

# Package: mvp (via r-universe)

June 29, 2026

**Type** Package

**Title** Fast Symbolic Multivariate Polynomials

**Version** 1.1-1

**Depends** R (>= 4.1.0)

**Suggests** knitr, rmarkdown, microbenchmark, testthat, spray (>= 1.1-1),  
magrittr, covr

**VignetteBuilder** knitr

**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>

**Description** Fast manipulation of symbolic multivariate polynomials using the 'Map' class of the Standard Template Library. The package uses print and coercion methods from the 'mpoly' package but offers speed improvements. It is comparable in speed to the 'spray' package for sparse arrays, but retains the symbolic benefits of 'mpoly'. To cite the package in publications, use Hankin 2022 <[doi:10.48550/ARXIV.2210.15991](https://doi.org/10.48550/ARXIV.2210.15991)>. Uses 'disordR' discipline.

**License** GPL (>= 2)

**Imports** Rcpp (>= 1.0-7), partitions, magic, digest, disordR (>= 0.9-7), numbers, mpoly (>= 1.1.0), methods

**LinkingTo** Rcpp

**URL** <https://github.com/RobinHankin/mvp>,  
<https://robinhankin.github.io/mvp/>

**BugReports** <https://github.com/RobinHankin/mvp/issues>

**LazyData** FALSE

**NeedsCompilation** yes

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**Repository** <https://cran.r-universe.dev>

**Date/Publication** 2026-06-29 14:31:29 UTC

**RemoteUrl** <https://github.com/cran/mvp>

**RemoteRef** HEAD

**RemoteSha** 6ee72ee5add445f1c81bf934bd13570111550954

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mvp-package

*Fast Symbolic Multivariate Polynomials*

---

## Description

Fast manipulation of symbolic multivariate polynomials using the 'Map' class of the Standard Template Library. The package uses print and coercion methods from the 'mpoly' package but offers speed improvements. It is comparable in speed to the 'spray' package for sparse arrays, but retains the symbolic benefits of 'mpoly'. To cite the package in publications, use Hankin 2022 <doi:10.48550/ARXIV.2210.15991>. Uses 'disordR' discipline.

**Details**

The DESCRIPTION file:

```

Package:      mvp
Type:        Package
Title:       Fast Symbolic Multivariate Polynomials
Version:     1.1-1
Authors@R:   person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@gmail.com")
Depends:     R (>= 4.1.0)
Suggests:   knitr, rmarkdown, microbenchmark, testthat, spray (>= 1.1-1), magrittr, covr
VignetteBuilder: knitr
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Description: Fast manipulation of symbolic multivariate polynomials using the 'Map' class of the Standard Template Library
License:     GPL (>= 2)
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LinkingTo:   Rcpp
URL:        https://github.com/RobinHankin/mvp, https://robinhankin.github.io/mvp/
BugReports:  https://github.com/RobinHankin/mvp/issues
LazyData:   FALSE
Author:     Robin K. S. Hankin [aut, cre] (ORCID: <https://orcid.org/0000-0001-5982-0415>)

```

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**Author(s)**

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 Maintainer: Robin K. S. Hankin <[hankin.robin@gmail.com](mailto:hankin.robin@gmail.com)>

**Examples**

```
(p <- as.mvp("1+x+x*y+x^5"))

p + as.mvp("a+b^6")

p^3

subs(p^4, x="a+b^2")
aderiv(p^2, x=4)
horner(p, 1:3)
```

---

all\_equal\_mvp

*Approximate equality of.mvp objects*


---

**Description**

Two multivariate polynomials  $x, y$  are held to be approximately equal if the coefficients of  $x - y$  are small compared to the coefficients of  $x$  and  $y$  separately. The comparison `all.equal()` is dispatched to `all_equal_mvp()` which uses `base::all.equal()`.

**Usage**

```
all_equal_mvp(target, current)
```

**Arguments**

target, current Objects of class.mvp

**Author(s)**

Robin K. S. Hankin

**Examples**

```
a <- rmvp()
a1 <- a + rmvp()/1e5
a2 <- a - rmvp()/1e5
all.equal(a1, a2)
```

---

allvars	<i>All variables in a multivariate polynomial</i>
---------	---

---

**Description**

Returns a character vector containing all the variables present in a.mvp object.

**Usage**

```
allvars(x)
```

**Arguments**

x                    object of class.mvp

**Note**

The character vector returned is not in any particular order. It might have been better for allvars() to return a disord object.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
p <- rmvp(5)
p
allvars(p)
```

---

as.function.mvp	<i>Functional form for multivariate polynomials</i>
-----------------	---

---

**Description**

Coerces a multivariate polynomial into a function

**Usage**

```
## S3 method for class 'mvp'
as.function(x, ...)
```

**Arguments**

x                    Multivariate polynomial  
 ...                  Further arguments (currently ignored)

**Author(s)**

Robin K. S. Hankin

**Examples**

```
p <- as.mvp("1+a^2 + a*b^2 + c")
p
f <- as.function(p)
f

f(a=1)
f(a=1,b=2)
f(a=1,b=2,c=3)          # coerces to a scalar
f(a=1,b=2,c=3,drop=FALSE) # formal.mvp object
```

---

coeffs

*Functionality for coeffs objects*


---

**Description**

Get and set the coefficients of an.mvp object.

**Usage**

```
coeffs(x)
vars(x)
powers(x)
coeffs(x) <- value
```

**Arguments**

x	Object of class disord
value	Object of class disord, or length-1 numeric vector

**Details**

Function coeffs() extracts the coefficients of an.mvp object. The coefficients may be set with coeffs<-( ), as in coeffs(M) <- value. These follow **disordR** discipline. The idea is that the methods allow arithmetic operators to be used for the coefficients of multivariate polynomials, bearing in mind that the order of coefficients is not determined.

“Pure” extraction and replacement, as in M[coeffs(M) > 9] [which returns an.mvp object] and M[Mod(coeffs(M)) <= 1] <- value [which modifies a] is implemented experimentally. The code for extraction is cute but not particularly efficient.

(much of the discussion below appears in the vignette of the **disordR** package).

Accessing elements of an `mvp` object is problematic because the order of the terms of an `mvp` object is not well-defined. This is because the `map` class of the STL does not specify an order for the key-value pairs (and indeed the actual order in which they are stored may be implementation dependent). The situation is similar to the `hyper2` package which uses the STL in a similar way.

A `coeffs` object is a vector of coefficients of a `mvp` object. But it is not a conventional vector; in a conventional vector, we can identify the first element unambiguously, and the second, and so on. An `mvp` is a map from terms to coefficients, and a map has no intrinsic ordering: the maps

```
{x -> 1, y -> 3, xy^3 -> 4}
```

and

```
{xy^3 -> 4, x -> 1, y -> 3}
```

are the same map and correspond to the same multinomial (symbolically,  $x + 3y + 4xy^3 = 4xy^3 + x + 3y$ ). Thus the coefficients of the multinomial might be  $c(1, 3, 4)$  or  $c(4, 1, 3)$ , or indeed any ordering. But note that any particular ordering imposes an ordering on the terms. If we choose  $c(1, 3, 4)$  then the terms are  $x, y, xy^3$ , and if we choose  $c(4, 1, 3)$  the terms are  $xy^3, x, y$ .

In the package, `coeffs()` returns an object of class `disord`. This class of object has a slot for the coefficients in the form of a numeric R vector, but also another slot which uses hash codes to prevent users from misusing the ordering of the numeric vector.

For example, a multinomial  $x+2y+3z$  might have coefficients  $c(1, 2, 3)$  or  $c(3, 1, 2)$ . Package idiom to extract the coefficients of a multivariate polynomial `a` is `coeffs(a)`; but this cannot return a standard numeric vector because a numeric vector has elements in a particular order, and the coefficients of a multivariate polynomial are stored in an implementation-specific (and thus unknown) order.

Suppose we have two multivariate polynomials, `a` as defined as above with  $a=x+2y+3z$  and  $b=x+3y+4z$ . Even though  $a+b$  is well-defined algebraically, and `coeffs(a+b)` will return a well-defined `mvp_coeffs` object, idiom such as `coeffs(a) + coeffs(b)` is not defined because there is no guarantee that the coefficients of the two multivariate polynomials are stored in the same order. We might have  $c(1, 2, 3)+c(1, 3, 4)=c(2, 5, 7)$  or  $c(1, 2, 3)+c(1, 4, 3)=c(2, 6, 6)$ , with neither being more “correct” than the other. In the package, `coeffs(a) + coeffs(b)` will return an error. In the same way `coeffs(a) + 1:3` is not defined and will return an error. Further, idiom such as `coeffs(a) <- 1:3` and `coeffs(a) <- coeffs(b)` are not defined and will return an error. However, note that `coeffs(a) + coeffs(a)` and `coeffs(a)+coeffs(a)^2` are fine, these returning a `mvp_coeffs` object specific to `a`.

Idiom such as `coeffs(a) <- coeffs(a)^2` is fine too, for one does not need to know the order of the coefficients on either side, so long as the order is the same on both sides. That would translate into idiomatic English: “the coefficient of each term of `a` becomes its square”; note that this operation is insensitive to the order of coefficients. The whole shebang is intended to make idiom such as `coeffs(a) <- coeffs(a)%%2` possible (so we can manipulate polynomials over finite rings, here  $\mathbb{Z}/2\mathbb{Z}$ ).

The replacement methods are defined so that an expression like `coeffs(a)[coeffs(a) > 5] <- 5` works as expected; the English idiom would be “Replace any coefficient greater than 5 with 5”.

To fix ideas, consider `a <- rmvp(8)`. Extraction presents issues; consider `coeffs(a)<5`. This object has Boolean elements but has the same ordering ambiguity as `coeffs(a)`. One might expect that

we could use this to extract elements of `coeffs(a)`, specifically elements less than 5. However, `coeffs(a)[coeffs(a)<5]` in isolation is meaningless: what can be done with such an object? However, it makes sense on the left hand side of an assignment, as long as the right hand side is a length-one vector. Idiom such as

- `coeffs(a)[coeffs(a)<5] <- 4+coeffs(a)[coeffs(a)<5]`
- `coeffs(a) <- pmax(a,3)`

is algebraically meaningful (“Add 4 to any element less than 5”; “coefficients become the pairwise maximum of themselves and 3”). The **disordR** package uses `pmaxdis()` rather than `pmax()` for technical reasons.

So the output of `coeffs(x)` is defined only up to an unknown rearrangement. The same considerations apply to the output of `vars()`, which returns a list of character vectors in an undefined order, and the output of `powers()`, which returns a numeric list whose elements are in an undefined order. However, even though the order of these three objects is undefined individually, their ordering is jointly consistent in the sense that the first element of `coeffs(x)` corresponds to the first element of `vars(x)` and the first element of `powers(x)`. The identity of this element is not defined—but whatever it is, the first element of all three accessor methods refers to it.

Note also that a single term (something like  $4a^3b^2c^6$ ) has the same issue: the variables are not stored in a well-defined order. This does not matter because the algebraic value of the term does not depend on the order in which the variables appear and this term would be equivalent to  $4b^2c^6a^3$ .

### Author(s)

Robin K. S. Hankin

### Examples

```
(x <- 5+rmvp(6))
(y <- 2+rmvp(6))

coeffs(x)^2
coeffs(y) <- coeffs(y)%%3 # fine, all coeffs of y now modulo 3
y

coeffs(y) <- 4
y

## Not run:
coeffs(x) <- coeffs(y) # not defined, will give an error
coeffs(x) <- seq_len(nterms(x)) # not defined, will give an error

## End(Not run)

## "Pure" extraction:
M <- as.mvp("4 -0.1 x + 7 x y z -3 x^5 + 0.3 y^5")
M[coeffs(M)>3]
M[Mod(coeffs(M) < 1] <- 0
M
```

---

 constant

*The constant term*


---

## Description

Get and set the constant term of an mvp object

## Usage

```
## S3 method for class 'mvp'
constant(x, ...)
## S3 replacement method for class 'mvp'
constant(x) <- value
## S3 method for class 'numeric'
constant(x, ...)
is.constant(x)
```

## Arguments

x	Object of class mvp
value	Scalar value for the constant
...	Further arguments, currently ignored

## Details

The constant term in a polynomial is the coefficient of the empty term. In an mvp object, the map  $\{ \} \rightarrow c$ , implies that  $c$  is the constant.

If  $x$  is an mvp object, `constant(x)` returns the value of the constant in the multivariate polynomial; if  $x$  is numeric, it returns a constant multivariate polynomial with value  $x$ .

Function `is.constant()` returns TRUE if its argument has no variables and FALSE otherwise.

## Author(s)

Robin K. S. Hankin

## Examples

```
a <- rmvp(5)+4
a
constant(a)
constant(a) <- 33
a

constant(0) # the zero mvp
```

---

 deriv *Differentiation of mvp objects*


---

**Description**

Differentiation of mvp objects

**Usage**

```
## S3 method for class 'mvp'
deriv(expr, v, ...)
## S3 method for class 'mvp'
aderiv(expr, ...)
```

**Arguments**

expr	Object of class mvp
v	Character vector. Elements denote variables to differentiate with respect to
...	Further arguments. In <code>deriv()</code> , a non-negative integer argument specifies the order of the differential, and in <code>aderiv()</code> the argument names specify the differentials and their values their respective orders

**Details**

`deriv(S,v)` returns  $\frac{\partial^r S}{\partial v_1 \partial v_2 \dots \partial v_r}$ . Here, v is a (character) vector of symbols.

`deriv(S,v,n)` returns the n-th derivative of S with respect to symbol v,  $\frac{\partial^n S}{\partial v^n}$ .

`aderiv()` uses the ellipsis construction with the names of the argument being the variable to be differentiated with respect to. Thus `aderiv(S,x=1,y=2)` returns  $\frac{\partial^3 S}{\partial x \partial y^2}$ .

**Author(s)**

Robin K. S. Hankin

**See Also**

[taylor](#)

**Examples**

```
(p <- rmvp(10,4,15,5))
deriv(p,"a")
deriv(p,"a",3)
deriv(p,letters[1:3])
deriv(p,rev(letters[1:3])) # should be the same

aderiv(p,a=1,b=2,c=1)
```

```
## verify the chain rule:
x <- rmvp(7,symbols=6)
v <- allvars(x)[1]
s <- as.mvp("1 + y - y^2 zz + y^3 z^2")
LHS <- subsmvp(deriv(x,v)*deriv(s,"y"),v,s) # dx/ds*ds/dy
RHS <- deriv(subsmvp(x,v,s),"y")           # dx/dy

LHS - RHS # should be zero
```

---

 drop

*Drop empty variables*


---

## Description

Convert an.mvp object which is a pure constant into a scalar whose value is the coefficient of the empty term.

A few functions in the package (notably `subs()`) take a `drop` argument that behaves much like the `drop` argument in base extraction.

Function `drop()` is an S4 generic, which is why the package calls `setOldClass()`.

Function `drop()` was formerly called `lose()`.

## Usage

```
## S4 method for signature 'mvp'
drop(x)
```

## Arguments

`x`                    Object of class.mvp

## Author(s)

Robin K. S. Hankin

## See Also

[subs](#)

## Examples

```
(m1 <- as.mvp("1+bish +bash^2 + bosh^3"))
(m2 <- as.mvp("bish +bash^2 + bosh^3"))

m1-m2            # an.mvp object
drop(m1-m2)     # numeric
```

---

 horner

*Horner's method*


---

**Description**

Horner's method for multivariate polynomials

**Usage**

horner(P, v)

**Arguments**

P	Multivariate polynomial
v	Numeric vector of coefficients

**Details**

Given a polynomial

$$p(x) = a_0 + a_1 + a_2x^2 + \cdots + a_nx^n$$

it is possible to express  $p(x)$  in the algebraically equivalent form

$$p(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + xa_n)\cdots))$$

which is much more efficient for evaluation, as it requires only  $n$  multiplications and  $n$  additions, and this is optimal. But this is not implemented here because it's efficient. It is implemented because it works if  $x$  is itself a (multivariate) polynomial, and that is the second coolest thing ever. The coolest thing ever is the Reduce() function.

**Author(s)**

Robin K. S. Hankin

**See Also**

[oom](#)

**Examples**

```
horner("x", 1:5)
horner("x+y", 1:3)

w <- as.mvp("x+y^2")
stopifnot(1 + 2*w + 3*w^2 == horner(w, 1:3)) # note off-by-one issue

"x+y+x*y" |> horner(1:3) |> horner(1:2)
```

---

invert	<i>Replace symbols with their reciprocals</i>
--------	---

---

**Description**

Given an mvp object, replace one or more symbols with their reciprocals

**Usage**

```
invert(p, v)
```

**Arguments**

p	Object (coerced to) mvp form
v	Character vector of symbols to be replaced with their reciprocal; missing interpreted as replace all symbols

**Author(s)**

Robin K. S. Hankin

**See Also**

[subs](#)

**Examples**

```
invert("x")  
  
(P <- as.mvp("1+a+6*a^2 -7*a*b"))  
invert(P, "a")
```

---

kahle	<i>A sparse multivariate polynomial</i>
-------	---

---

**Description**

A sparse multivariate polynomial inspired by Kahle (2013)

**Usage**

```
kahle(n = 26, r = 1, p = 1, coeffs = 1, symbols = letters)
```

**Arguments**

n	Number of different symbols to use
r	Number of symbols in a single term
p	Power of each symbol in each terms
coeffs	Coefficients of the terms
symbols	Alphabet of symbols

**Author(s)**

Robin K. S. Hankin

**References**

David Kahle 2013. “**mpoly**: multivariate polynomials in R”. *R Journal*, volume 5/1.

**See Also**

[special](#)

**Examples**

```
kahle() # a+b+...+z
kahle(r=2,p=1:2) # Kahle's original example

## example where.mvp runs faster than spray (mvp does not need a 200x200 matrix):
k <- kahle(200,r=3,p=1:3,symbols=paste("x",sprintf("%02d",1:200),sep=""))
system.time(ignore <- k^2)
#system.time(ignore <-.mvp_to_spray(k)^2) # needs spray package loaded
```

---

knight

*Chess knight*

---

**Description**

Generating function for a chess knight on an infinite  $d$ -dimensional chessboard

**Usage**

```
knight(d, can_stay_still = FALSE)
```

**Arguments**

d	Dimension of the board
can_stay_still	Boolean, with default FALSE meaning that the knight is obliged to move and FALSE meaning that it has the option of remaining on its square

**Note**

The function is a slight modification of `spray::knight()`.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
knight(2)          # regular chess knight on a regular chess board
knight(2, TRUE)   # regular chess knight that can stay still

# Q: how many ways are there for a 3D knight to return to its starting
# square after four moves?

# A:
constant(knight(3)^4)

# Q ...and how many ways in four moves or fewer?

# A1:
constant(knight(3, TRUE)^4)

# A2:
constant((1+knight(3))^4)
```

---

laurent_factor	<i>Laurent factor</i>
----------------	-----------------------

---

**Description**

Given a multivariate polynomial  $P$ , possibly with symbols raised to a negative power, return the minimal one-term polynomial required to lift  $P$  to a standard multivariate polynomial:  $P * \text{laurent\_factor}(P)$  is guaranteed to have no negative powers.

**Usage**

```
laurent_factor(P)
```

**Arguments**

`P`                      Object of class `mvp`

**Details**

The returned polynomial has coefficient 1.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
laurent_factor(invert(rmvp()))
P <- rmvp() * invert(rmvp())
P * laurent_factor(P)
```

---

letters

*Single-letter symbols*

---

**Description**

Variables a, b, ... z are given their.mvp semantic meaning.

**Usage**

```
data(lettersymbols)
```

**Details**

Twenty-six variables a-z are defined as their.mvp semantic equivalent:

```
a <- as.mvp("a")
...
z <- as.mvp("z")
```

These objects can be generated by running script inst/symb.Rmd, which includes some further discussion and technical documentation and creates file lettersymbols.rda which resides in the data/ directory.

Letters c, q, and t might pose difficulties.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
data(lettersymbols)
(a+b)*(a-b)

(x + y + z)^3 - 3*(x + y + z)*(x*y + x*z + y*z) + 3*x*y*z
```

---

lowlevel

*Low level functions*

---

## Description

Various low-level functions that call the C routines

## Usage

```
mvp_substitute(allnames,allpowers,coefficients,v,values)
mvp_substitute_mvp(allnames1, allpowers1, coefficients1, allnames2, allpowers2,
  coefficients2, v)
mvp_vectorised_substitute(allnames, allpowers, coefficients, M, nrows, ncols, v)
mvp_prod(allnames1,allpowers1,coefficients1,allnames2,allpowers2,coefficients2)
mvp_add(allnames1, allpowers1, coefficients1, allnames2, allpowers2,coefficients2)
simplify(allnames,allpowers,coefficients)
mvp_deriv(allnames, allpowers, coefficients, v)
mvp_power(allnames, allpowers, coefficients, n)
```

## Arguments

allnames, allpowers, coefficients, allnames1, allpowers1, coefficients1,  
allnames2, allpowers2, coefficients2, v, values, n, M, nrows, ncols  
Variables sent to the C routines

## Details

These functions call the functions defined in `RcppExports.R`

## Note

These functions are not intended for the end-user. Use the syntactic sugar (as in `a+b` or `a*b` or `a^n`), or functions like `mvp_plus_mvp()`, which are more user-friendly.

## Author(s)

Robin K. S. Hankin

---

`mmvp`*Spray-style specification of.mvp objects*

---

## Description

Create an `mvp` object using an index matrix and a vector of coefficients, **spray**-style.

## Usage

```
mmvp(M, coeffs, vars)
```

## Arguments

<code>M</code>	Integer matrix with rows corresponding to terms of a multivariate polynomial
<code>coeffs</code>	Numeric vector of coefficients; if missing, default to a vector of 1s
<code>vars</code>	Character vector of variable names

## Details

Function `mmvp()` uses syntax similar to `spray::spray()` but adapted for the **mvp** package.

If argument `vars` is missing, variables are named after the `colnames` of `M`.

The function has special dispensation if `M` is a **partitions** object. So, `mmvp(parts(5))` and `mmvp(blockparts(1:5, 3))` and `mmvp(compositions(4, 3))` and `mmvp(multiset(c(1, 2, 2, 3)))` work as expected [returning homogeneous polynomials].

## Value

Returns a multivariate polynomial, an object of class `mvp`

## Author(s)

Robin K. S. Hankin

## Examples

```
mmvp(matrix(1:35, 5, 7))
```

```
mmvp(matrix(rpois(21,1), ncol=3), 1:7)
```

---

mpoly

*Conversion to and from mpoly form*

---

### Description

The **mpoly** package by David Kahle provides similar functionality to this package, and the functions documented here convert between mpoly and.mvp objects. The.mvp package uses `mpoly::mp()` to convert character strings to.mvp objects.

### Usage

```
mpoly_to.mvp(m)
## S3 method for class 'mvp'
as.mpoly(x,...)
```

### Arguments

<code>m</code>	object of class.mvp
<code>x</code>	object of class.mpoly
<code>...</code>	further arguments, currently ignored

### Author(s)

Robin K. S. Hankin

### Examples

```
x <- rmvp(5)
x == mpoly_to.mvp(mpoly::as.mpoly(x))      # should be TRUE
```

---

mvp

*Multivariate polynomials,.mvp objects*

---

### Description

Create, test for, and coerce to,.mvp objects

**Usage**

```

mvp(vars, powers, coeffs)
is_ok_mvp(vars, powers, coeffs)
is.mvp(x)
as.mvp(x)
## S3 method for class 'character'
as.mvp(x)
## S3 method for class 'list'
as.mvp(x)
## S3 method for class 'mpoly'
as.mvp(x)
## S3 method for class 'mvp'
as.mvp(x)
## S3 method for class 'numeric'
as.mvp(x)

```

**Arguments**

vars	List of variables comprising each term of an mvp object
powers	List of powers corresponding to the variables of the vars argument
coeffs	Numeric vector corresponding to the coefficients to each element of the var and powers lists
x	Object to be coerced to or tested for being class mvp

**Details**

Function `mvp()` is the formal creation mechanism for mvp objects. However, it is not very user-friendly; it is better to use `as.mvp()` in day-to-day use.

Function `is_ok_mvp()` checks for consistency of its arguments.

**Note**

Interactive development environments such as RStudio may invoke subsetting operations internally for object inspection, particularly the Environment pane. These calls may trigger warnings on the console, typically:

```

Warning message:
In jj[] <- value :
  number of items to replace is not a multiple of replacement length

```

Such warnings are expected in interactive inspection contexts and do not indicate an error in user code or package functionality. Just ignore them.

**Author(s)**

Robin K. S. Hankin

**Examples**

```

mvp(list("x", c("x","y"), "a", c("y","x")), list(1,1:2,3,c(-1,4)), 1:4)

## Note how the terms appear in an arbitrary order, as do
## the symbols within a term.

kahle <- mvp(
  vars = split(cbind(letters,letters[c(26,1:25)]),rep(seq_len(26),each=2)),
  powers = rep(list(1:2),26),
  coeffs = 1:26
)
kahle
## again note arbitrary order of terms and symbols within a term

## Standard arithmetic rules apply:

a <- as.mvp("1 + 4*x*y + 7*z")
b <- as.mvp("-7*z + 3*x^34 - 2*z*x")

a+b
a*b^2

(a+b)*(a-b) == a^2-b^2 # should be TRUE

## variable "xy" is distinct from "x*y":

as.mvp("x + y + xy")^2
as.mvp(paste(state.name[1:5],collapse="+"))^2

```

---

oom

*One over one minus a multivariate polynomial*


---

**Description**

Uses Taylor's theorem to give one over one minus a multipol:

$$\frac{1}{1-x} = 1 + x + x^2 + \dots$$

Function oom() requires an argument n and returns

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n$$

**Usage**

```
oom(P, n)
```

**Arguments**

n	Order of expansion
P	Multivariate polynomial

**Author(s)**

Robin K. S. Hankin

**See Also**

[horner](#)

**Examples**

```
oom("x", 5)
oom("x", 5) * as.mvp("1-x") # 1 + 0(x^6)
```

```
oom("x+y", 4)
```

```
"x+y" |> oom(5) |> aderiv(x=2, y=1)
```

---

Ops.mvp

*Arithmetic Ops Group Methods for.mvp objects*

---

**Description**

Allows arithmetic operators to be used for multivariate polynomials such as addition, multiplication, integer powers, etc.

**Usage**

```
## S3 method for class 'mvp'
Ops(e1, e2)
mvp_negative(S)
mvp_reciprocal(S)
mvp_times_mvp(S1, S2)
mvp_times_scalar(S, x)
mvp_plus_mvp(S1, S2)
mvp_plus_numeric(S, x)
mvp_eq_mvp(S1, S2)
mvp_modulo(S1, S2)
```

**Arguments**

e1, e2, S, S1, S2	Objects of class.mvp
x	Scalar, length one numeric vector

## Details

The function `Ops.mvp()` passes unary and binary arithmetic operators “+”, “-”, “\*” and “^” to the appropriate specialist function.

The most interesting operator is “\*”, which is passed to `mvp_times.mvp()`. I guess “+” is quite interesting too.

The caret “^” denotes arithmetic exponentiation, as in  $x^3 = x * x * x$ . As an experimental feature, this is (sort of) vectorised: if  $n$  is a vector, then  $a^n$  returns the sum of  $a$  raised to the power of each element of  $n$ . For example,  $a^c(n1, n2, n3)$  is  $a^{n1} + a^{n2} + a^{n3}$ . Internally,  $n$  is tabulated in the interests of efficiency, so  $a^c(0, 2, 5, 5, 5) = 1 + a^2 + 3a^5$  is evaluated with only a single fifth power. Similar functionality is implemented in the **freealg** package.

Function `mvp_reciprocal()` returns the reciprocal of a one-term `mvp` object. If its argument is not a one-term `mvp` object, an error is returned.

Function `mvp_modulo()` reduces an `mvp` object using modular arithmetic. Here, `mvp_modulo(S, p)` returns  $S$  modulo  $p$  (despite the argument names) where  $p$  is prime. Package idiom includes `P %% n`.

## Value

The high-level functions documented here return an object of `mvp`, the low-level functions documented at `lowlevel.Rd` return lists. But don’t use the low-level functions.

## Note

Function `mvp_modulo()` is distinctly sub-optimal and `inst/mvp_modulo.Rmd` details ideas for better implementation.

## Author(s)

Robin K. S. Hankin

## See Also

[lowlevel](#), [laurent\\_factor](#)

## Examples

```
(p1 <- rmvp(3))
(p2 <- rmvp(3))

p1*p2

p1+p2

p1^3

p1*(p1+p2) == p1^2+p1*p2 # should be TRUE
```

---

print

*Print methods for.mvp objects*

---

### Description

Print methods for.mvp objects: to print, an.mvp object is coerced to.mpoly form and the.mpoly print method used.

### Usage

```
## S3 method for class 'mvp'  
print(x, ...)
```

### Arguments

x	Object of class.mvp, coerced to.mpoly form
...	Further arguments

### Details

See.mvp.Rd for notes on interactive inspection in Rstudio.

### Value

Returns its argument invisibly

### Author(s)

Robin K. S. Hankin

### See Also

[mvp](#)

### Examples

```
a <- rmvp(4)  
a  
print(a)  
print(a,stars=TRUE)  
print(a,varorder=rev(letters))
```

**Description**

Random multivariate polynomials, intended as quick “get you going” examples of mvp objects

**Usage**

```
rhmv(n=7, size=4, pow=6, symbols=6)
rmvp(n=7, size=4, pow=6, symbols=6)
rmvpp(n=30, size=9, pow=20, symbols=15)
rmvppp(n=100, size=15, pow=99, symbols)
```

**Arguments**

n	Number of terms to generate
size	Maximum number of symbols in each term
pow	Maximum power of each symbol
symbols	Symbols to use; if numeric, interpret as the first symbols letters of the alphabet

**Details**

Function rhmv() returns a random homogeneous mvp. Function rmvp() returns a possibly nonhomogeneous mvp and functions rmvpp() and rmvppp() return, by default, progressively more complicated mvp objects. Function rmvppp() returns a polynomial with multi-letter variable names.

**Value**

Returns a multivariate polynomial, an object of class mvp

**Author(s)**

Robin K. S. Hankin

**Examples**

```
rhmv()
rmvp()
rmvpp()
rmvppp()
```

series

*Decomposition of multivariate polynomials by powers***Description**

Power series of multivariate polynomials, in various forms

**Usage**

```
trunc(S,n)
truncall(S,n)
trunc1(S,...)
series(S,v,showsymb=TRUE)
## S3 method for class 'series'
print(x,...)
onevarpow(S,...)
taylor(S,vx,va,debug=FALSE)
mvp_taylor_onevar(allnames,allpowers,coefficients, v, n)
mvp_taylor_allvars(allnames,allpowers,coefficients, n)
mvp_taylor_onepower_onevar(allnames, allpowers, coefficients, v, n)
mvp_to_series(allnames, allpowers, coefficients, v)
```

**Arguments**

S	Object of class mvp
n	Non-negative integer specifying highest order to be retained
v	Variable to take Taylor series with respect to. If missing, total power of each term is used (except for series() where it is mandatory)
x, ...	Object of class series and further arguments, passed to the print method; in trunc1() a list of variables to truncate
showsymb	In function series(), Boolean, with default TRUE meaning to substitute variables like x_m_foo with (x-foo) for readability reasons; see the vignette for a discussion
vx, va, debug	In function taylor(), names of variables to take series with respect to; and a Boolean with default FALSE meaning to return the mvp and TRUE meaning to return the string that is passed to eval()
allnames, allpowers, coefficients	Components of mvp objects

**Details**

Function onevarpow() returns just the terms in which the symbols corresponding to the named arguments have powers equal to the arguments' powers. Thus:

```

onevarpow(as.mvp("x*y*z + 3*x*y^2 + 7*x*y^2*z^6 + x*y^3"),x=1,y=2)
mvp object algebraically equal to
3 + 7 z^6

```

Above, we see that only the terms with  $x^1y^2$  have been extracted, corresponding to arguments  $x=1, y=2$ .

Function `series()` returns a power series expansion of powers of variable  $v$ . The value returned is a list of three elements named `mvp`, `varpower`, and `variablename`. The first element is a list of `mvp` objects and the second is an integer vector of powers of variable  $v$  (element `variablename` is a character string holding the variable name, argument  $v$ ).

Function `trunc(S, n)` returns the terms of  $S$  with the sum of the powers of the variables  $\leq n$ . Alternatively, it discards all terms with total power  $> n$ .

Function `trunc1()` is similar to `trunc()`. It takes a `mvp` object and an arbitrary number of named arguments, with names corresponding to variables and their values corresponding to the highest power in that variable to be retained. Thus `trunc1(S, x=2, y=4)` will discard any term with variable  $x$  raised to the power 3 or above, and also any term with variable  $y$  raised to the power 5 or above. The highest power of  $x$  will be 2 and the highest power of  $y$  will be 4.

Function `truncall(S, n)` discards any term of  $S$  with any variable raised to a power greater than  $n$ .

Function `series()` returns an object of class `series`; the print method for `series` objects is sensitive to the value of `getOption("mvp_mult_symbol")`; set this to `"*` to get `mpoly`-compatible output.

Function `taylor()` is a convenience wrapper for `series()`.

Functions `mvp_taylor_onevar()`, `mvp_taylor_allvars()` and `mvp_to_series()` are low-level helper functions that are not intended for the user.

### Author(s)

Robin K. S. Hankin

### See Also

[deriv](#)

### Examples

```

trunc(as.mvp("1+x")^6,2)

trunc(as.mvp("1+x+y")^3,2)      # discards all terms with total power>2
trunc1(as.mvp("1+x+y")^3,x=2)   # terms like y^3 are treated as constants

trunc(as.mvp("1+x+y^2")^3,3)    # discards x^2y^2 term (total power=4>3)
truncall(as.mvp("1+x+y^2")^3,3) # retains x^2y^2 term (all vars to power 2)

onevarpow(as.mvp("1+x*x*y^2 + z*y^2*x"),x=1,y=2)

(p2 <- rmvp(10))
series(p2,"a")

```

```

# Works well with pipes:

f <- function(n){as.mvp(sub('n',n,'1+x^n*y'))}
Reduce(`*`,lapply(1:6,f)) |> series('y')
Reduce(`*`,lapply(1:6,f)) |> series('x')

(p <- horner("x+y",1:4))
onevarpow(p,x=2) # coefficient of x^2
onevarpow(p,x=3) # coefficient of x^3

p |> trunc(2)
p |> trunc1(x=2)
(p |> subs(x="x+dx") -p) |> trunc1(dx=2)

# Nice example of Horner's method:
(p <- as.mvp("x + y + 3*x*y"))
trunc(horner(p,1:5)*(1-p)^2,4) # should be 1

## Third order taylor expansion of f(x)=sin(x+y) for x=1.1, about x=1:
(sinxpy <- horner("x+y",c(0,1,0,-1/6,0,+1/120,0,-1/5040,0,1/362880))) # sin(x+y)
dx <- as.mvp("dx")
t3 <- sinxpy + aderiv(sinxpy,x=1)*dx + aderiv(sinxpy,x=2)*dx^2/2 + aderiv(sinxpy,x=3)*dx^3/6
t3 <- t3 |> subs(x=1,dx=0.1) # t3 = Taylor expansion of sin(y+1.1)
t3 |> subs(y=0.3) - sin(1.4) # numeric; should be small

```

---

special

*Various functions to create simple multivariate polynomials*


---

## Description

Various functions to create simple.mvp objects such as single-term, homogeneous, and constant multivariate polynomials.

## Usage

```

product(v,symbols=letters)
homog(d,power=1,symbols=letters)
linear(x,power=1,symbols=letters)
xyz(n,symbols=letters)
numeric_to_mvp(x)

```

## Arguments

d, n	An integer; generally, the dimension or arity of the resulting.mvp object
v, power	Integer vector of powers
x	Numeric vector of coefficients
symbols	Character vector for the symbols

**Value**

All functions documented here return a mvp object

**Note**

The functions here are related to their equivalents in the multicol and spray packages, but are not exactly the same.

Function constant() is documented at constant.Rd, but is listed below for convenience.

**Author(s)**

Robin K. S. Hankin

**See Also**

[constant](#), [zero](#)

**Examples**

```
product(1:3)      # a * b^2 * c^3
homog(3)         # a + b + c
homog(3,2)       # a^2 + a b + a c + b^2 + b c + c^2
linear(1:3)      # 1*a + 2*b + 3*c
constant(5)      # 5
xyz(5)           # a*b*c*d*e
```

---

subs

*Substitution*

---

**Description**

Substitute symbols in an mvp object for numbers or other multivariate polynomials

**Usage**

```
subs(S, ..., drop = TRUE)
subvec(S, ...)
subsmvp(S, v, X)
varchange(S, ...)
varchange_formal(S, old, new)
namechanger(x, old, new)
```

**Arguments**

S, X	Multivariate polynomials
...	Named arguments corresponding to variables to substitute
drop	Boolean with default TRUE meaning to return a scalar (the constant) in place of a constant.mvp object
v	A string corresponding to the variable to substitute
old, new, x	The old and new variable names respectively; x is a character vector

**Details**

Function `subs()` substitutes variables for.mvp objects, using a natural R idiom. Observe that this type of substitution is sensitive to order:

```
> p <- as.mvp("a b^2")
> subs(p, a="b", b="x")
.mvp object algebraically equal to
x^3
> subs(p, b="x", a="b") # same arguments, different order
.mvp object algebraically equal to
b x^2
```

Function `subsmvp()` is a lower-level function, not really intended for the end-user. It takes a.mvp object and substitutes another.mvp object for a specific symbol.

Function `subvec()` substitutes **all** the symbols of S with numerical values. It is vectorised in its ellipsis arguments with recycling rules and names behaviour inherited from `cbind()`. However, if the first element of ... is a matrix, then this is interpreted by rows, with symbol names given by the matrix column names; further arguments are ignored. Unlike `subs()`, this function is generally only useful if all symbols are given a value; unassigned symbols take a value of zero.

Function `varchange()` makes a *formal* variable substitution. It is useful because it can take non-standard variable names such as "(a-b)" or "?", and is used in `taylor()`. Function `varchange_formal()` does the same task, but takes two character vectors, `old` and `new`, which might be more convenient than passing named arguments. Remember that non-standard names might need to be quoted; also you might need to escape some characters, see the examples. Function `namechanger()` is a low-level helper function that uses regular expression idiom to substitute variable names.

There is limited support for substitution for variables raised to negative powers.

**Value**

Functions `subs()` and `subsmvp()` return a multivariate polynomial unless `drop` is TRUE in which case a length one numeric vector is returned. Function `subvec()` returns a numeric vector (sic! the output inherits its order from the arguments).

**Author(s)**

Robin K. S. Hankin

**See Also**[drop](#)**Examples**

```

p <- rmvp(6, 2, 2, letters[1:3])
p
subs(p, a=1)
subs(p, a=1, b=2)

subs(p, a="1+b x^3", b="1-y")
subs(p, a=1, b=2, c=3, drop=FALSE)

do.call(subs, c(list(as.mvp("z")), rep(c(z="C+z^2"),5)))

subvec(p, a=1, b=2, c=1:5) # supply a named list of vectors

M <- matrix(sample(1:3, 26*3, replace=TRUE), ncol=26)
colnames(M) <- letters
rownames(M) <- c("Huey", "Dewie", "Louie")
subvec(kahle(r=3, p=1:3), M) # supply a matrix

varchange(as.mvp("1+x+xy + x*y"), x="newx") # variable xy unchanged

kahle(5,3,1:3) |> subs(a="a + delta")

varchange(p, a="]") # nonstandard variable names OK

varchange_formal(p, "\\]", "a")

## OEIS A004019:
x <- as.mvp("x")
p <- (1+x)^2
sapply(1:5,function(n){constant(do.call(subs, c(list(x), rep(list(x=p),n))))})

## substitution for negative powers is implemented where defined:

x <- as.mvp("x")
y <- as.mvp("y")
P <- 5 + x + y + 1/x + x/y^4 + y/x^4*7
subs(P, x=1.1)
subs(P, x="x*z", y="a*b")

```

**Description**

Summary methods for.mvp objects and extraction of typical terms

**Usage**

```
## S3 method for class 'mvp'
summary(object, ...)
## S3 method for class 'summary.mvp'
print(x, ...)
rtypical(object,n=3)
```

**Arguments**

x, object	Multivariate polynomial, class.mvp
n	In.rtypical(), number of terms (in addition to the constant) to select
...	Further arguments, currently ignored

**Details**

The summary method prints out a list of interesting facts about an.mvp object such as the longest term or highest power. Function.rtypical() extracts the constant if present, and a.random selection of terms of its argument.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
summary(rmvp(40))
rtypical(rmvp(40))
```

---

zero

*The zero polynomial*

---

**Description**

Test for a multivariate polynomial being zero

**Usage**

```
is.zero(x)
```

**Arguments**

x	Object of class.mvp
---	---------------------

### Details

Function `is.zero()` returns TRUE if `x` is indeed the zero polynomial. It is defined as `length(vars(x))==0` for reasons of efficiency, but conceptually it returns `x==constant(0)`.

(Use `constant(0)` to create the zero polynomial).

### Note

I would have expected the zero polynomial to be problematic (cf the **freegroup** and **permutations** packages, where similar issues require extensive special case treatment). But it seems to work fine, which is a testament to the robust coding in the STL.

A general mvp object is something like

```
{{"x" -> 3, "y" -> 5} -> 6, {"x" -> 1, "z" -> 8} -> -7}}
```

which would be  $6x^3y^5 - 7xz^8$ . The zero polynomial is just `{}`. Neat, eh?

### Author(s)

Robin K. S. Hankin

### See Also

[constant](#)

### Examples

```
constant(0)
```

```
t1 <- as.mvp("x+y")
```

```
t2 <- as.mvp("x-y")
```

```
stopifnot(is.zero(t1*t2-as.mvp("x^2-y^2")))
```

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