for (Generalized) Linear Models

Plots that check the homogeneity of variance use standardized Pearson's residuals for generalized linear models, and standardized residuals for linear models. The plots for the normality of residuals (with overlayed normal curve) and for the linearity assumption use the default residuals for lm and glm (which are deviance residuals for glm). The Q-Q plots use simulated residuals (see [simulate\\_residuals\(\)](#page-105-1)) for non-Gaussian models and standardized residuals for linear models.

#### Troubleshooting

For models with many observations, or for more complex models in general, generating the plot might become very slow. One reason might be that the underlying graphic engine becomes slow for plotting many data points. In such cases, setting the argument show\_dots = FALSE might help. Furthermore, look at the check argument and see if some of the model checks could be skipped, which also increases performance.

## **Note**

This function just prepares the data for plotting. To create the plots, see needs to be installed. Furthermore, this function suppresses all possible warnings. In case you observe suspicious plots, please refer to the dedicated functions (like check\_collinearity(), check\_normality() etc.) to get informative messages and warnings.

## <span id="page-28-0"></span>check\_multimodal 29

## See Also

```
Other functions to check model assumptions and and assess model quality: check_autocorrelation(),
check_collinearity(), check_convergence(), check_heteroscedasticity(), check_homogeneity(),
check_outliers(), check_overdispersion(), check_predictions(), check_singularity(),
check_zeroinflation()
```
## Examples

```
m \le -1m(mpg ~ wt + cyl + gear + disp, data = mtcars)
check_model(m)
data(sleepstudy, package = "lme4")
m <- lme4::lmer(Reaction ~ Days + (Days | Subject), sleepstudy)
check_model(m, panel = FALSE)
```
check\_multimodal *Check if a distribution is unimodal or multimodal*

#### **Description**

For univariate distributions (one-dimensional vectors), this functions performs a Ameijeiras-Alonso et al. (2018) excess mass test. For multivariate distributions (data frames), it uses mixture modelling. However, it seems that it always returns a significant result (suggesting that the distribution is multimodal). A better method might be needed here.

### Usage

```
check_multimodal(x, ...)
```
#### Arguments



## References

• Ameijeiras-Alonso, J., Crujeiras, R. M., and Rodríguez-Casal, A. (2019). Mode testing, critical bandwidth and excess mass. Test, 28(3), 900-919.

## Examples

```
# Univariate
x <- rnorm(1000)
check_multimodal(x)
x <- c(rnorm(1000), rnorm(1000, 2))
check_multimodal(x)
# Multivariate
m <- data.frame(
 x = rnorm(200),
  y = rbeta(200, 2, 1)
)
plot(m$x, m$y)
check_multimodal(m)
m <- data.frame(
  x = c(rnorm(100), rnorm(100, 4)),y = c(rbeta(100, 2, 1), rbeta(100, 1, 4))
)
plot(m$x, m$y)
check_multimodal(m)
```
<span id="page-29-1"></span>check\_normality *Check model for (non-)normality of residuals.*

## Description

Check model for (non-)normality of residuals.

## Usage

```
check_normality(x, ...)
```

```
## S3 method for class 'merMod'
check_normality(x, effects = c("fixed", "random"), ...)
```


<span id="page-29-0"></span>

## <span id="page-30-0"></span>check\_outliers 31

#### Details

check\_normality() calls stats::shapiro.test and checks the standardized residuals (or studentized residuals for mixed models) for normal distribution. Note that this formal test almost always yields significant results for the distribution of residuals and visual inspection (e.g. Q-Q plots) are preferable. For generalized linear models, no formal statistical test is carried out. Rather, there's only a plot() method for GLMs. This plot shows a half-normal Q-Q plot of the absolute value of the standardized deviance residuals is shown (in line with changes in plot.lm() for R 4.3+).

## Value

The p-value of the test statistics. A p-value  $< 0.05$  indicates a significant deviation from normal distribution.

#### **Note**

For mixed-effects models, studentized residuals, and *not* standardized residuals, are used for the test. There is also a plot()[-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

#### See Also

[see::plot.see\\_check\\_normality\(\)](#page-0-0) for options to customize the plot.

#### Examples

```
m \ll - \ln(mpg \sim wt + cyl + gear + disp, data = mtcars)check_normality(m)
```

```
# plot results
x <- check_normality(m)
plot(x)
```
# QQ-plot plot(check\_normality(m), type = "qq")

# PP-plot plot(check\_normality(m), type = "pp")

<span id="page-30-1"></span>check\_outliers *Outliers detection (check for influential observations)*

## Description

Checks for and locates influential observations (i.e., "outliers") via several distance and/or clustering methods. If several methods are selected, the returned "Outlier" vector will be a composite outlier score, made of the average of the binary (0 or 1) results of each method. It represents the probability of each observation of being classified as an outlier by at least one method. The decision rule used by default is to classify as outliers observations which composite outlier score is superior or equal to 0.5 (i.e., that were classified as outliers by at least half of the methods). See the Details section below for a description of the methods.

#### Usage

```
check_outliers(x, ...)
## Default S3 method:
check_outliers(
  x,
 method = c("cook", "pareto"),threshold = NULL,
  ID = NULL,verbose = TRUE,
  ...
)
## S3 method for class 'numeric'
check_utliers(x, method = "zscore_robust", threshold = NULL, ...)
## S3 method for class 'data.frame'
check_outliers(x, method = "mahalanobis", threshold = NULL, ID = NULL, ...)## S3 method for class 'performance_simres'
check_outliers(
  x,
  type = "default",
  iterations = 100,
  alternative = "two.sided",
  ...
)
```




#### Details

Outliers can be defined as particularly influential observations. Most methods rely on the computation of some distance metric, and the observations greater than a certain threshold are considered outliers. Importantly, outliers detection methods are meant to provide information to consider for the researcher, rather than to be an automatized procedure which mindless application is a substitute for thinking.

An example sentence for reporting the usage of the composite method could be:

*"Based on a composite outlier score (see the 'check\_outliers' function in the 'performance' R package; Lüdecke et al., 2021) obtained via the joint application of multiple outliers detection algorithms (Z-scores, Iglewicz, 1993; Interquartile range (IQR); Mahalanobis distance, Cabana, 2019; Robust Mahalanobis distance, Gnanadesikan and Kettenring, 1972; Minimum Covariance Determinant, Leys et al., 2018; Invariant Coordinate Selection, Archimbaud et al., 2018; OPTICS, Ankerst et al., 1999; Isolation Forest, Liu et al. 2008; and Local Outlier Factor, Breunig et al., 2000), we excluded n participants that were classified as outliers by at least half of the methods used."*

## Value

A logical vector of the detected outliers with a nice printing method: a check (message) on whether outliers were detected or not. The information on the distance measure and whether or not an observation is considered as outlier can be recovered with the [as.data.frame](#page-0-0) function. Note that the function will (silently) return a vector of FALSE for non-supported data types such as character strings.

#### Model-specific methods

• Cook's Distance: Among outlier detection methods, Cook's distance and leverage are less common than the basic Mahalanobis distance, but still used. Cook's distance estimates the variations in regression coefficients after removing each observation, one by one (Cook, 1977). Since Cook's distance is in the metric of an F distribution with p and n-p degrees of freedom, the median point of the quantile distribution can be used as a cut-off (Bollen, 1985). A common approximation or heuristic is to use 4 divided by the numbers of observations, which usually corresponds to a lower threshold (i.e., more outliers are detected). This only works for frequentist models. For Bayesian models, see pareto.

• Pareto: The reliability and approximate convergence of Bayesian models can be assessed using the estimates for the shape parameter k of the generalized Pareto distribution. If the estimated tail shape parameter k exceeds 0.5, the user should be warned, although in practice the authors of the loo package observed good performance for values of k up to 0.7 (the default threshold used by performance).

#### Univariate methods

- Z-scores ("zscore", "zscore\_robust"): The Z-score, or standard score, is a way of describing a data point as deviance from a central value, in terms of standard deviations from the mean ("zscore") or, as it is here the case ("zscore\_robust") by default (Iglewicz, 1993), in terms of Median Absolute Deviation (MAD) from the median (which are robust measures of dispersion and centrality). The default threshold to classify outliers is 1.959 (threshold  $=$  list("zscore" = 1.959)), corresponding to the 2.5% (qnorm(0.975)) most extreme observations (assuming the data is normally distributed). Importantly, the Z-score method is univariate: it is computed column by column. If a data frame is passed, the Z-score is calculated for each variable separately, and the maximum (absolute) Z-score is kept for each observations. Thus, all observations that are extreme on at least one variable might be detected as outliers. Thus, this method is not suited for high dimensional data (with many columns), returning too liberal results (detecting many outliers).
- IQR ("iqr"): Using the IQR (interquartile range) is a robust method developed by John Tukey, which often appears in box-and-whisker plots (e.g., in [ggplot2::geom\\_boxplot\)](#page-0-0). The interquartile range is the range between the first and the third quartiles. Tukey considered as outliers any data point that fell outside of either 1.5 times (the default threshold is 1.7) the IQR below the first or above the third quartile. Similar to the Z-score method, this is a univariate method for outliers detection, returning outliers detected for at least one column, and might thus not be suited to high dimensional data. The distance score for the IQR is the absolute deviation from the median of the upper and lower IQR thresholds. Then, this value is divided by the IQR threshold, to "standardize" it and facilitate interpretation.
- CI ("ci", "eti", "hdi", "bci"): Another univariate method is to compute, for each variable, some sort of "confidence" interval and consider as outliers values lying beyond the edges of that interval. By default, "ci" computes the Equal-Tailed Interval ("eti"), but other types of intervals are available, such as Highest Density Interval ("hdi") or the Bias Corrected and Accelerated Interval ("bci"). The default threshold is 0.95, considering as outliers all observations that are outside the 95% CI on any of the variable. See [bayestestR::ci\(\)](#page-0-0) for more details about the intervals. The distance score for the CI methods is the absolute deviation from the median of the upper and lower CI thresholds. Then, this value is divided by the difference between the upper and lower CI bounds divided by two, to "standardize" it and facilitate interpretation.

#### Multivariate methods

• Mahalanobis Distance: Mahalanobis distance (Mahalanobis, 1930) is often used for multivariate outliers detection as this distance takes into account the shape of the observations. The default threshold is often arbitrarily set to some deviation (in terms of SD or MAD) from the mean (or median) of the Mahalanobis distance. However, as the Mahalanobis distance can be approximated by a Chi squared distribution (Rousseeuw and Van Zomeren, 1990), we can use the alpha quantile of the chi-square distribution with k degrees of freedom (k being the number of columns). By default, the alpha threshold is set to 0.025 (corresponding to the 2.5\) Cabana, 2019). This criterion is a natural extension of the median plus or minus a coefficient times the MAD method (Leys et al., 2013).

- Robust Mahalanobis Distance: A robust version of Mahalanobis distance using an Orthogonalized Gnanadesikan-Kettenring pairwise estimator (Gnanadesikan and Kettenring, 1972). Requires the bigutilsr package. See the [bigutilsr::dist\\_ogk\(\)](#page-0-0) function.
- Minimum Covariance Determinant (MCD): Another robust version of Mahalanobis. Leys et al. (2018) argue that Mahalanobis Distance is not a robust way to determine outliers, as it uses the means and covariances of all the data - including the outliers - to determine individual difference scores. Minimum Covariance Determinant calculates the mean and covariance matrix based on the most central subset of the data (by default,  $66\$ ) is deemed to be a more robust method of identifying and removing outliers than regular Mahalanobis distance. This method has a percentage\_central argument that allows specifying the breakdown point (0.75, the default, is recommended by Leys et al. 2018, but a commonly used alternative is 0.50).
- Invariant Coordinate Selection (ICS): The outlier are detected using ICS, which by default uses an alpha threshold of 0.025 (corresponding to the 2.5\ value for outliers classification. Refer to the help-file of [ICSOutlier::ics.outlier\(\)](#page-0-0) to get more details about this procedure. Note that method = "ics" requires both **ICS** and **ICSOutlier** to be installed, and that it takes some time to compute the results. You can speed up computation time using parallel computing. Set the number of cores to use with options(mc.cores = 4) (for example).
- **OPTICS:** The Ordering Points To Identify the Clustering Structure (OPTICS) algorithm (Ankerst et al., 1999) is using similar concepts to DBSCAN (an unsupervised clustering technique that can be used for outliers detection). The threshold argument is passed as minPts, which corresponds to the minimum size of a cluster. By default, this size is set at 2 times the number of columns (Sander et al., 1998). Compared to the other techniques, that will always detect several outliers (as these are usually defined as a percentage of extreme values), this algorithm functions in a different manner and won't always detect outliers. Note that method = "optics" requires the **dbscan** package to be installed, and that it takes some time to compute the results.
- Local Outlier Factor: Based on a K nearest neighbors algorithm, LOF compares the local density of a point to the local densities of its neighbors instead of computing a distance from the center (Breunig et al., 2000). Points that have a substantially lower density than their neighbors are considered outliers. A LOF score of approximately 1 indicates that density around the point is comparable to its neighbors. Scores significantly larger than 1 indicate outliers. The default threshold of 0.025 will classify as outliers the observations located at qnorm(1-0.025)  $\star$  SD) of the log-transformed LOF distance. Requires the **dbscan** package.

#### Methods for simulated residuals

The approach for detecting outliers based on simulated residuals differs from the traditional methods and may not be detecting outliers as expected. Literally, this approach compares observed to simulated values. However, we do not know the deviation of the observed data to the model expectation, and thus, the term "outlier" should be taken with a grain of salt. It refers to "simulation outliers". Basically, the comparison tests whether on observed data point is outside the simulated range. It is strongly recommended to read the related documentations in the **DHARMa** package, e.g. ?DHARMa::testOutliers.

#### Threshold specification

Default thresholds are currently specified as follows:

```
list(
  \text{zscore} = \text{stats} : \text{qnorm}(p = 1 - 0.001 / 2),\text{zscore\_robust} = \text{stats::qnorm}(p = 1 - 0.001 / 2),iqr = 1.7,
  ci = 1 - 0.001,
  eti = 1 - 0.001,hdi = 1 - 0.001,bci = 1 - 0.001,
  \text{cook} = \text{stats}:qf(\emptyset.5, \text{ ncol}(x), \text{ nrow}(x) - \text{ncol}(x)),pareto = 0.7,
  mahalanobis = stats::qchisq(p = 1 - 0.001, df = ncol(x)),mahalanobis_robust = stats::qchi(q) = 1 - 0.001, df = ncol(x)),
  mcd = stats::qchisq(p = 1 - 0.001, df = ncol(x)),ics = 0.001,optics = 2 * \text{ncol}(x),
  lof = 0.001)
```
#### Meta-analysis models

For meta-analysis models (e.g. objects of class rma from the *metafor* package or metagen from package *meta*), studies are defined as outliers when their confidence interval lies outside the confidence interval of the pooled effect.

#### Note

There is also a plot()[-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/) Please note that the range of the distance-values along the y-axis is re-scaled to range from 0 to 1.

## References

- Archimbaud, A., Nordhausen, K., and Ruiz-Gazen, A. (2018). ICS for multivariate outlier detection with application to quality control. *Computational Statistics and Data Analysis*, *128*, 184-199. [doi:10.1016/j.csda.2018.06.011](https://doi.org/10.1016/j.csda.2018.06.011)
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- Liu, F. T., Ting, K. M., and Zhou, Z. H. (2008, December). Isolation forest. In 2008 Eighth IEEE International Conference on Data Mining (pp. 413-422). IEEE.
- Lüdecke, D., Ben-Shachar, M. S., Patil, I., Waggoner, P., and Makowski, D. (2021). performance: An R package for assessment, comparison and testing of statistical models. *Journal of Open Source Software*, *6*(60), 3139. [doi:10.21105/joss.03139](https://doi.org/10.21105/joss.03139)
- Thériault, R., Ben-Shachar, M. S., Patil, I., Lüdecke, D., Wiernik, B. M., and Makowski, D. (2023). Check your outliers! An introduction to identifying statistical outliers in R with easystats. *Behavior Research Methods*, 1-11. [doi:10.3758/s1342802402356w](https://doi.org/10.3758/s13428-024-02356-w)
- Rousseeuw, P. J., and Van Zomeren, B. C. (1990). Unmasking multivariate outliers and leverage points. *Journal of the American Statistical association*, *85*(411), 633-639.

## See Also

[see::plot.see\\_check\\_outliers\(\)](#page-0-0) for options to customize the plot.

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-0)), [check\\_collinearity\(](#page-7-0)), [check\\_convergence\(](#page-10-0)), [check\\_heteroscedasticity\(](#page-20-0)), [check\\_homogeneity\(](#page-21-0)), [check\\_model\(](#page-24-0)), [check\\_overdispersion\(](#page-37-0)), [check\\_predictions\(](#page-39-0)), [check\\_singularity\(](#page-43-0)), [check\\_zeroinflation\(](#page-47-0))

```
data \leq mtcars # Size nrow(data) = 32
```

```
# For single variables ------------------------------------------------------
outliers_list <- check_outliers(data$mpg) # Find outliers
outliers_list # Show the row index of the outliers
as.numeric(outliers_list) # The object is a binary vector...
filtered_data <- data[!outliers_list, ] # And can be used to filter a data frame
nrow(filtered_data) # New size, 28 (4 outliers removed)
# Find all observations beyond +/- 2 SD
check_outliers(data$mpg, method = "zscore", threshold = 2)
# For dataframes ------------------------------------------------------
check_outliers(data) # It works the same way on data frames
# You can also use multiple methods at once
outliers_list <- check_outliers(data, method = c(
  "mahalanobis",
  "iqr",
  "zscore"
))
outliers_list
```

```
# Using `as.data.frame()`, we can access more details!
outliers_info <- as.data.frame(outliers_list)
head(outliers_info)
outliers_info$Outlier # Including the probability of being an outlier
# And we can be more stringent in our outliers removal process
filtered_data <- data[outliers_info$Outlier < 0.1, ]
# We can run the function stratified by groups using `{datawizard}` package:
group_iris <- datawizard::data_group(iris, "Species")
check_outliers(group_iris)
# nolint start
# nolint end
# You can also run all the methods
check_outliers(data, method = "all", verbose = FALSE)
# For statistical models ---------------------------------------------
# select only mpg and disp (continuous)
mt1 <- mtcars[, c(1, 3, 4)]
# create some fake outliers and attach outliers to main df
mt2 <- rbind(mt1, data.frame(
  mpg = c(37, 40), disp = c(300, 400),
  hp = c(110, 120)))
# fit model with outliers
model \leq lm(disp \sim mpg + hp, data = mt2)
outliers_list <- check_outliers(model)
plot(outliers_list)
insight::get_data(model)[outliers_list, ] # Show outliers data
```
<span id="page-37-0"></span>check\_overdispersion *Check overdispersion (and underdispersion) of GL(M)M's*

## Description

check\_overdispersion() checks generalized linear (mixed) models for overdispersion (and underdispersion).

## Usage

```
check_overdispersion(x, ...)
## S3 method for class 'performance_simres'
check_overdispersion(x, alternative = c("two.sided", "less", "greater"), ...)
```
#### **Arguments**



#### Details

Overdispersion occurs when the observed variance is higher than the variance of a theoretical model. For Poisson models, variance increases with the mean and, therefore, variance usually (roughly) equals the mean value. If the variance is much higher, the data are "overdispersed". A less common case is underdispersion, where the variance is much lower than the mean.

## Value

A list with results from the overdispersion test, like chi-squared statistics, p-value or dispersion ratio.

#### Interpretation of the Dispersion Ratio

If the dispersion ratio is close to one, a Poisson model fits well to the data. Dispersion ratios larger than one indicate overdispersion, thus a negative binomial model or similar might fit better to the data. Dispersion ratios much smaller than one indicate underdispersion. A p-value < .05 indicates either overdispersion or underdispersion (the first being more common).

#### Overdispersion in Poisson Models

For Poisson models, the overdispersion test is based on the code from *Gelman and Hill (2007), page 115*.

#### Overdispersion in Negative Binomial or Zero-Inflated Models

For negative binomial (mixed) models or models with zero-inflation component, the overdispersion test is based simulated residuals (see [simulate\\_residuals\(\)](#page-105-0)).

## Overdispersion in Mixed Models

For merMod- and glmmTMB-objects, check\_overdispersion() is based on the code in the [GLMM](http://bbolker.github.io/mixedmodels-misc/glmmFAQ.html) [FAQ,](http://bbolker.github.io/mixedmodels-misc/glmmFAQ.html) section *How can I deal with overdispersion in GLMMs?*. Note that this function only returns an *approximate* estimate of an overdispersion parameter. Using this approach would be inaccurate for zero-inflated or negative binomial mixed models (fitted with glmmTMB), thus, in such cases, the overdispersion test is based on [simulate\\_residuals\(\)](#page-105-0) (which is identical to check\_overdispersion(simulate\_residuals(model))).

#### How to fix Overdispersion

Overdispersion can be fixed by either modeling the dispersion parameter, or by choosing a different distributional family (like Quasi-Poisson, or negative binomial, see *Gelman and Hill (2007), pages 115-116*).

#### Tests based on simulated residuals

For certain models, resp. model from certain families, tests are based on simulated residuals (see [simulate\\_residuals\(\)](#page-105-0)). These are usually more accurate for testing such models than the traditionally used Pearson residuals. However, when simulating from more complex models, such as mixed models or models with zero-inflation, there are several important considerations. Arguments specified in ... are passed to [simulate\\_residuals\(\)](#page-105-0), which relies on [DHARMa::simulateResiduals\(\)](#page-0-0) (and therefore, arguments in ... are passed further down to *DHARMa*). The defaults in DHARMa are set on the most conservative option that works for all models. However, in many cases, the help advises to use different settings in particular situations or for particular models. It is recommended to read the 'Details' in ?DHARMa::simulateResiduals closely to understand the implications of the simulation process and which arguments should be modified to get the most accurate results.

#### References

- Bolker B et al. (2017): [GLMM FAQ.](http://bbolker.github.io/mixedmodels-misc/glmmFAQ.html)
- Gelman, A., and Hill, J. (2007). Data analysis using regression and multilevel/hierarchical models. Cambridge; New York: Cambridge University Press.

#### See Also

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-0)), [check\\_collinearity\(](#page-7-0)), [check\\_convergence\(](#page-10-0)), [check\\_heteroscedasticity\(](#page-20-0)), [check\\_homogeneity\(](#page-21-0)), [check\\_model\(](#page-24-0)), [check\\_outliers\(](#page-30-0)), [check\\_predictions\(](#page-39-0)), [check\\_singularity\(](#page-43-0)), [check\\_zeroinflation\(](#page-47-0))

## Examples

```
data(Salamanders, package = "glmmTMB")
m \leq -g \ln(\text{count} \leq \text{sp} + \text{mined}, \text{ family} = \text{poisson}, \text{ data} = \text{Salamanders})check_overdispersion(m)
```
<span id="page-39-0"></span>check\_predictions *Posterior predictive checks*

#### Description

Posterior predictive checks mean "simulating replicated data under the fitted model and then comparing these to the observed data" (*Gelman and Hill, 2007, p. 158*). Posterior predictive checks can be used to "look for systematic discrepancies between real and simulated data" (*Gelman et al. 2014, p. 169*).

performance provides posterior predictive check methods for a variety of frequentist models (e.g., lm, merMod, glmmTMB, ...). For Bayesian models, the model is passed to [bayesplot::pp\\_check\(\)](#page-0-0).

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## Usage

```
check_predictions(object, ...)
## Default S3 method:
check_predictions(
 object,
 iterations = 50,
 check_range = FALSE,
 re_formula = NULL,
 bandwidth = "nrd",
  type = "density",
  verbose = TRUE,
  ...
\mathcal{L}posterior_predictive_check(object, ...)
```
check\_posterior\_predictions(object, ...)

## Arguments



## Details

An example how posterior predictive checks can also be used for model comparison is Figure 6 from *Gabry et al. 2019, Figure 6*.

The model shown in the right panel (b) can simulate new data that are more similar to the observed outcome than the model in the left panel (a). Thus, model (b) is likely to be preferred over model (a).

## Value

A data frame of simulated responses and the original response vector.

## Note

Every model object that has a simulate()-method should work with check\_predictions(). On R 3.6.0 and higher, if **bayesplot** (or a package that imports **bayesplot** such as **rstanarm** or **brms**) is loaded, pp\_check() is also available as an alias for check\_predictions().

#### References

- Gabry, J., Simpson, D., Vehtari, A., Betancourt, M., and Gelman, A. (2019). Visualization in Bayesian workflow. Journal of the Royal Statistical Society: Series A (Statistics in Society), 182(2), 389–402. https://doi.org/10.1111/rssa.12378
- Gelman, A., and Hill, J. (2007). Data analysis using regression and multilevel/hierarchical models. Cambridge; New York: Cambridge University Press.
- Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., and Rubin, D. B. (2014). Bayesian data analysis. (Third edition). CRC Press.
- Gelman, A., Hill, J., and Vehtari, A. (2020). Regression and Other Stories. Cambridge University Press.

## See Also

[simulate\\_residuals\(\)](#page-105-0) and [check\\_residuals\(\)](#page-42-0). See also [see::print.see\\_performance\\_pp\\_check\(\)](#page-0-0) for options to customize the plot.

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-0)), [check\\_collinearity\(](#page-7-0)), [check\\_convergence\(](#page-10-0)), [check\\_heteroscedasticity\(](#page-20-0)), [check\\_homogeneity\(](#page-21-0)), [check\\_model\(](#page-24-0)), [check\\_outliers\(](#page-30-0)), [check\\_overdispersion\(](#page-37-0)), [check\\_singularity\(](#page-43-0)), [check\\_zeroinflation\(](#page-47-0))

```
# linear model
model \leq lm(mpg \sim disp, data = mtcars)
check_predictions(model)
# discrete/integer outcome
set.seed(99)
d <- iris
d$skewed <- rpois(150, 1)
model \leq glm(skewed ~ Species + Petal.Length + Petal.Width,
 family = poisson(),
 data = d
\lambdacheck_predictions(model, type = "discrete_both")
```
<span id="page-42-0"></span>

#### **Description**

check\_residuals() checks generalized linear (mixed) models for uniformity of randomized quantile residuals, which can be used to identify typical model misspecification problems, such as over/underdispersion, zero-inflation, and residual spatial and temporal autocorrelation.

#### Usage

```
check_residuals(x, ...)
## Default S3 method:
check_residuals(x, alternative = c("two.sided", "less", "greater"), ...)
```
#### Arguments



#### Details

Uniformity of residuals is checked using a Kolmogorov-Smirnov test. There is a plot() method to visualize the distribution of the residuals. The test for uniformity basically tests to which extent the observed values deviate from the model expectations (i.e. simulated values). In this sense, the check\_residuals() function has similar goals like [check\\_predictions\(\)](#page-39-0).

#### Value

The p-value of the test statistics.

#### Tests based on simulated residuals

For certain models, resp. model from certain families, tests like [check\\_zeroinflation\(\)](#page-47-0) or [check\\_overdispersion\(\)](#page-37-0) are based on simulated residuals. These are usually more accurate for such tests than the traditionally used Pearson residuals. However, when simulating from more complex models, such as mixed models or models with zero-inflation, there are several important considerations. simulate\_residuals() relies on [DHARMa::simulateResiduals\(\)](#page-0-0), and additional arguments specified in ... are passed further down to that function. The defaults in DHARMa are set on the most conservative option that works for all models. However, in many cases, the help advises to use different settings in particular situations or for particular models. It is recommended to read the 'Details' in ?DHARMa::simulateResiduals closely to understand the implications of the simulation process and which arguments should be modified to get the most accurate results.

## See Also

```
simulate_residuals(), check_zeroinflation(), check_overdispersion() and check_predictions().
see::plot.see_performance_simres() for options to customize the plot.
```
#### Examples

```
dat <- DHARMa::createData(sampleSize = 100, overdispersion = 0.5, family = poisson())
m \leq glm(observedResponse \sim Environment1, family = poisson(), data = dat)
res <- simulate_residuals(m)
check_residuals(res)
```
<span id="page-43-0"></span>check\_singularity *Check mixed models for boundary fits*

## **Description**

Check mixed models for boundary fits.

## Usage

```
check_singularity(x, tolerance = 1e-05, ...)
```
#### Arguments



#### Details

If a model is "singular", this means that some dimensions of the variance-covariance matrix have been estimated as exactly zero. This often occurs for mixed models with complex random effects structures.

"While singular models are statistically well defined (it is theoretically sensible for the true maximum likelihood estimate to correspond to a singular fit), there are real concerns that (1) singular fits correspond to overfitted models that may have poor power; (2) chances of numerical problems and mis-convergence are higher for singular models (e.g. it may be computationally difficult to compute profile confidence intervals for such models); (3) standard inferential procedures such as Wald statistics and likelihood ratio tests may be inappropriate." (*lme4 Reference Manual*)

There is no gold-standard about how to deal with singularity and which random-effects specification to choose. Beside using fully Bayesian methods (with informative priors), proposals in a frequentist framework are:

• avoid fitting overly complex models, such that the variance-covariance matrices can be estimated precisely enough (*Matuschek et al. 2017*)

- use some form of model selection to choose a model that balances predictive accuracy and overfitting/type I error (*Bates et al. 2015*, *Matuschek et al. 2017*)
- "keep it maximal", i.e. fit the most complex model consistent with the experimental design, removing only terms required to allow a non-singular fit (*Barr et al. 2013*)
- since version 1.1.9, the glmmTMB package allows to use priors in a frequentist framework, too. One recommendation is to use a Gamma prior (*Chung et al. 2013*). The mean may vary from 1 to very large values (like 1e8), and the shape parameter should be set to a value of 2.5. You can then update() your model with the specified prior. In glmmTMB, the code would look like this:

```
# "model" is an object of class gmmmTMB
prior <- data.frame(
 prior = "gamma(1, 2.5)", # mean can be 1, but even 1e8
 class = "ranef" \# for random effects\lambdamodel_with_priors <- update(model, priors = prior)
```
Large values for the mean parameter of the Gamma prior have no large impact on the random effects variances in terms of a "bias". Thus, if 1 doesn't fix the singular fit, you can safely try larger values.

Note the different meaning between singularity and convergence: singularity indicates an issue with the "true" best estimate, i.e. whether the maximum likelihood estimation for the variance-covariance matrix of the random effects is positive definite or only semi-definite. Convergence is a question of whether we can assume that the numerical optimization has worked correctly or not.

#### Value

TRUE if the model fit is singular.

## **References**

- Bates D, Kliegl R, Vasishth S, Baayen H. Parsimonious Mixed Models. arXiv:1506.04967, June 2015.
- Barr DJ, Levy R, Scheepers C, Tily HJ. Random effects structure for confirmatory hypothesis testing: Keep it maximal. Journal of Memory and Language, 68(3):255-278, April 2013.
- Chung Y, Rabe-Hesketh S, Dorie V, Gelman A, and Liu J. 2013. "A Nondegenerate Penalized Likelihood Estimator for Variance Parameters in Multilevel Models." Psychometrika 78 (4): 685–709. [doi:10.1007/s1133601393282](https://doi.org/10.1007/s11336-013-9328-2)
- Matuschek H, Kliegl R, Vasishth S, Baayen H, Bates D. Balancing type I error and power in linear mixed models. Journal of Memory and Language, 94:305-315, 2017.
- lme4 Reference Manual, <https://cran.r-project.org/package=lme4>

#### See Also

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-0)), [check\\_collinearity\(](#page-7-0)), [check\\_convergence\(](#page-10-0)), [check\\_heteroscedasticity\(](#page-20-0)), [check\\_homogeneity\(](#page-21-0)), [check\\_model\(](#page-24-0)), [check\\_outliers\(](#page-30-0)), [check\\_overdispersion\(](#page-37-0)), [check\\_predictions\(](#page-39-0)), [check\\_zeroinflation\(](#page-47-0))

## Examples

```
data(sleepstudy, package = "lme4")
set.seed(123)
sleepstudy$mygrp <- sample(1:5, size = 180, replace = TRUE)
sleepstudy$mysubgrp <- NA
for (i in 1:5) {
  filter_group <- sleepstudy$mygrp == i
  sleepstudy$mysubgrp[filter_group] <-
    sample(1:30, size = sum(filter_group), replace = TRUE)
}
model <- lme4::lmer(
  Reaction \sim Days + (1 | mygrp / mysubgrp) + (1 | Subject),
  data = sleepstudy
\lambdacheck_singularity(model)
## Not run:
# Fixing singularity issues using priors in glmmTMB
# Example taken from `vignette("priors", package = "glmmTMB")`
dat <- readRDS(system.file(
  "vignette_data",
  "gophertortoise.rds",
  package = "glmmTMB"
))
model <- glmmTMB::glmmTMB(
  shells \sim prev + offset(log(Area)) + factor(year) + (1 | Site),
  family = poisson,
  data = dat
)
# singular fit
check_singularity(model)
# impose Gamma prior on random effects parameters
prior <- data.frame(
  prior = "gamma(1, 2.5)", # mean can be 1, but even 1e8
 class = "ranef" # for random effects
\lambdamodel_with_priors <- update(model, priors = prior)
# no singular fit
check_singularity(model_with_priors)
## End(Not run)
```
check\_sphericity *Check model for violation of sphericity*

## check\_symmetry 47

## Description

Check model for violation of sphericity. For [Bartlett's Test of Sphericity](#page-17-0) (used for correlation matrices and factor analyses), see [check\\_sphericity\\_bartlett.](#page-17-1)

#### Usage

```
check_sphericity(x, ...)
```
## Arguments



## Value

Invisibly returns the p-values of the test statistics. A p-value  $< 0.05$  indicates a violation of sphericity.

## Examples

```
data(Soils, package = "carData")
soils.mod <- lm(
  cbind(pH, N, Dens, P, Ca, Mg, K, Na, Conduc) ~ Block + Contour * Depth,
  data = Soils
\mathcal{L}check_sphericity(Manova(soils.mod))
```
check\_symmetry *Check distribution symmetry*

## Description

Uses Hotelling and Solomons test of symmetry by testing if the standardized nonparametric skew  $(\frac{(Mean - Median)}{SD})$  is different than 0.

This is an underlying assumption of Wilcoxon signed-rank test.

## Usage

check\_symmetry(x, ...)

## Arguments



## Examples

```
V <- suppressWarnings(wilcox.test(mtcars$mpg))
check_symmetry(V)
```
<span id="page-47-0"></span>check\_zeroinflation *Check for zero-inflation in count models*

## Description

check\_zeroinflation() checks whether count models are over- or underfitting zeros in the outcome.

## Usage

```
check_zeroinflation(x, ...)
## Default S3 method:
check\_zeroinflation(x, tolerance = 0.05, ...)## S3 method for class 'performance_simres'
check_zeroinflation(
 x,
  tolerance = 0.1,
  alternative = c("two.sided", "less", "greater"),
  ...
)
```
## **Arguments**



## Details

If the amount of observed zeros is larger than the amount of predicted zeros, the model is underfitting zeros, which indicates a zero-inflation in the data. In such cases, it is recommended to use negative binomial or zero-inflated models.

In case of negative binomial models, models with zero-inflation component, or hurdle models, the results from check\_zeroinflation() are based on [simulate\\_residuals\(\)](#page-105-0), i.e. check\_zeroinflation(simulate\_residuals()) is internally called if necessary.

#### Value

A list with information about the amount of predicted and observed zeros in the outcome, as well as the ratio between these two values.

## Tests based on simulated residuals

For certain models, resp. model from certain families, tests are based on simulated residuals (see [simulate\\_residuals\(\)](#page-105-0)). These are usually more accurate for testing such models than the traditionally used Pearson residuals. However, when simulating from more complex models, such as mixed models or models with zero-inflation, there are several important considerations. Arguments specified in ... are passed to [simulate\\_residuals\(\)](#page-105-0), which relies on [DHARMa::simulateResiduals\(\)](#page-0-0) (and therefore, arguments in ... are passed further down to *DHARMa*). The defaults in DHARMa are set on the most conservative option that works for all models. However, in many cases, the help advises to use different settings in particular situations or for particular models. It is recommended to read the 'Details' in ?DHARMa::simulateResiduals closely to understand the implications of the simulation process and which arguments should be modified to get the most accurate results.

#### See Also

Other functions to check model assumptions and and assess model quality: [check\\_autocorrelation\(](#page-5-0)), [check\\_collinearity\(](#page-7-0)), [check\\_convergence\(](#page-10-0)), [check\\_heteroscedasticity\(](#page-20-0)), [check\\_homogeneity\(](#page-21-0)), [check\\_model\(](#page-24-0)), [check\\_outliers\(](#page-30-0)), [check\\_overdispersion\(](#page-37-0)), [check\\_predictions\(](#page-39-0)), [check\\_singularity\(](#page-43-0))

## Examples

```
data(Salamanders, package = "glmmTMB")
m < - glm(count \sim spp + mined, family = poisson, data = Salamanders)
check_zeroinflation(m)
# for models with zero-inflation component, it's better to carry out
# the check for zero-inflation using simulated residuals
m <- glmmTMB::glmmTMB(
 count \sim spp + mined,
 ziformula = \sim mined + spp,
 family = poisson,
 data = Salamanders
)
res <- simulate_residuals(m)
check_zeroinflation(res)
```
classify\_distribution *Classify the distribution of a model-family using machine learning*

#### Description

Classify the distribution of a model-family using machine learning

## Details

The trained model to classify distributions, which is used by the check\_distribution() function.

<span id="page-49-0"></span>compare\_performance *Compare performance of different models*

## Description

compare\_performance() computes indices of model performance for different models at once and hence allows comparison of indices across models.

## Usage

```
compare_performance(
  ...,
 metrics = "all",
 rank = FALSE,
 estimator = "ML",
  verbose = TRUE
)
```
# **Arguments**



## Details

Model Weights: When information criteria (IC) are requested in metrics (i.e., any of "all", "common", "AIC", "AICc", "BIC", "WAIC", or "LOOIC"), model weights based on these criteria are also computed. For all IC except LOOIC, weights are computed as  $w = \exp(-0.5 \times \text{delta\_ic})$ / sum(exp(-0.5 \* delta\_ic)), where delta\_ic is the difference between the model's IC value and the smallest IC value in the model set (Burnham and Anderson, 2002). For LOOIC, weights are computed as "stacking weights" using [loo::stacking\\_weights\(\)](#page-0-0).

Ranking Models: When rank = TRUE, a new column Performance\_Score is returned. This score ranges from 0\ performance. Note that all score value do not necessarily sum up to 100\ Rather, calculation is based on normalizing all indices (i.e. rescaling them to a range from 0 to 1), and taking the mean value of all indices for each model. This is a rather quick heuristic, but might be helpful as exploratory index.

In particular when models are of different types (e.g. mixed models, classical linear models, logistic regression, ...), not all indices will be computed for each model. In case where an index can't be calculated for a specific model type, this model gets an NA value. All indices that have any NAs are excluded from calculating the performance score.

There is a plot()-method for compare\_performance(), which creates a "spiderweb" plot, where the different indices are normalized and larger values indicate better model performance. Hence, points closer to the center indicate worse fit indices (see [online-documentation](https://easystats.github.io/see/articles/performance.html) for more details).

REML versus ML estimator: By default, estimator = "ML", which means that values from information criteria (AIC, AICc, BIC) for specific model classes (like models from *lme4*) are based on the ML-estimator, while the default behaviour of AIC() for such classes is setting REML = TRUE. This default is intentional, because comparing information criteria based on REML fits is usually not valid (it might be useful, though, if all models share the same fixed effects - however, this is usually not the case for nested models, which is a prerequisite for the LRT). Set estimator  $=$  "REML" explicitly return the same (AIC/...) values as from the defaults in AIC.merMod().

## Value

A data frame with one row per model and one column per "index" (see metrics).

#### Note

There is also a plot()[-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

## References

Burnham, K. P., and Anderson, D. R. (2002). *Model selection and multimodel inference: A practical information-theoretic approach* (2nd ed.). Springer-Verlag. [doi:10.1007/b97636](https://doi.org/10.1007/b97636)

```
data(iris)
lm1 <- lm(Sepal.Length ~ Species, data = iris)
lm2 <- lm(Sepal.Length ~ Species + Petal.Length, data = iris)
lm3 \leq lm(Sepal.length \leq Species * Petal.length, data = iris)compare_performance(lm1, lm2, lm3)
compare_performance(lm1, lm2, lm3, rank = TRUE)
m1 < - \ln(mpg \sim wt + cyl, data = mtcars)m2 \leq -g \ln(vs \sim wt + mpg, data = mtcars, family = "binomial")m3 <- lme4::lmer(Petal.Length ~ Sepal.Length + (1 | Species), data = iris)
compare_performance(m1, m2, m3)
```
## Description

Compute various measures of internal consistencies for tests or item-scales of questionnaires.

#### Usage

cronbachs\_alpha(x, ...)

## Arguments



## Details

The Cronbach's Alpha value for x. A value closer to 1 indicates greater internal consistency, where usually following rule of thumb is applied to interpret the results:  $\alpha$  < 0.5 is unacceptable, 0.5 <  $\alpha$  $< 0.6$  is poor,  $0.6 < \alpha < 0.7$  is questionable,  $0.7 < \alpha < 0.8$  is acceptable, and everything  $> 0.8$  is good or excellent.

## Value

The Cronbach's Alpha value for x.

## References

Bland, J. M., and Altman, D. G. Statistics notes: Cronbach's alpha. BMJ 1997;314:572. 10.1136/bmj.314.7080.572

```
data(mtcars)
x <- mtcars[, c("cyl", "gear", "carb", "hp")]
cronbachs_alpha(x)
```
display.performance\_model

*Print tables in different output formats*

## Description

Prints tables (i.e. data frame) in different output formats. print\_md() is a alias for display(format = "markdown").

## Usage

```
## S3 method for class 'performance_model'
display(object, format = "markdown", digits = 2, caption = NULL, \ldots)
## S3 method for class 'performance_model'
print_md(
  x,
  digits = 2,
  caption = "Indices of model performance",
  layout = "horizontal",
  ...
\lambda## S3 method for class 'compare_performance'
print_md(
  x,
 digits = 2,
  caption = "Comparison of Model Performance Indices",
  layout = "horizontal",
  ...
)
```
## Arguments



## Details

display() is useful when the table-output from functions, which is usually printed as formatted text-table to console, should be formatted for pretty table-rendering in markdown documents, or if knitted from rmarkdown to PDF or Word files. See [vignette](https://easystats.github.io/parameters/articles/model_parameters_formatting.html) for examples.

## Value

A character vector. If format = "markdown", the return value will be a character vector in markdowntable format.

## Examples

```
model \leq -\ln(mpg \sim wt + cyl, data = mtcars)mp <- model_performance(model)
display(mp)
```
<span id="page-53-0"></span>icc *Intraclass Correlation Coefficient (ICC)*

## Description

This function calculates the intraclass-correlation coefficient (ICC) - sometimes also called *variance partition coefficient* (VPC) or *repeatability* - for mixed effects models. The ICC can be calculated for all models supported by [insight::get\\_variance\(\)](#page-0-0). For models fitted with the brms-package, icc() might fail due to the large variety of models and families supported by the brms-package. In such cases, an alternative to the ICC is the variance\_decomposition(), which is based on the posterior predictive distribution (see 'Details').

## Usage

```
icc(
  model,
 by_group = FALSE,
  tolerance = 1e-05,
  ci = NULL,iterations = 100,
  ci_method = NULL,
  null_model = NULL,
  approximation = "lognormal",
 model_component = NULL,
  verbose = TRUE,
  ...
)
```
variance\_decomposition(model, re\_formula = NULL, robust = TRUE, ci = 0.95, ...)

## Arguments





## Details

## Interpretation:

The ICC can be interpreted as "the proportion of the variance explained by the grouping structure in the population". The grouping structure entails that measurements are organized into groups (e.g., test scores in a school can be grouped by classroom if there are multiple classrooms and each classroom was administered the same test) and ICC indexes how strongly measurements in the same group resemble each other. This index goes from 0, if the grouping conveys no information, to 1, if all observations in a group are identical (*Gelman and Hill, 2007, p. 258*). In other word, the ICC - sometimes conceptualized as the measurement repeatability - "can also be interpreted as the expected correlation between two randomly drawn units that are in the same group" *(Hox 2010: 15)*, although this definition might not apply to mixed models with more complex random effects structures. The ICC can help determine whether a mixed model is even necessary: an ICC of zero (or very close to zero) means the observations within clusters are no more similar than observations from different clusters, and setting it as a random factor might not be necessary.

#### Difference with R2:

The coefficient of determination R2 (that can be computed with  $r2()$ ) quantifies the proportion of variance explained by a statistical model, but its definition in mixed model is complex (hence, different methods to compute a proxy exist). ICC is related to R2 because they are both ratios of variance components. More precisely, R2 is the proportion of the explained variance (of the full model), while the ICC is the proportion of explained variance that can be attributed to the random effects. In simple cases, the ICC corresponds to the difference between the *conditional R2* and the *marginal R2* (see [r2\\_nakagawa\(\)](#page-99-0)).

## Calculation:

The ICC is calculated by dividing the random effect variance,  $\sigma_i^2$ , by the total variance, i.e. the sum of the random effect variance and the residual variance,  $\sigma_{\epsilon}^2$ .

## Adjusted and unadjusted ICC:

icc() calculates an adjusted and an unadjusted ICC, which both take all sources of uncertainty (i.e. of *all random effects*) into account. While the *adjusted ICC* only relates to the random effects, the *unadjusted ICC* also takes the fixed effects variances into account, more precisely, the fixed effects variance is added to the denominator of the formula to calculate the ICC (see *Nakagawa et al. 2017*). Typically, the *adjusted* ICC is of interest when the analysis of random effects is of interest. icc() returns a meaningful ICC also for more complex random effects structures, like models with random slopes or nested design (more than two levels) and is applicable for models with other distributions than Gaussian. For more details on the computation of the variances, see ?insight::get\_variance.

#### ICC for unconditional and conditional models:

Usually, the ICC is calculated for the null model ("unconditional model"). However, according to *Raudenbush and Bryk (2002)* or *Rabe-Hesketh and Skrondal (2012)* it is also feasible to compute the ICC for full models with covariates ("conditional models") and compare how much, e.g., a level-2 variable explains the portion of variation in the grouping structure (random intercept).

#### ICC for specific group-levels:

The proportion of variance for specific levels related to the overall model can be computed by setting by\_group = TRUE. The reported ICC is the variance for each (random effect) group compared to the total variance of the model. For mixed models with a simple random intercept, this is identical to the classical (adjusted) ICC.

#### Variance decomposition for brms-models:

If model is of class brmsfit, icc() might fail due to the large variety of models and families supported by the brms package. In such cases, variance\_decomposition() is an alternative ICC measure. The function calculates a variance decomposition based on the posterior predictive distribution. In this case, first, the draws from the posterior predictive distribution *not conditioned* on group-level terms (posterior\_predict(..., re\_formula = NA)) are calculated as well as draws from this distribution *conditioned* on *all random effects* (by default, unless specified else in re\_formula) are taken. Then, second, the variances for each of these draws are calculated. The "ICC" is then the ratio between these two variances. This is the recommended way to analyse random-effect-variances for non-Gaussian models. It is then possible to compare variances across models, also by specifying different group-level terms via the re\_formula-argument.

Sometimes, when the variance of the posterior predictive distribution is very large, the variance ratio in the output makes no sense, e.g. because it is negative. In such cases, it might help to use robust = TRUE.

## Value

A list with two values, the adjusted ICC and the unadjusted ICC. For variance\_decomposition(), a list with two values, the decomposed ICC as well as the credible intervals for this ICC.

## Supported models and model families

The single variance components that are required to calculate the marginal and conditional r-squared values are calculated using the [insight::get\\_variance\(\)](#page-0-0) function. The results are validated against the solutions provided by *Nakagawa et al. (2017)*, in particular examples shown in the Supplement 2 of the paper. Other model families are validated against results from the MuMIn package. This means that the r-squared values returned by r2\_nakagawa() should be accurate and reliable for following mixed models or model families:

- Bernoulli (logistic) regression
- Binomial regression (with other than binary outcomes)
- Poisson and Quasi-Poisson regression
- Negative binomial regression (including nbinom1, nbinom2 and nbinom12 families)
- Gaussian regression (linear models)
- Gamma regression
- Tweedie regression
- Beta regression
- Ordered beta regression

Following model families are not yet validated, but should work:

• Zero-inflated and hurdle models

- Beta-binomial regression
- Compound Poisson regression
- Generalized Poisson regression
- Log-normal regression
- Skew-normal regression

Extracting variance components for models with zero-inflation part is not straightforward, because it is not definitely clear how the distribution-specific variance should be calculated. Therefore, it is recommended to carefully inspect the results, and probably validate against other models, e.g. Bayesian models (although results may be only roughly comparable).

Log-normal regressions (e.g. lognormal() family in **glmmTMB** or gaussian("log")) often have a very low fixed effects variance (if they were calculated as suggested by *Nakagawa et al. 2017*). This results in very low ICC or r-squared values, which may not be meaningful.

## References

- Hox, J. J. (2010). Multilevel analysis: techniques and applications (2nd ed). New York: Routledge.
- Nakagawa, S., Johnson, P. C. D., and Schielzeth, H. (2017). The coefficient of determination R2 and intra-class correlation coefficient from generalized linear mixed-effects models revisited and expanded. Journal of The Royal Society Interface, 14(134), 20170213.
- Rabe-Hesketh, S., and Skrondal, A. (2012). Multilevel and longitudinal modeling using Stata (3rd ed). College Station, Tex: Stata Press Publication.
- Raudenbush, S. W., and Bryk, A. S. (2002). Hierarchical linear models: applications and data analysis methods (2nd ed). Thousand Oaks: Sage Publications.

```
model <- lme4::lmer(Sepal.Length ~ Petal.Length + (1 | Species), data = iris)
icc(model)
# ICC for specific group-levels
data(sleepstudy, package = "lme4")
set.seed(12345)
sleepstudy$grp <- sample(1:5, size = 180, replace = TRUE)
sleepstudy$subgrp <- NA
for (i in 1:5) {
 filter_group <- sleepstudy$grp == i
 sleepstudy$subgrp[filter_group] <-
    sample(1:30, size = sum(filter_group), replace = TRUE)
}
model <- lme4::lmer(
 Reaction \sim Days + (1 | grp / subgrp) + (1 | Subject),
 data = sleepstudy
)
icc(model, by_group = TRUE)
```
## Description

Compute various measures of internal consistencies for tests or item-scales of questionnaires.

## Usage

```
item_difficulty(x, maximum_value = NULL)
```
## Arguments



## Details

*Item difficutly* of an item is defined as the quotient of the sum actually achieved for this item of all and the maximum achievable score. This function calculates the item difficulty, which should range between 0.2 and 0.8. Lower values are a signal for more difficult items, while higher values close to one are a sign for easier items. The ideal value for item difficulty is  $p + (1 - p) / 2$ , where  $p = 1$  $\prime$  max(x). In most cases, the ideal item difficulty lies between 0.5 and 0.8.

## Value

A data frame with three columns: The name(s) of the item(s), the item difficulties for each item, and the ideal item difficulty.

#### References

- Bortz, J., and Döring, N. (2006). Quantitative Methoden der Datenerhebung. In J. Bortz and N. Döring, Forschungsmethoden und Evaluation. Springer: Berlin, Heidelberg: 137–293
- Kelava A, Moosbrugger H (2020). Deskriptivstatistische Itemanalyse und Testwertbestimmung. In: Moosbrugger H, Kelava A, editors. Testtheorie und Fragebogenkonstruktion. Berlin, Heidelberg: Springer, 143–158

```
data(mtcars)
x <- mtcars[, c("cyl", "gear", "carb", "hp")]
item_difficulty(x)
```
item\_discrimination *Discrimination of Questionnaire Items*

## Description

Compute various measures of internal consistencies for tests or item-scales of questionnaires.

#### Usage

```
item_discrimination(x, standardize = FALSE)
```
## Arguments



## Details

This function calculates the item discriminations (corrected item-total correlations for each item of x with the remaining items) for each item of a scale. The absolute value of the item discrimination indices should be above 0.2. An index between 0.2 and 0.4 is considered as "fair", while a satisfactory index ranges from 0.4 to 0.7. Items with low discrimination indices are often ambiguously worded and should be examined. Items with negative indices should be examined to determine why a negative value was obtained (e.g. reversed answer categories regarding positive and negative poles).

## Value

A data frame with the item discrimination (*corrected item-total correlations*) for each item of the scale.

## References

• Kelava A, Moosbrugger H (2020). Deskriptivstatistische Itemanalyse und Testwertbestimmung. In: Moosbrugger H, Kelava A, editors. Testtheorie und Fragebogenkonstruktion. Berlin, Heidelberg: Springer, 143–158

```
data(mtcars)
x <- mtcars[, c("cyl", "gear", "carb", "hp")]
item_discrimination(x)
```
#### **Description**

Compute various measures of internal consistencies for tests or item-scales of questionnaires.

#### Usage

 $item\_intercor(x, method = c("pearson", "spearman", "kendall"))$ 

## Arguments



## Details

This function calculates a mean inter-item-correlation, i.e. a correlation matrix of x will be computed (unless x is already a matrix as returned by the cor() function) and the mean of the sum of all items' correlation values is returned. Requires either a data frame or a computed cor() object.

"Ideally, the average inter-item correlation for a set of items should be between 0.20 and 0.40, suggesting that while the items are reasonably homogeneous, they do contain sufficiently unique variance so as to not be isomorphic with each other. When values are lower than 0.20, then the items may not be representative of the same content domain. If values are higher than 0.40, the items may be only capturing a small bandwidth of the construct." *(Piedmont 2014)*

## Value

The mean inter-item-correlation value for x.

## References

Piedmont RL. 2014. Inter-item Correlations. In: Michalos AC (eds) Encyclopedia of Quality of Life and Well-Being Research. Dordrecht: Springer, 3303-3304. [doi:10.1007/9789400707535\\_1493](https://doi.org/10.1007/978-94-007-0753-5_1493)

```
data(mtcars)
x <- mtcars[, c("cyl", "gear", "carb", "hp")]
item_intercor(x)
```
#### Description

Compute various measures of internal consistencies for tests or item-scales of questionnaires.

#### Usage

```
item\_reliability(x, standardize = FALSE, digits = 3)
```
## Arguments



## Details

This function calculates the item discriminations (corrected item-total correlations for each item of x with the remaining items) and the Cronbach's alpha for each item, if it was deleted from the scale. The absolute value of the item discrimination indices should be above 0.2. An index between 0.2 and 0.4 is considered as "fair", while an index above 0.4 (or below -0.4) is "good". The range of satisfactory values is from 0.4 to 0.7. Items with low discrimination indices are often ambiguously worded and should be examined. Items with negative indices should be examined to determine why a negative value was obtained (e.g. reversed answer categories regarding positive and negative poles).

## Value

A data frame with the corrected item-total correlations (*item discrimination*, column item\_discrimination) and Cronbach's Alpha (if item deleted, column alpha\_if\_deleted) for each item of the scale, or NULL if data frame had too less columns.

```
data(mtcars)
x <- mtcars[, c("cyl", "gear", "carb", "hp")]
item_reliability(x)
```
## item\_split\_half *Split-Half Reliability*

#### Description

Compute various measures of internal consistencies for tests or item-scales of questionnaires.

#### Usage

item\_split\_half(x, digits = 3)

## Arguments



## Details

This function calculates the split-half reliability for items in x, including the Spearman-Brown adjustment. Splitting is done by selecting odd versus even columns in x. A value closer to 1 indicates greater internal consistency.

## Value

A list with two elements: the split-half reliability splithalf and the Spearman-Brown corrected split-half reliability spearmanbrown.

## References

- Spearman C. 1910. Correlation calculated from faulty data. British Journal of Psychology (3): 271-295. [doi:10.1111/j.20448295.1910.tb00206.x](https://doi.org/10.1111/j.2044-8295.1910.tb00206.x)
- Brown W. 1910. Some experimental results in the correlation of mental abilities. British Journal of Psychology (3): 296-322. [doi:10.1111/j.20448295.1910.tb00207.x](https://doi.org/10.1111/j.2044-8295.1910.tb00207.x)

```
data(mtcars)
x <- mtcars[, c("cyl", "gear", "carb", "hp")]
item_split_half(x)
```
## Description

Compute LOOIC (leave-one-out cross-validation (LOO) information criterion) and ELPD (expected log predictive density) for Bayesian regressions. For LOOIC and ELPD, smaller and larger values are respectively indicative of a better fit.

#### Usage

looic(model, verbose = TRUE)

## Arguments



## Value

A list with four elements, the ELPD, LOOIC and their standard errors.

## Examples

```
model <- suppressWarnings(rstanarm::stan_glm(
  mpg \sim wt + cyl,data = mtcars,
  chains = 1,
  iter = 500,
  refresh = 0))
looic(model)
```
<span id="page-63-0"></span>model\_performance *Model Performance*

#### Description

See the documentation for your object's class:

- [Frequentist Regressions](#page-68-0)
- [Instrumental Variables Regressions](#page-64-0)
- [Mixed models](#page-69-0)
- [Bayesian models](#page-72-0)
- [CFA / SEM lavaan models](#page-66-0)
- [Meta-analysis models](#page-70-0)

## Usage

```
model_performance(model, ...)
```

```
performance(model, ...)
```
#### Arguments



## Details

model\_performance() correctly detects transformed response and returns the "corrected" AIC and BIC value on the original scale. To get back to the original scale, the likelihood of the model is multiplied by the Jacobian/derivative of the transformation.

#### Value

A data frame (with one row) and one column per "index" (see metrics).

## See Also

[compare\\_performance\(\)](#page-49-0) to compare performance of many different models.

## Examples

```
model \leq -\ln(mpg \sim wt + cyl, data = mtcars)model_performance(model)
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
model_performance(model)
```
<span id="page-64-0"></span>model\_performance.ivreg

*Performance of instrumental variable regression models*

## Description

Performance of instrumental variable regression models

## Usage

```
## S3 method for class 'ivreg'
model_performance(model, metrics = "all", verbose = TRUE, ...)
```
## Arguments



## Details

model\_performance() correctly detects transformed response and returns the "corrected" AIC and BIC value on the original scale. To get back to the original scale, the likelihood of the model is multiplied by the Jacobian/derivative of the transformation.

model\_performance.kmeans

*Model summary for k-means clustering*

## Description

Model summary for k-means clustering

## Usage

## S3 method for class 'kmeans' model\_performance(model, verbose = TRUE, ...)

## Arguments



```
# a 2-dimensional example
x \leftarrow \text{rbind}(matrix(rnorm(100, sd = 0.3), ncol = 2),matrix(rnorm(100, mean = 1, sd = 0.3), ncol = 2))
\text{columns}(x) \leftarrow c("x", "y")model <- kmeans(x, 2)
model_performance(model)
```
<span id="page-66-0"></span>model\_performance.lavaan

*Performance of lavaan SEM / CFA Models*

#### **Description**

Compute indices of model performance for SEM or CFA models from the lavaan package.

## Usage

```
## S3 method for class 'lavaan'
model_performance(model, metrics = "all", verbose = TRUE, ...)
```
## **Arguments**



#### Details

## Indices of fit:

- Chisq: The model Chi-squared assesses overall fit and the discrepancy between the sample and fitted covariance matrices. Its p-value should be > .05 (i.e., the hypothesis of a perfect fit cannot be rejected). However, it is quite sensitive to sample size.
- GFI/AGFI: The (Adjusted) Goodness of Fit is the proportion of variance accounted for by the estimated population covariance. Analogous to R2. The GFI and the AGFI should be > .95 and > .90, respectively.
- NFI/NNFI/TLI: The (Non) Normed Fit Index. An NFI of 0.95, indicates the model of interest improves the fit by 95\ null model. The NNFI (also called the Tucker Lewis index; TLI) is preferable for smaller samples. They should be > .90 (Byrne, 1994) or > .95 (Schumacker and Lomax, 2004).
- CFI: The Comparative Fit Index is a revised form of NFI. Not very sensitive to sample size (Fan, Thompson, and Wang, 1999). Compares the fit of a target model to the fit of an independent, or null, model. It should be > .90.
- RMSEA: The Root Mean Square Error of Approximation is a parsimony-adjusted index. Values closer to 0 represent a good fit. It should be  $< .08$  or  $< .05$ . The p-value printed with it tests the hypothesis that RMSEA is less than or equal to .05 (a cutoff sometimes used for good fit), and thus should be not significant.
- RMR/SRMR: the (Standardized) Root Mean Square Residual represents the square-root of the difference between the residuals of the sample covariance matrix and the hypothesized model. As the RMR can be sometimes hard to interpret, better to use SRMR. Should be < .08.
- RFI: the Relative Fit Index, also known as RHO1, is not guaranteed to vary from 0 to 1. However, RFI close to 1 indicates a good fit.
- IFI: the Incremental Fit Index (IFI) adjusts the Normed Fit Index (NFI) for sample size and degrees of freedom (Bollen's, 1989). Over 0.90 is a good fit, but the index can exceed 1.
- PNFI: the Parsimony-Adjusted Measures Index. There is no commonly agreed-upon cutoff value for an acceptable model for this index. Should be > 0.50.

See the documentation for ?lavaan::fitmeasures.

What to report: Kline (2015) suggests that at a minimum the following indices should be reported: The model chi-square, the RMSEA, the CFI and the SRMR.

#### Value

A data frame (with one row) and one column per "index" (see metrics).

## References

- Byrne, B. M. (1994). Structural equation modeling with EQS and EQS/Windows. Thousand Oaks, CA: Sage Publications.
- Tucker, L. R., and Lewis, C. (1973). The reliability coefficient for maximum likelihood factor analysis. Psychometrika, 38, 1-10.
- Schumacker, R. E., and Lomax, R. G. (2004). A beginner's guide to structural equation modeling, Second edition. Mahwah, NJ: Lawrence Erlbaum Associates.
- Fan, X., B. Thompson, and L. Wang (1999). Effects of sample size, estimation method, and model specification on structural equation modeling fit indexes. Structural Equation Modeling, 6, 56-83.
- Kline, R. B. (2015). Principles and practice of structural equation modeling. Guilford publications.

```
# Confirmatory Factor Analysis (CFA) ---------
data(HolzingerSwineford1939, package = "lavaan")
structure <- " visual =- x1 + x2 + x3textual =~x4 + x5 + x6speed = x7 + x8 + x9 "
model <- lavaan::cfa(structure, data = HolzingerSwineford1939)
model_performance(model)
```
<span id="page-68-0"></span>model\_performance.lm *Performance of Regression Models*

## **Description**

Compute indices of model performance for regression models.

## Usage

```
## S3 method for class 'lm'
model_performance(model, metrics = "all", verbose = TRUE, ...)
```
## Arguments



## Details

Depending on model, following indices are computed:

- AIC: Akaike's Information Criterion, see ?stats:: AIC
- AICc: Second-order (or small sample) AIC with a correction for small sample sizes
- BIC: Bayesian Information Criterion, see ?stats::BIC
- R2: r-squared value, see [r2\(\)](#page-87-0)
- **R2\_adj**: adjusted r-squared, see [r2\(\)](#page-87-0)
- RMSE: root mean squared error, see [performance\\_rmse\(\)](#page-82-0)
- SIGMA: residual standard deviation, see [insight::get\\_sigma\(\)](#page-0-0)
- LOGLOSS: Log-loss, see [performance\\_logloss\(\)](#page-78-0)
- SCORE\_LOG: score of logarithmic proper scoring rule, see [performance\\_score\(\)](#page-85-0)
- SCORE\_SPHERICAL: score of spherical proper scoring rule, see [performance\\_score\(\)](#page-85-0)
- PCP: percentage of correct predictions, see [performance\\_pcp\(\)](#page-81-0)

model\_performance() correctly detects transformed response and returns the "corrected" AIC and BIC value on the original scale. To get back to the original scale, the likelihood of the model is multiplied by the Jacobian/derivative of the transformation.

#### Value

A data frame (with one row) and one column per "index" (see metrics).

## Examples

```
model \leq -\ln(mpg \sim wt + cyl, data = mtcars)model_performance(model)
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
model_performance(model)
```
<span id="page-69-0"></span>model\_performance.merMod

*Performance of Mixed Models*

## Description

Compute indices of model performance for mixed models.

## Usage

```
## S3 method for class 'merMod'
model_performance(
 model,
 metrics = "all",estimator = "REML",
 verbose = TRUE,
  ...
)
```
## Arguments



## Details

Intraclass Correlation Coefficient (ICC): This method returns the *adjusted ICC* only, as this is typically of interest when judging the variance attributed to the random effects part of the model (see also [icc\(\)](#page-53-0)).

REML versus ML estimator: The default behaviour of model\_performance() when computing AIC or BIC of linear mixed model from package lme4 is the same as for AIC() or BIC() (i.e. estimator = "REML"). However, for model comparison using compare\_performance() sets estimator = "ML" by default, because *comparing* information criteria based on REML fits is usually not valid (unless all models have the same fixed effects). Thus, make sure to set the correct estimator-value when looking at fit-indices or comparing model fits.

Other performance indices: Furthermore, see 'Details' in [model\\_performance.lm\(\)](#page-68-0) for more details on returned indices.

## Value

A data frame (with one row) and one column per "index" (see metrics).

## Examples

```
model <- lme4::lmer(Petal.Length ~ Sepal.Length + (1 | Species), data = iris)
model_performance(model)
```
<span id="page-70-0"></span>model\_performance.rma *Performance of Meta-Analysis Models*

#### Description

Compute indices of model performance for meta-analysis model from the metafor package.

#### Usage

```
## S3 method for class 'rma'
model_performance(
  model,
  metrics = "all",estimator = "ML",
  verbose = TRUE,
  ...
\mathcal{L}
```
#### Arguments





## **Details**

## Indices of fit:

- AIC Akaike's Information Criterion, see ?stats:: AIC
- BIC Bayesian Information Criterion, see ?stats::BIC
- I2: For a random effects model, I2 estimates (in percent) how much of the total variability in the effect size estimates can be attributed to heterogeneity among the true effects. For a mixed-effects model, I2 estimates how much of the unaccounted variability can be attributed to residual heterogeneity.
- H2: For a random-effects model, H2 estimates the ratio of the total amount of variability in the effect size estimates to the amount of sampling variability. For a mixed-effects model, H2 estimates the ratio of the unaccounted variability in the effect size estimates to the amount of sampling variability.
- TAU2: The amount of (residual) heterogeneity in the random or mixed effects model.
- CochransQ (QE): Test for (residual) Heterogeneity. Without moderators in the model, this is simply Cochran's *Q*-test.
- Omnibus (QM): Omnibus test of parameters.
- R2: Pseudo-R2-statistic, which indicates the amount of heterogeneity accounted for by the moderators included in a fixed-effects model.

See the documentation for ?metafor::fitstats.

## Value

A data frame (with one row) and one column per "index" (see metrics).

```
data(dat.bcg, package = "metadat")
dat <- metafor::escalc(
 measure = "RR",
 ai = tpos,bi = tneg,ci = cpos,di = cneg,data = dat.bcg)
model <- metafor::rma(yi, vi, data = dat, method = "REML")
model_performance(model)
```
model\_performance.stanreg

*Performance of Bayesian Models*

## Description

Compute indices of model performance for (general) linear models.

#### Usage

```
## S3 method for class 'stanreg'
model_performance(model, metrics = "all", verbose = TRUE, ...)
## S3 method for class 'BFBayesFactor'
model_performance(
 model,
 metrics = "all".verbose = TRUE,
  average = FALSE,
 prior_odds = NULL,
  ...
)
```
## Arguments



## Details

Depending on model, the following indices are computed:

- ELPD: expected log predictive density. Larger ELPD values mean better fit. See [looic\(\)](#page-63-0).
- LOOIC: leave-one-out cross-validation (LOO) information criterion. Lower LOOIC values mean better fit. See [looic\(\)](#page-63-0).
- WAIC: widely applicable information criterion. Lower WAIC values mean better fit. See ?loo::waic.
- R2: r-squared value, see [r2\\_bayes\(\)](#page-88-0).
- R2\_adjusted: LOO-adjusted r-squared, see [r2\\_loo\(\)](#page-94-0).
- RMSE: root mean squared error, see [performance\\_rmse\(\)](#page-82-0).
- SIGMA: residual standard deviation, see [insight::get\\_sigma\(\)](#page-0-0).
- LOGLOSS: Log-loss, see [performance\\_logloss\(\)](#page-78-0).
- SCORE\_LOG: score of logarithmic proper scoring rule, see [performance\\_score\(\)](#page-85-0).
- SCORE\_SPHERICAL: score of spherical proper scoring rule, see [performance\\_score\(\)](#page-85-0).
- PCP: percentage of correct predictions, see [performance\\_pcp\(\)](#page-81-0).

## Value

A data frame (with one row) and one column per "index" (see metrics).

#### References

Gelman, A., Goodrich, B., Gabry, J., and Vehtari, A. (2018). R-squared for Bayesian regression models. The American Statistician, The American Statistician, 1-6.

## See Also

[r2\\_bayes](#page-88-0)

## Examples

```
model <- suppressWarnings(rstanarm::stan_glm(
 mpg \sim wt + cy1,
  data = mtcars,
 chains = 1,
  iter = 500,
  refresh = 0))
model_performance(model)
model <- suppressWarnings(rstanarm::stan_glmer(
  mpg \sim wt + cyl + (1 | gear),data = mtcars,
  chains = 1,
  iter = 500,
  refresh = 0))
model_performance(model)
```
performance\_accuracy *Accuracy of predictions from model fit*

#### Description

This function calculates the predictive accuracy of linear or logistic regression models.

## Usage

```
performance_accuracy(
 model,
 method = c("cv", "boot"),k = 5,
 n = 1000,ci = 0.95,verbose = TRUE
)
```
## Arguments



## Details

For linear models, the accuracy is the correlation coefficient between the actual and the predicted value of the outcome. For logistic regression models, the accuracy corresponds to the AUC-value, calculated with the bayestestR::auc()-function.

The accuracy is the mean value of multiple correlation resp. AUC-values, which are either computed with cross-validation or non-parametric bootstrapping (see argument method). The standard error is the standard deviation of the computed correlation resp. AUC-values.

## Value

A list with three values: The Accuracy of the model predictions, i.e. the proportion of accurately predicted values from the model, its standard error, SE, and the Method used to compute the accuracy.

#### Examples

```
model \leq - \ln(\text{mpg} \sim \text{wt} + \text{cyl}, \text{data} = \text{mtcars})performance_accuracy(model)
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
performance_accuracy(model)
```
performance\_aicc *Compute the AIC or second-order AIC*

#### Description

Compute the AIC or the second-order Akaike's information criterion (AICc). performance\_aic() is a small wrapper that returns the AIC, however, for models with a transformed response variable, performance\_aic() returns the corrected AIC value (see 'Examples'). It is a generic function that also works for some models that don't have a AIC method (like Tweedie models). performance\_aicc() returns the second-order (or "small sample") AIC that incorporates a correction for small sample sizes.

#### Usage

```
performance_aicc(x, ...)
performance_aic(x, ...)
## Default S3 method:
performance\_aic(x, estimator = "ML", verbose = TRUE, ...)## S3 method for class 'lmerMod'
performance\_aic(x, estimator = "REML", verbose = TRUE, ...)
```
#### Arguments



## Details

performance\_aic() correctly detects transformed response and, unlike stats::AIC(), returns the "corrected" AIC value on the original scale. To get back to the original scale, the likelihood of the model is multiplied by the Jacobian/derivative of the transformation.

```
performance_cv 77
```
## Value

Numeric, the AIC or AICc value.

#### References

- Akaike, H. (1973) Information theory as an extension of the maximum likelihood principle. In: Second International Symposium on Information Theory, pp. 267-281. Petrov, B.N., Csaki, F., Eds, Akademiai Kiado, Budapest.
- Hurvich, C. M., Tsai, C.-L. (1991) Bias of the corrected AIC criterion for underfitted regression and time series models. Biometrika 78, 499–509.

## Examples

```
m < - \ln(mpg \sim wt + cyl + gear + disp, data = mtears)AIC(m)
performance_aicc(m)
# correct AIC for models with transformed response variable
data("mtcars")
mtcars$mpg <- floor(mtcars$mpg)
model <- lm(log(mpg) ~ factor(cyl), mtcars)
# wrong AIC, not corrected for log-transformation
AIC(model)
# performance_aic() correctly detects transformed response and
# returns corrected AIC
performance_aic(model)
```
performance\_cv *Cross-validated model performance*

#### Description

This function cross-validates regression models in a user-supplied new sample or by using holdout (train-test), k-fold, or leave-one-out cross-validation.

#### Usage

```
performance_cv(
  model,
  data = NULL,method = c("holdout", "k_fold", "loo"),
  metrics = "all",
  prop = 0.3,
  k = 5,stack = TRUE,
  verbose = TRUE,
```
... )

## Arguments



## Value

A data frame with columns for each metric requested, as well as k if method = "holdout" and the Method used for cross-validation. If method = "holdout" and stack = TRUE, the standard error (standard deviation across holdout folds) for each metric is also included.

## Examples

```
model \leq Im(mpg \sim wt + cyl, data = mtcars)performance_cv(model)
```
performance\_hosmer *Hosmer-Lemeshow goodness-of-fit test*

## Description

Check model quality of logistic regression models.

## Usage

```
performance_hosmer(model, n_bins = 10)
```
#### Arguments



#### Details

A well-fitting model shows *no* significant difference between the model and the observed data, i.e. the reported p-value should be greater than 0.05.

#### Value

An object of class hoslem\_test with following values: chisq, the Hosmer-Lemeshow chi-squared statistic; df, degrees of freedom and p.value the p-value for the goodness-of-fit test.

## References

Hosmer, D. W., and Lemeshow, S. (2000). Applied Logistic Regression. Hoboken, NJ, USA: John Wiley and Sons, Inc. [doi:10.1002/0471722146](https://doi.org/10.1002/0471722146)

#### Examples

```
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
performance_hosmer(model)
```
<span id="page-78-0"></span>performance\_logloss *Log Loss*

## Description

Compute the log loss for models with binary outcome.

#### Usage

```
performance_logloss(model, verbose = TRUE, ...)
```
#### Arguments



## Details

Logistic regression models predict the probability of an outcome of being a "success" or "failure" (or 1 and 0 etc.). performance\_logloss() evaluates how good or bad the predicted probabilities are. High values indicate bad predictions, while low values indicate good predictions. The lower the log-loss, the better the model predicts the outcome.

## Value

Numeric, the log loss of model.

## See Also

[performance\\_score\(\)](#page-85-0)

## Examples

```
data(mtcars)
m \leq -g \ln(\text{formula} = \text{vs} \sim \text{hp} + \text{wt}, \text{ family} = \text{binomial}, \text{ data} = \text{mtcars})performance_logloss(m)
```
performance\_mae *Mean Absolute Error of Models*

## Description

Compute mean absolute error of models.

#### Usage

```
performance_mae(model, ...)
```

```
mae(model, ...)
```
## Arguments



#### Value

Numeric, the mean absolute error of model.

## Examples

```
data(mtcars)
m <- lm(mpg \sim hp + gear, data = mtcars)performance_mae(m)
```
performance\_mse *Mean Square Error of Linear Models*

## Description

Compute mean square error of linear models.

## Usage

```
performance_mse(model, ...)
```

```
mse(model, ...)
```
## Arguments



#### Details

The mean square error is the mean of the sum of squared residuals, i.e. it measures the average of the squares of the errors. Less technically speaking, the mean square error can be considered as the variance of the residuals, i.e. the variation in the outcome the model doesn't explain. Lower values (closer to zero) indicate better fit.

## Value

Numeric, the mean square error of model.

## Examples

```
data(mtcars)
m \le - \ln(mpg \sim hp + gear, data = mtcars)performance_mse(m)
```
<span id="page-81-0"></span>

#### Description

Percentage of correct predictions (PCP) for models with binary outcome.

### Usage

performance\_pcp(model, ci =  $0.95$ , method = "Herron", verbose = TRUE)

#### Arguments



## Details

method = "Gelman-Hill" (or "gelman\_hill") computes the PCP based on the proposal from *Gelman and Hill 2017, 99*, which is defined as the proportion of cases for which the deterministic prediction is wrong, i.e. the proportion where the predicted probability is above  $0.5$ , although  $y=0$ (and vice versa) (see also *Herron 1999, 90*).

method = "Herron" (or "herron") computes a modified version of the PCP (*Herron 1999, 90-92*), which is the sum of predicted probabilities, where  $y=1$ , plus the sum of  $1$  - predicted probabilities, where y=0, divided by the number of observations. This approach is said to be more accurate.

The PCP ranges from 0 to 1, where values closer to 1 mean that the model predicts the outcome better than models with an PCP closer to 0. In general, the PCP should be above 0.5 (i.e. 50\ Furthermore, the PCP of the full model should be considerably above the null model's PCP.

The likelihood-ratio test indicates whether the model has a significantly better fit than the null-model  $(in such cases, p < 0.05).$ 

## Value

A list with several elements: the percentage of correct predictions of the full and the null model, their confidence intervals, as well as the chi-squared and p-value from the Likelihood-Ratio-Test between the full and null model.

#### References

- Herron, M. (1999). Postestimation Uncertainty in Limited Dependent Variable Models. Political Analysis, 8, 83–98.
- Gelman, A., and Hill, J. (2007). Data analysis using regression and multilevel/hierarchical models. Cambridge; New York: Cambridge University Press, 99.

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## Examples

```
data(mtcars)
m \leq glm(formula = vs \sim hp + wt, family = binomial, data = mtcars)performance_pcp(m)
performance_pcp(m, method = "Gelman-Hill")
```
<span id="page-82-0"></span>performance\_rmse *Root Mean Squared Error*

#### Description

Compute root mean squared error for (mixed effects) models, including Bayesian regression models.

## Usage

```
performance_rmse(
 model,
 normalized = FALSE,
 ci = NULL,iterations = 100,
 ci_method = NULL,
 verbose = TRUE,
  ...
\mathcal{L}rmse(
 model,
 normalized = FALSE,
 ci = NULL,iterations = 100,
 ci_method = NULL,
  verbose = TRUE,
  ...
)
```
## Arguments





#### Details

The RMSE is the square root of the variance of the residuals and indicates the absolute fit of the model to the data (difference between observed data to model's predicted values). It can be interpreted as the standard deviation of the unexplained variance, and is in the same units as the response variable. Lower values indicate better model fit.

The normalized RMSE is the proportion of the RMSE related to the range of the response variable. Hence, lower values indicate less residual variance.

#### Value

Numeric, the root mean squared error.

## Examples

```
data(Orthodont, package = "nlme")
m \le -\text{ n}lme::lme(distance \sim age, data = Orthodont)
# RMSE
performance_rmse(m, normalized = FALSE)
# normalized RMSE
performance_rmse(m, normalized = TRUE)
```
performance\_roc *Simple ROC curve*

#### Description

This function calculates a simple ROC curves of x/y coordinates based on response and predictions of a binomial model.

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#### Usage

performance\_roc(x, ..., predictions, new\_data)

#### Arguments



#### Value

A data frame with three columns, the x/y-coordinate pairs for the ROC curve (Sensitivity and Specificity), and a column with the model name.

## Note

There is also a  $plot()$ [-method](https://easystats.github.io/see/articles/performance.html) implemented in the see[-package.](https://easystats.github.io/see/)

#### Examples

```
library(bayestestR)
data(iris)
set.seed(123)
iris$y \le rbinom(nrow(iris), size = 1, .3)
folds \leq sample(nrow(iris), size = nrow(iris) / 8, replace = FALSE)
test_data <- iris[folds, ]
train_data <- iris[-folds, ]
model <- glm(y ~ Sepal.Length + Sepal.Width, data = train_data, family = "binomial")
as.data.frame(performance_roc(model, new_data = test_data))
roc <- performance_roc(model, new_data = test_data)
area_under_curve(roc$Specificity, roc$Sensitivity)
if (interactive()) {
  m1 <- glm(y ~ Sepal.Length + Sepal.Width, data = iris, family = "binomial")
  m2 < - glm(y \sim Sepal. Length + Petal. Width, data = iris, family = "binomial")
  m3 <- glm(y ~ Sepal.Length + Species, data = iris, family = "binomial")<br>performance_roc(m1, m2, m3)<br># if you have `see` package installed, you can also plot comparison of
  performance_roc(m1, m2, m3)
  # ROC curves for different models
  if (require("see")) plot(performance_roc(m1, m2, m3))
}
```
#### Description

Compute residual standard error of linear models.

## Usage

```
performance_rse(model)
```
## Arguments

model A model.

## Details

The residual standard error is the square root of the residual sum of squares divided by the residual degrees of freedom.

## Value

Numeric, the residual standard error of model.

## Examples

data(mtcars)  $m$  <-  $lm(mpg \sim hp + gear, data = mtcars)$ performance\_rse(m)

<span id="page-85-0"></span>performance\_score *Proper Scoring Rules*

#### Description

Calculates the logarithmic, quadratic/Brier and spherical score from a model with binary or count outcome.

#### Usage

performance\_score(model, verbose = TRUE, ...)

## Arguments



## Details

Proper scoring rules can be used to evaluate the quality of model predictions and model fit. performance\_score() calculates the logarithmic, quadratic/Brier and spherical scoring rules. The spherical rule takes values in the interval  $[0, 1]$ , with values closer to 1 indicating a more accurate model, and the logarithmic rule in the interval [-Inf, 0], with values closer to 0 indicating a more accurate model.

For stan\_lmer() and stan\_glmer() models, the predicted values are based on posterior\_predict(), instead of predict(). Thus, results may differ more than expected from their non-Bayesian counterparts in lme4.

## Value

A list with three elements, the logarithmic, quadratic/Brier and spherical score.

## Note

Code is partially based on [GLMMadaptive::scoring\\_rules\(\).](https://drizopoulos.github.io/GLMMadaptive/reference/scoring_rules.html)

#### References

Carvalho, A. (2016). An overview of applications of proper scoring rules. Decision Analysis 13, 223–242. [doi:10.1287/deca.2016.0337](https://doi.org/10.1287/deca.2016.0337)

#### See Also

[performance\\_logloss\(\)](#page-78-0)

#### Examples

```
## Dobson (1990) Page 93: Randomized Controlled Trial :
counts <- c(18, 17, 15, 20, 10, 20, 25, 13, 12)
outcome \leq gl(3, 1, 9)
treatment \leq gl(3, 3)
model \leq glm(counts \sim outcome + treatment, family = poisson())
performance_score(model)
data(Salamanders, package = "glmmTMB")
model <- glmmTMB::glmmTMB(
  count \sim spp + mined + (1 | site),
  zi = ~ spp + mined,
  family = <i>nbinom2()</i>,data = Salamanders
)
performance_score(model)
```
#### <span id="page-87-0"></span>Description

Calculate the R2, also known as the coefficient of determination, value for different model objects. Depending on the model, R2, pseudo-R2, or marginal / adjusted R2 values are returned.

### Usage

```
r2(model, ...)
## Default S3 method:
r2(model, ci = NULL, verbose = TRUE, ...)
## S3 method for class 'mlm'
r2(mod 1, multivariate = TRUE, ...)## S3 method for class 'merMod'
r2(model, ci = NULL, tolerance = 1e-05, ...)
```
#### Arguments



#### Value

Returns a list containing values related to the most appropriate R2 for the given model (or NULL if no R2 could be extracted). See the list below:

- Logistic models: [Tjur's R2](#page-103-0)
- General linear models: [Nagelkerke's R2](#page-98-0)

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- Multinomial Logit: [McFadden's R2](#page-95-0)
- Models with zero-inflation: [R2 for zero-inflated models](#page-104-0)
- Mixed models: [Nakagawa's R2](#page-99-0)
- Bayesian models: [R2 bayes](#page-88-0)

## Note

If there is no r2()-method defined for the given model class, r2() tries to return a "generic" rquared value, calculated as following: 1-sum((y-y\_hat)^2)/sum((y-y\_bar)^2)

#### See Also

```
r2_bayes(), r2_coxsnell(), r2_kullback(), r2_loo(), r2_mcfadden(), r2_nagelkerke(),
r2_nakagawa(), r2_tjur(), r2_xu(), r2_zeroinflated(), and r2_mlm().
```
## Examples

```
# Pseudo r-quared for GLM
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
r2(model)
# r-squared including confidence intervals
model \leq - \ln(mpg \sim wt + hp, data = mtcars)r2(model, ci = 0.95)
model <- lme4::lmer(Sepal.Length ~ Petal.Length + (1 | Species), data = iris)
r2(model)
```
<span id="page-88-0"></span>r2\_bayes *Bayesian R2*

# Description

Compute R2 for Bayesian models. For mixed models (including a random part), it additionally computes the R2 related to the fixed effects only (marginal R2). While r2\_bayes() returns a single R2 value, r2\_posterior() returns a posterior sample of Bayesian R2 values.

#### Usage

```
r2_bayes(model, robust = TRUE, ci = 0.95, verbose = TRUE, ...)
r2_posterior(model, ...)
## S3 method for class 'brmsfit'
r2_posterior(model, verbose = TRUE, ...)
## S3 method for class 'stanreg'
```

```
r2_posterior(model, verbose = TRUE, ...)
```

```
## S3 method for class 'BFBayesFactor'
r2_posterior(model, average = FALSE, prior_odds = NULL, verbose = TRUE, ...)
```
#### Arguments



#### Details

r2\_bayes() returns an "unadjusted" R2 value. See [r2\\_loo\(\)](#page-94-0) to calculate a LOO-adjusted R2, which comes conceptually closer to an adjusted R2 measure.

For mixed models, the conditional and marginal R2 are returned. The marginal R2 considers only the variance of the fixed effects, while the conditional R2 takes both the fixed and random effects into account. Technically, since r2\_bayes() relies on [rstantools::bayes\\_R2\(\)](#page-0-0), the "marginal" R2 calls bayes\_R2(re.form = NA), while the "conditional" R2 calls bayes\_R2(re.form = NULL). The re.form argument is passed to [rstantools::posterior\\_epred\(\)](#page-0-0), which is internally called in bayes\_R2().

Note that for "marginal" and "conditional", we refer to the wording suggested by *Nakagawa et al. 2017*. Thus, we don't use the term "marginal" in the sense that the random effects are integrated out, but are "ignored".

r2\_posterior() is the actual workhorse for r2\_bayes() and returns a posterior sample of Bayesian R2 values.

#### Value

A list with the Bayesian R2 value. For mixed models, a list with the Bayesian R2 value and the marginal Bayesian R2 value. The standard errors and credible intervals for the R2 values are saved as attributes.

#### References

- Gelman, A., Goodrich, B., Gabry, J., and Vehtari, A. (2018). R-squared for Bayesian regression models. The American Statistician, 1–6. [doi:10.1080/00031305.2018.1549100](https://doi.org/10.1080/00031305.2018.1549100)
- Nakagawa, S., Johnson, P. C. D., and Schielzeth, H. (2017). The coefficient of determination R2 and intra-class correlation coefficient from generalized linear mixed-effects models revisited and expanded. Journal of The Royal Society Interface, 14(134), 20170213.

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## Examples

```
library(performance)
model <- suppressWarnings(rstanarm::stan_glm(
  mpg \sim wt + cyl,data = mtcars,
  chains = 1,
  iter = 500.
  refresh = 0,
  show_messages = FALSE
))
r2_bayes(model)
model <- suppressWarnings(rstanarm::stan_lmer(
  Petal.Length ~ Petal.Width + (1 | Species),
 data = iris,
  chains = 1,
  iter = 500,
  refresh = 0))
r2_bayes(model)
model <- suppressWarnings(brms::brm(
  mpg \sim wt + cy1,
 data = mtcars,
  silent = 2,
  refresh = 0))
r2_bayes(model)
model <- suppressWarnings(brms::brm(
  Petal.Length ~ Petal.Width + (1 | Species),
  data = iris,
  silent = 2,
  refresh = 0))
```
r2\_bayes(model)

<span id="page-90-0"></span>r2\_coxsnell *Cox & Snell's R2*

#### Description

Calculates the pseudo-R2 value based on the proposal from *Cox & Snell (1989)*.

#### Usage

r2\_coxsnell(model, ...)

#### **Arguments**



#### Details

This index was proposed by *Cox and Snell (1989, pp. 208-9)* and, apparently independently, by *Magee (1990)*; but had been suggested earlier for binary response models by *Maddala (1983)*. However, this index achieves a maximum of less than 1 for discrete models (i.e. models whose likelihood is a product of probabilities) which have a maximum of 1, instead of densities, which can become infinite *(Nagelkerke, 1991)*.

## Value

A named vector with the R2 value.

#### References

- Cox, D. R., Snell, E. J. (1989). Analysis of binary data (Vol. 32). Monographs on Statistics and Applied Probability.
- Magee, L. (1990). R 2 measures based on Wald and likelihood ratio joint significance tests. The American Statistician, 44(3), 250-253.
- Maddala, G. S. (1986). Limited-dependent and qualitative variables in econometrics (No. 3). Cambridge university press.
- Nagelkerke, N. J. (1991). A note on a general definition of the coefficient of determination. Biometrika, 78(3), 691-692.

#### Examples

```
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
r2_coxsnell(model)
```
r2\_efron *Efron's R2*

## Description

Calculates Efron's pseudo R2.

#### Usage

r2\_efron(model)

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#### Arguments

model Generalized linear model.

## Details

Efron's R2 is calculated by taking the sum of the squared model residuals, divided by the total variability in the dependent variable. This R2 equals the squared correlation between the predicted values and actual values, however, note that model residuals from generalized linear models are not generally comparable to those of OLS.

#### Value

The R2 value.

#### References

Efron, B. (1978). Regression and ANOVA with zero-one data: Measures of residual variation. Journal of the American Statistical Association, 73, 113-121.

## Examples

```
## Dobson (1990) Page 93: Randomized Controlled Trial:
counts <- c(18, 17, 15, 20, 10, 20, 25, 13, 12) #
outcome \leq gl(3, 1, 9)
treatment \leq gl(3, 3)
model \leq glm(counts \sim outcome + treatment, family = poisson())
r2_efron(model)
```
r2\_ferrari *Ferrari's and Cribari-Neto's R2*

## Description

Calculates Ferrari's and Cribari-Neto's pseudo R2 (for beta-regression models).

## Usage

```
r2_ferrari(model, ...)
## Default S3 method:
r2_ferrari(model, correct_bounds = FALSE, ...)
```
## Arguments



## Value

A list with the pseudo R2 value.

## References

• Ferrari, S., and Cribari-Neto, F. (2004). Beta Regression for Modelling Rates and Proportions. Journal of Applied Statistics, 31(7), 799–815. [doi:10.1080/0266476042000214501](https://doi.org/10.1080/0266476042000214501)

#### Examples

```
data("GasolineYield", package = "betareg")
model <- betareg::betareg(yield ~ batch + temp, data = GasolineYield)
r2_ferrari(model)
```
<span id="page-93-0"></span>

## Description

Calculates the Kullback-Leibler-divergence-based R2 for generalized linear models.

## Usage

```
r2_kullback(model, ...)
## S3 method for class 'glm'
r2_kullback(model, adjust = TRUE, ...)
```
#### Arguments



## Value

A named vector with the R2 value.

#### $r2\_\text{loo}$  95

#### References

Cameron, A. C. and Windmeijer, A. G. (1997) An R-squared measure of goodness of fit for some common nonlinear regression models. Journal of Econometrics, 77: 329-342.

## Examples

```
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
r2_kullback(model)
```
<span id="page-94-0"></span>r2\_loo *LOO-adjusted R2*

## Description

Compute LOO-adjusted R2.

#### Usage

```
r2\_loo(\text{model}, \text{robust} = \text{TRUE}, \text{ci} = 0.95, \text{verbose} = \text{TRUE}, \dots)
```

```
r2_loo_posterior(model, ...)
```
## S3 method for class 'brmsfit' r2\_loo\_posterior(model, verbose = TRUE, ...)

```
## S3 method for class 'stanreg'
r2_loo_posterior(model, verbose = TRUE, ...)
```
## Arguments



## Details

r2\_loo() returns an "adjusted" R2 value computed using a leave-one-out-adjusted posterior distribution. This is conceptually similar to an adjusted/unbiased R2 estimate in classical regression modeling. See [r2\\_bayes\(\)](#page-88-0) for an "unadjusted" R2.

Mixed models are not currently fully supported.

r2\_loo\_posterior() is the actual workhorse for r2\_loo() and returns a posterior sample of LOOadjusted Bayesian R2 values.

## Value

A list with the Bayesian R2 value. For mixed models, a list with the Bayesian R2 value and the marginal Bayesian R2 value. The standard errors and credible intervals for the R2 values are saved as attributes.

A list with the LOO-adjusted R2 value. The standard errors and credible intervals for the R2 values are saved as attributes.

## Examples

```
model <- suppressWarnings(rstanarm::stan_glm(
  mpg \sim wt + cy1,
  data = mtcars,
  chains = 1,
  iter = 500,
  refresh = 0,
  show_messages = FALSE
))
r2_loo(model)
```
<span id="page-95-0"></span>r2\_mcfadden *McFadden's R2*

#### Description

Calculates McFadden's pseudo R2.

#### Usage

```
r2_mcfadden(model, ...)
```
## Arguments



#### Value

For most models, a list with McFadden's R2 and adjusted McFadden's R2 value. For some models, only McFadden's R2 is available.

#### References

- McFadden, D. (1987). Regression-based specification tests for the multinomial logit model. Journal of econometrics, 34(1-2), 63-82.
- McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior.

## r2\_mckelvey 97

#### Examples

```
if (require("mlogit")) {
 data("Fishing", package = "mlogit")
 Fish \le mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")
 model \leq mlogit(mode \sim price + catch, data = Fish)
 r2_mcfadden(model)
}
```
r2\_mckelvey *McKelvey & Zavoinas R2*

## Description

Calculates McKelvey and Zavoinas pseudo R2.

#### Usage

r2\_mckelvey(model)

#### Arguments

model Generalized linear model.

#### Details

McKelvey and Zavoinas R2 is based on the explained variance, where the variance of the predicted response is divided by the sum of the variance of the predicted response and residual variance. For binomial models, the residual variance is either  $pi^2/3$  for logit-link and 1 for probit-link. For poisson-models, the residual variance is based on log-normal approximation, similar to the *distribution-specific variance* as described in ?insight::get\_variance.

#### Value

The R2 value.

#### References

McKelvey, R., Zavoina, W. (1975), "A Statistical Model for the Analysis of Ordinal Level Dependent Variables", Journal of Mathematical Sociology 4, S. 103–120.

#### Examples

```
## Dobson (1990) Page 93: Randomized Controlled Trial:
counts <- c(18, 17, 15, 20, 10, 20, 25, 13, 12) #
outcome <- gl(3, 1, 9)
treatment \leq gl(3, 3)
model \leq glm(counts \sim outcome + treatment, family = poisson())
r2_mckelvey(model)
```
<span id="page-97-0"></span>

#### Description

Calculates two multivariate R2 values for multivariate linear regression.

#### Usage

r2\_mlm(model, ...)

#### Arguments



#### Details

The two indexes returned summarize model fit for the set of predictors given the system of responses. As compared to the default [r2](#page-87-0) index for multivariate linear models, the indexes returned by this function provide a single fit value collapsed across all responses.

The two returned indexes were proposed by *Van den Burg and Lewis (1988)* as an extension of the metrics proposed by *Cramer and Nicewander (1979)*. Of the numerous indexes proposed across these two papers, only two metrics, the  $R_{xy}$  and  $P_{xy}$ , are recommended for use by *Azen and Budescu (2006)*.

For a multivariate linear regression with p predictors and q responses where  $p > q$ , the  $R_{xy}$  index is computed as:

$$
R_{xy} = 1 - \prod_{i=1}^{p} (1 - \rho_i^2)
$$

Where  $\rho$  is a canonical variate from a [canonical correlation](#page-0-0) between the predictors and responses. This metric is symmetric and its value does not change when the roles of the variables as predictors or responses are swapped.

The  $P_{xy}$  is computed as:

$$
P_{xy} = \frac{q - trace(\mathbf{S_{YY}^{-1}S_{YY|X}})}{q}
$$

Where  $S_{YY}$  is the matrix of response covariances and  $S_{YY|X}$  is the matrix of residual covariances given the predictors. This metric is asymmetric and can change depending on which variables are considered predictors versus responses.

#### Value

A named vector with the R2 values.

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#### Author(s)

Joseph Luchman

#### References

- Azen, R., & Budescu, D. V. (2006). Comparing predictors in multivariate regression models: An extension of dominance analysis. Journal of Educational and Behavioral Statistics, 31(2), 157-180.
- Cramer, E. M., & Nicewander, W. A. (1979). Some symmetric, invariant measures of multivariate association. Psychometrika, 44, 43-54.
- Van den Burg, W., & Lewis, C. (1988). Some properties of two measures of multivariate association. Psychometrika, 53, 109-122.

#### Examples

```
model \leq lm(cbind(qsec, drat) \sim wt + mpg + cyl, data = mtcars)
r2_mlm(model)
model_swap <- lm(cbind(wt, mpg, cyl) ~ qsec + drat, data = mtcars)
r2_mlm(model_swap)
```
<span id="page-98-0"></span>r2\_nagelkerke *Nagelkerke's R2*

#### Description

Calculate Nagelkerke's pseudo-R2.

## Usage

```
r2_nagelkerke(model, ...)
```
## Arguments



## Value

A named vector with the R2 value.

#### References

Nagelkerke, N. J. (1991). A note on a general definition of the coefficient of determination. Biometrika, 78(3), 691-692.

## Examples

```
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
r2_nagelkerke(model)
```
<span id="page-99-0"></span>r2\_nakagawa *Nakagawa's R2 for mixed models*

## Description

Compute the *marginal* and *conditional* r-squared value for mixed effects models with complex random effects structures.

## Usage

```
r2_nakagawa(
 model,
 by_group = FALSE,tolerance = 1e-08,
 ci = NULL,iterations = 100,
  ci_method = NULL,
  null_model = NULL,
  approximation = "lognormal",
 model_component = NULL,
 verbose = TRUE,
  ...
\mathcal{L}
```
## Arguments



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- null\_model Optional, a null model to compute the random effect variances, which is passed to [insight::get\\_variance\(\)](#page-0-0). Usually only required if calculation of r-squared or ICC fails when null\_model is not specified. If calculating the null model takes longer and you already have fit the null model, you can pass it here, too, to speed up the process.
- approximation Character string, indicating the approximation method for the distribution-specific (observation level, or residual) variance. Only applies to non-Gaussian models. Can be "lognormal" (default), "delta" or "trigamma". For binomial models, the default is the *theoretical* distribution specific variance, however, it can also be "observation\_level". See *Nakagawa et al. 2017*, in particular supplement 2, for details.

```
model_component
```
For models that can have a zero-inflation component, specify for which component variances should be returned. If NULL or "full" (the default), both the conditional and the zero-inflation component are taken into account. If "conditional", only the conditional component is considered.

verbose Toggle warnings and messages.

... Arguments passed down to lme4::bootMer() or boot::boot() for bootstrapped ICC, R2, RMSE etc.; for variance\_decomposition(), arguments are passed down to brms::posterior\_predict().

#### Details

Marginal and conditional r-squared values for mixed models are calculated based on *Nakagawa et al. (2017)*. For more details on the computation of the variances, see [insight::get\\_variance\(\)](#page-0-0). The random effect variances are actually the mean random effect variances, thus the r-squared value is also appropriate for mixed models with random slopes or nested random effects (see *Johnson, 2014*).

- Conditional R2: takes both the fixed and random effects into account.
- Marginal R2: considers only the variance of the fixed effects.

The contribution of random effects can be deduced by subtracting the marginal R2 from the conditional R2 or by computing the [icc\(\)](#page-53-0).

#### Value

A list with the conditional and marginal R2 values.

#### Supported models and model families

The single variance components that are required to calculate the marginal and conditional r-squared values are calculated using the [insight::get\\_variance\(\)](#page-0-0) function. The results are validated against the solutions provided by *Nakagawa et al. (2017)*, in particular examples shown in the Supplement 2 of the paper. Other model families are validated against results from the MuMIn package. This means that the r-squared values returned by r2\_nakagawa() should be accurate and reliable for following mixed models or model families:

- Bernoulli (logistic) regression
- Binomial regression (with other than binary outcomes)
- Poisson and Quasi-Poisson regression
- Negative binomial regression (including nbinom1, nbinom2 and nbinom12 families)
- Gaussian regression (linear models)
- Gamma regression
- Tweedie regression
- Beta regression
- Ordered beta regression

Following model families are not yet validated, but should work:

- Zero-inflated and hurdle models
- Beta-binomial regression
- Compound Poisson regression
- Generalized Poisson regression
- Log-normal regression
- Skew-normal regression

Extracting variance components for models with zero-inflation part is not straightforward, because it is not definitely clear how the distribution-specific variance should be calculated. Therefore, it is recommended to carefully inspect the results, and probably validate against other models, e.g. Bayesian models (although results may be only roughly comparable).

Log-normal regressions (e.g. lognormal() family in **glmmTMB** or gaussian("log")) often have a very low fixed effects variance (if they were calculated as suggested by *Nakagawa et al. 2017*). This results in very low ICC or r-squared values, which may not be meaningful.

## References

- Hox, J. J. (2010). Multilevel analysis: techniques and applications (2nd ed). New York: Routledge.
- Johnson, P. C. D. (2014). Extension of Nakagawa and Schielzeth's R2 GLMM to random slopes models. Methods in Ecology and Evolution, 5(9), 944–946. [doi:10.1111/2041210X.12225](https://doi.org/10.1111/2041-210X.12225)
- Nakagawa, S., and Schielzeth, H. (2013). A general and simple method for obtaining R2 from generalized linear mixed-effects models. Methods in Ecology and Evolution, 4(2), 133–142. [doi:10.1111/j.2041210x.2012.00261.x](https://doi.org/10.1111/j.2041-210x.2012.00261.x)
- Nakagawa, S., Johnson, P. C. D., and Schielzeth, H. (2017). The coefficient of determination R2 and intra-class correlation coefficient from generalized linear mixed-effects models revisited and expanded. Journal of The Royal Society Interface, 14(134), 20170213.

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#### Examples

```
model <- lme4::lmer(Sepal.Length ~ Petal.Length + (1 | Species), data = iris)
r2_nakagawa(model)
r2_nakagawa(model, by_group = TRUE)
```
r2\_somers *Somers' Dxy rank correlation for binary outcomes*

## Description

Calculates the Somers' Dxy rank correlation for logistic regression models.

#### Usage

```
r2_somers(model)
```
## Arguments

model A logistic regression model.

## Value

A named vector with the R2 value.

## References

Somers, R. H. (1962). A new asymmetric measure of association for ordinal variables. American Sociological Review. 27 (6).

## Examples

```
if (require("correlation") && require("Hmisc")) {
 model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
 r2_somers(model)
}
```
<span id="page-103-0"></span>

## Description

This method calculates the Coefficient of Discrimination D (also known as Tjur's R2; Tjur, 2009) for generalized linear (mixed) models for binary outcomes. It is an alternative to other pseudo-R2 values like Nagelkerke's R2 or Cox-Snell R2. The Coefficient of Discrimination D can be read like any other (pseudo-)R2 value.

## Usage

r2\_tjur(model, ...)

#### Arguments



## Value

A named vector with the R2 value.

#### References

Tjur, T. (2009). Coefficients of determination in logistic regression models - A new proposal: The coefficient of discrimination. The American Statistician, 63(4), 366-372.

#### Examples

```
model \leq glm(vs \sim wt + mpg, data = mtcars, family = "binomial")
r2_tjur(model)
```
<span id="page-103-1"></span>

r2\_xu *Xu' R2 (Omega-squared)*

## Description

Calculates Xu' Omega-squared value, a simple R2 equivalent for linear mixed models.

#### Usage

r2\_xu(model)

## r2\_zeroinflated 105

#### Arguments

model A linear (mixed) model.

#### Details

r2\_xu() is a crude measure for the explained variance from linear (mixed) effects models, which is originally denoted as  $\Omega^2$ .

#### Value

The R2 value.

## References

Xu, R. (2003). Measuring explained variation in linear mixed effects models. Statistics in Medicine, 22(22), 3527–3541. [doi:10.1002/sim.1572](https://doi.org/10.1002/sim.1572)

#### Examples

```
model <- lm(Sepal.Length ~ Petal.Length + Species, data = iris)
r2_xu(model)
```
<span id="page-104-0"></span>r2\_zeroinflated *R2 for models with zero-inflation*

#### Description

Calculates R2 for models with zero-inflation component, including mixed effects models.

#### Usage

```
r2_zeroinflated(model, method = c("default", "correlation"))
```
#### Arguments



## Details

The default-method calculates an R2 value based on the residual variance divided by the total variance. For method = "correlation", R2 is a correlation-based measure, which is rather crude. It simply computes the squared correlation between the model's actual and predicted response.

#### Value

For the default-method, a list with the R2 and adjusted R2 values. For method = "correlation", a named numeric vector with the correlation-based R2 value.

## Examples

```
if (require("pscl")) {
 data(bioChemists)
 model <- zeroinfl(
   art \sim fem + mar + kid5 + ment | kid5 + phd,
   data = bioChemists
 \lambdar2_zeroinflated(model)
}
```
simulate\_residuals *Simulate randomized quantile residuals from a model*

## Description

Returns simulated residuals from a model. This is useful for checking the uniformity of residuals, in particular for non-Gaussian models, where the residuals are not expected to be normally distributed.

#### Usage

```
simulate_residuals(x, iterations = 250, ...)
## S3 method for class 'performance_simres'
residuals(object, quantile_function = NULL, outlier_values = NULL, ...)
```
#### Arguments



## Details

This function is a small wrapper around [DHARMa::simulateResiduals\(\)](#page-0-0). It basically only sets plot = FALSE and adds an additional class attribute ("performance\_sim\_res"), which allows using the DHARMa object in own plotting functions from the see package. See also vignette("DHARMa"). There is a plot() method to visualize the distribution of the residuals.

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#### Value

Simulated residuals, which can be further processed with [check\\_residuals\(\)](#page-42-0). The returned object is of class DHARMa and performance\_simres.

## Tests based on simulated residuals

For certain models, resp. model from certain families, tests like [check\\_zeroinflation\(\)](#page-47-0) or [check\\_overdispersion\(\)](#page-37-0) are based on simulated residuals. These are usually more accurate for such tests than the traditionally used Pearson residuals. However, when simulating from more complex models, such as mixed models or models with zero-inflation, there are several important considerations. simulate\_residuals() relies on [DHARMa::simulateResiduals\(\)](#page-0-0), and additional arguments specified in ... are passed further down to that function. The defaults in DHARMa are set on the most conservative option that works for all models. However, in many cases, the help advises to use different settings in particular situations or for particular models. It is recommended to read the 'Details' in ?DHARMa::simulateResiduals closely to understand the implications of the simulation process and which arguments should be modified to get the most accurate results.

### References

- Hartig, F., & Lohse, L. (2022). DHARMa: Residual Diagnostics for Hierarchical (Multi-Level / Mixed) Regression Models (Version 0.4.5). Retrieved from https://CRAN.R-project.org/package=DHARMa
- Dunn, P. K., & Smyth, G. K. (1996). Randomized Quantile Residuals. Journal of Computational and Graphical Statistics, 5(3), 236. [doi:10.2307/1390802](https://doi.org/10.2307/1390802)

### See Also

[check\\_residuals\(\)](#page-42-0), [check\\_zeroinflation\(\)](#page-47-0), [check\\_overdispersion\(\)](#page-37-0) and [check\\_predictions\(\)](#page-39-0). See also [see::plot.see\\_performance\\_simres\(\)](#page-0-0) for options to customize the plot.

#### Examples

```
m \le - \ln(mpg \sim wt + cyl + gear + disp, data = mtcars)simulate_residuals(m)
# extract residuals
head(residuals(simulate_residuals(m)))
```
test\_bf *Test if models are different*

#### Description

Testing whether models are "different" in terms of accuracy or explanatory power is a delicate and often complex procedure, with many limitations and prerequisites. Moreover, many tests exist, each coming with its own interpretation, and set of strengths and weaknesses.

The test\_performance() function runs the most relevant and appropriate tests based on the type of input (for instance, whether the models are *nested* or not). However, it still requires the user to understand what the tests are and what they do in order to prevent their misinterpretation. See the *Details* section for more information regarding the different tests and their interpretation.

#### Usage

```
test_bf(...)
## Default S3 method:
test_bf(..., reference = 1, text_length = NULL)test_likelihoodratio(..., estimator = "ML", verbose = TRUE)
test_lrt(..., estimator = "ML", verbose = TRUE)test_performance(..., reference = 1, verbose = TRUE)
test_vuong(..., verbose = TRUE)
test_wald(..., verbose = TRUE)
```
## Arguments



#### Details

## Nested vs. Non-nested Models:

Model's "nesting" is an important concept of models comparison. Indeed, many tests only make sense when the models are *"nested",* i.e., when their predictors are nested. This means that all the *fixed effects* predictors of a model are contained within the *fixed effects* predictors of a larger model (sometimes referred to as the encompassing model). For instance, model 1 ( $y \sim x1 + x2$ ) is "nested" within model2 ( $y \sim x1 + x2 + x3$ ). Usually, people have a list of nested models, for instance m1 (y ~ 1), m2 (y ~ x1), m3 (y ~ x1 + x2), m4 (y ~ x1 + x2 + x3), and it is conventional that they are "ordered" from the smallest to largest, but it is up to the user to reverse the order from largest to smallest. The test then shows whether a more parsimonious model, or whether adding
<span id="page-108-0"></span>a predictor, results in a significant difference in the model's performance. In this case, models are usually compared *sequentially*: m2 is tested against m1, m3 against m2, m4 against m3, etc.

Two models are considered as *"non-nested"* if their predictors are different. For instance, model1  $(y \sim x1 + x2)$  and model2  $(y \sim x3 + x4)$ . In the case of non-nested models, all models are usually compared against the same *reference* model (by default, the first of the list).

Nesting is detected via the insight::is\_nested\_models() function. Note that, apart from the nesting, in order for the tests to be valid, other requirements have often to be the fulfilled. For instance, outcome variables (the response) must be the same. You cannot meaningfully test whether apples are significantly different from oranges!

#### Estimator of the standard deviation:

The estimator is relevant when comparing regression models using test\_likelihoodratio(). If estimator =  $"OLS",$  then it uses the same method as anova $(..., test = "LRT")$  implemented in base R, i.e., scaling by n-k (the unbiased OLS estimator) and using this estimator under the alternative hypothesis. If estimator =  $mL$ , which is for instance used by  $lrtest(...)$  in package lmtest, the scaling is done by n (the biased ML estimator) and the estimator under the null hypothesis. In moderately large samples, the differences should be negligible, but it is possible that OLS would perform slightly better in small samples with Gaussian errors. For estimator = "REML", the LRT is based on the REML-fit log-likelihoods of the models. Note that not all types of estimators are available for all model classes.

# REML versus ML estimator:

When estimator = "ML", which is the default for linear mixed models (unless they share the same fixed effects), values from information criteria (AIC, AICc) are based on the ML-estimator, while the default behaviour of AIC() may be different (in particular for linear mixed models from lme4, which sets REML = TRUE). This default in test\_likelihoodratio() intentional, because comparing information criteria based on REML fits requires the same fixed effects for all models, which is often not the case. Thus, while anova.merMod() automatically refits all models to REML when performing a LRT, test\_likelihoodratio() checks if a comparison based on REML fits is indeed valid, and if so, uses REML as default (else, ML is the default). Set the estimator argument explicitely to override the default behaviour.

#### Tests Description:

- Bayes factor for Model Comparison test\_bf(): If all models were fit from the same data, the returned BF shows the Bayes Factor (see bayestestR::bayesfactor\_models()) for each model against the reference model (which depends on whether the models are nested or not). Check out [this vignette](https://easystats.github.io/bayestestR/articles/bayes_factors.html#bayesfactor_models) for more details.
- Wald's F-Test test\_wald(): The Wald test is a rough approximation of the Likelihood Ratio Test. However, it is more applicable than the LRT: you can often run a Wald test in situations where no other test can be run. Importantly, this test only makes statistical sense if the models are nested.

Note: this test is also available in base R through the [anova\(\)](#page-0-0) function. It returns an F-value column as a statistic and its associated p-value.

• Likelihood Ratio Test (LRT) - test\_likelihoodratio(): The LRT tests which model is a better (more likely) explanation of the data. Likelihood-Ratio-Test (LRT) gives usually somewhat close results (if not equivalent) to the Wald test and, similarly, only makes sense for nested models. However, maximum likelihood tests make stronger assumptions than method of moments tests like the F-test, and in turn are more efficient. Agresti (1990) suggests that <span id="page-109-0"></span>you should use the LRT instead of the Wald test for small sample sizes (under or about 30) or if the parameters are large.

Note: for regression models, this is similar to anova(..., test="LRT") (on models) or lmtest::lrtest(...), depending on the estimator argument. For lavaan models (SEM, CFA), the function calls lavaan::lavTestLRT().

For models with transformed response variables (like  $log(x)$  or  $sqrt(x)$ ),  $logLik()$  returns a wrong log-likelihood. However, test\_likelihoodratio() calls insight::get\_loglikelihood() with check\_response=TRUE, which returns a corrected log-likelihood value for models with transformed response variables. Furthermore, since the LRT only accepts nested models (i.e. models that differ in their fixed effects), the computed log-likelihood is always based on the ML estimator, not on the REML fits.

- Vuong's Test test\_vuong(): Vuong's (1989) test can be used both for nested and nonnested models, and actually consists of two tests.
	- The Test of Distinguishability (the Omega2 column and its associated p-value) indicates whether or not the models can possibly be distinguished on the basis of the observed data. If its p-value is significant, it means the models are distinguishable.
	- The Robust Likelihood Test (the LR column and its associated p-value) indicates whether each model fits better than the reference model. If the models are nested, then the test works as a robust LRT. The code for this function is adapted from the **nonnest2** package, and all credit go to their authors.

#### Value

A data frame containing the relevant indices.

## References

- Vuong, Q. H. (1989). Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica, 57, 307-333.
- Merkle, E. C., You, D., & Preacher, K. (2016). Testing non-nested structural equation models. Psychological Methods, 21, 151-163.

# See Also

[compare\\_performance\(\)](#page-49-0) to compare the performance indices of many different models.

#### Examples

```
# Nested Models
# -------------
m1 <- lm(Sepal.Length ~ Petal.Width, data = iris)
m2 <- lm(Sepal.Length ~ Petal.Width + Species, data = iris)
m3 \leq Im(Sepal.Length \sim Petal.Width \times Species, data = iris)test_performance(m1, m2, m3)
test_bf(m1, m2, m3)
test_wald(m1, m2, m3) # Equivalent to anova(m1, m2, m3)
# Equivalent to lmtest::lrtest(m1, m2, m3)
```
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```
test_likelihoodratio(m1, m2, m3, estimator = "ML")
# Equivalent to anova(m1, m2, m3, test='LRT')
test_likelihoodratio(m1, m2, m3, estimator = "OLS")
if (require("CompQuadForm")) {
 test_vuong(m1, m2, m3) # nonnest2::vuongtest(m1, m2, nested=TRUE)
 # Non-nested Models
 # -----------------
 m1 <- lm(Sepal.Length ~ Petal.Width, data = iris)
 m2 <- lm(Sepal.Length ~ Petal.Length, data = iris)
 m3 <- lm(Sepal.Length ~ Species, data = iris)
 test_performance(m1, m2, m3)
 test_bf(m1, m2, m3)
 test_vuong(m1, m2, m3) # nonnest2::vuongtest(m1, m2)
}
# Tweak the output
# ----------------
test_performance(m1, m2, m3, include_formula = TRUE)
# SEM / CFA (lavaan objects)
# --------------------------
# Lavaan Models
if (require("lavaan")) {
 structure <- " visual =- x1 + x2 + x3textual =~x4 + x5 + x6speed = x7 + x8 + x9visual ~~ textual + speed "
 m1 <- lavaan::cfa(structure, data = HolzingerSwineford1939)
 structure <- " visual =- x1 + x2 + x3textual = x4 + x5 + x6speed = x7 + x8 + x9visual ~~ 0 * textual + speed "
 m2 <- lavaan::cfa(structure, data = HolzingerSwineford1939)
 structure <- " visual =- x1 + x2 + x3textual =~x4 + x5 + x6speed = x7 + x8 + x9visual \sim 0 * textual + 0 * speed "
 m3 <- lavaan::cfa(structure, data = HolzingerSwineford1939)
 test_likelihoodratio(m1, m2, m3)
 # Different Model Types
 # ---------------------
```

```
if (require("lme4") && require("mgcv")) {
   m1 <- lm(Sepal.Length ~ Petal.Length + Species, data = iris)
   m2 <- lmer(Sepal.Length ~ Petal.Length + (1 | Species), data = iris)
   m3 <- gam(Sepal.Length ~ s(Petal.Length, by = Species) + Species, data = iris)
   test_performance(m1, m2, m3)
 }
}
```
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