Package: multiwave (via r-universe)

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Title Estimation of Multivariate Long-Memory Models Parameters

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Depends signal, R (>= 3.5)

Description Computation of an estimation of the long-memory parameters and the long-run covariance matrix using a multivariate model (Lobato (1999) <doi:10.1016/S0304-4076(98)00038-4>; Shimotsu (2007) <doi:10.1016/j.jeconom.2006.01.003>). Two semi-parametric methods are implemented: a Fourier based approach (Shimotsu (2007) <doi:10.1016/j.jeconom.2006.01.003>) and a wavelet based approach (Achard and Gannaz (2016) <doi:10.1111/jtsa.12170>; Achard and Gannaz (2024) <doi:10.1111/jtsa.12719>). Real and complex wavelets are implemented.

License GPL (>= 2)

LazyData true

NeedsCompilation no

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Contents

`multiwave-package` *Estimation of multivariate long-memory models parameters: memory parameters and long-run covariance matrix (also called fractal connectivity).*

Description

This package computes an estimation of the long-memory parameters and the long-run covariance matrix using a multivariate model (Lobato, 1999; Shimotsu 2007). Two semi-parametric methods are implemented: a Fourier based approach (Shimotsu 2007) and a wavelet based approach (Achard and Gannaz 2014; Achard and Gannaz (2024) <doi:10.1111/jtsa.12719>). Real and complex wavelets are implemented.

Details

Package:	multiwave
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License:	GPL (>= 2)

Author(s)

Sophie Achard and Irene Gannaz

Maintainer: Sophie Achard <sophie.achard@gipsa-lab.fr>, Irene Gannaz <irene.gannaz@insa-lyon.fr>

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. Journal of Time Series Analysis, Vol 45, pp 421-443.

Examples

```
rho<-0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d<-c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)

x <- resp$x
long_run_cov <- resp$long_run_cov

#### Compute wavelets this is also included in the functions without _wav
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(1,11)

if(is.matrix(x)){
    N <- dim(x)[1]
    k <- dim(x)[2]
}else{
```

```
    N <- length(x)
    k <- 1
}
mat_x <- as.matrix(x,dim=c(N,k))

## Wavelet decomposition
xwav <- matrix(0,N,k)
    for(j in 1:k){
        xx <- mat_x[,j]
        resw <- DWTexact(xx,filter)
        xwav_temp <- resw$dwt
        index <- resw$indmaxband
        Jmax <- resw$Jmax
        xwav[1:index[Jmax],j] <- xwav_temp;
    }
## we free some memory
new_xwav <- matrix(0,min(index[Jmax],N),k)
    if(index[Jmax]<N){
        new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
    }
    xwav <- new_xwav
    index <- c(0,index)


##### Compute the wavelet functions
res_psi <- psi_hat_exact(filter,J)
psih<-res_psi$psih
grid<-res_psi$grid


##### Estimate using Fourier #############

m <- floor(N^{0.65}) ## default value of Shimotsu
res_mfw <- mfw(x,m)
res_d_mfw<-res_mfw$d
res_rho_mfw<-res_mfw$cov[1,2]

### Eval MFW

res_mfw_eval <- mfw_eval(d,x,m)
res_mfw_cov_eval <- mfw_cov_eval(d,x,m)

###### Estimate using Wavelets #############

## Using xwav

if(dim(xwav)[2]==1) xwav<-as.vector(xwav)
res_mww_wav <- mww_wav(xwav,index,psih,grid,LU)

### Eval MWW_wav

res_mww_wav_eval <- mww_wav_eval(d,xwav,index,LU)
res_mww_wav_cov_eval <- mww_wav_cov_eval(d,xwav,index,psih,grid,LU)
```

```
## Using directly the time series

res_mww <- mww(x,filter,LU)
res_d_mww<-res_mww$d
res_rho_mww<-res_mww$cov[1,2]

### Eval MWW_wav

res_mww_eval <- mww_eval(d,x,filter,LU)
res_mww_cov_eval <- mww_cov_eval(d,x,filter,LU)
```

`brainHCP` *Time series obtained by an fMRI experiment on the brain*

Description

Time series for each region of interest in the brain. These series are obtained by SPM preprocessing.

Usage

```
data(brainHCP)
```

Format

A data frame with 1200 observations on the following 89 variables.

Source

contact S. Achard (sophie.achard@gipsa-lab.fr)

References

M. Termenon, A. Jaillard, C. Delon-Martin, S. Achard (2016) Reliability of graph analysis of resting state fMRI using test-retest dataset from the Human Connectome Project, *Neuroimage*, Vol 142, pages 172-187.

Examples

```
data(brainHCP)
## maybe str(brainHCP) ; plot(brainHCP) ...
```

`compute_nj` *Wavelets coefficients utilities*

Description

Computes the number of wavelet coefficients at each scale.

Usage

```
compute_nj(n, N)
```

Arguments

`n`	sample size.
`N`	filter length.

Value

`nj`	number of coefficients at each scale.
`J`	Number of scales.

Author(s)

S. Achard and I. Gannaz

References

G. Fay, E. Moulines, F. Roueff, M. S. Taqqu (2009) Estimators of long-memory: Fourier versus wavelets. *Journal of Econometrics*, vol. 151, N. 2, pages 159-177.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`DWTexact`, `scaling_filter`

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
n <- 5^10
N <- length(filter)
compute_nj(n,N)
```

`convmtx` *Convolution matrix*

Description

Returns the convolution matrix, A, associated to the filter v such that the product of A and an n-element vector, x, is the convolution of v and x.

Usage

```
convmtx(v,n)
```

Arguments

`v`	A filter.
`n`	Size of the convolution matrix.

Value

The convolution matrix, A, associated to the filter v such that the product of A and an n-element vector, x, is the convolution of v and x.

Author(s)

Achard, Clausel, Gannaz, Roueff (2017)

References

S. Achard, M. Clausel, I. Gannaz, F. Roueff (2020). New results on approximate Hilbert pairs of wavelet filters with common factor structure. *Applied and Computational Harmonic Analysis*, Vol 49, N.3, pp 1025-1045.

`DWTcomplex` *Exact discrete wavelet decomposition with common-factor wavelets*

Description

Computes the discrete wavelet transform of the data using the pyramidal algorithm.

Usage

```
DWTcomplex(x, filter, real)
```

Arguments

`x`	vector of raw data
`filter`	Common-factor wavelet filters, as returned by the `hwlet` function, or real wavelet filters, as returned by the `sclaing_filter` function.
`real`	Precise if the filter is a real filter (obtained with `scaling_filter`) or a complex filter (obtained with `hwlet`). The default value is FALSE

Value

`dwt`	computable Wavelet coefficients without taking into account the border effect.
`indmaxband`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\texttt{dwt}(k)$ for $k \in [\texttt{indmaxband}(j-1)+1, \texttt{indmaxband}(j)]$ and for $j = 1$, $\texttt{dwt}(k)$ for $k \in [1, \texttt{indmaxband}(1)]$.
`Jmax`	largest available scale index (=length of `indmaxband`).

Author(s)

S. Achard and I. Gannaz

References

S. Achard, M. Clausel, I. Gannaz, F. Roueff (2020). New results on approximate Hilbert pairs of wavelet filters with common factor structure. *Applied and Computational Harmonic Analysis*, Vol 49, N.3, pp 1025-1045.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`hwlet`

Examples

```
filter_complex <- hwlet(M=4,L=4,type='const')
u <- rnorm(2^10,0,1)
x <- vfracdiff(u,d=0.2)

 resw <- DWTcomplex(x,filter_complex)
  xwav <- resw$dwt
  index <- resw$indmaxband
  Jmax <- resw$Jmax

## Wavelet scale 1
ws_1 <- xwav[1:index[1]]
## Wavelet scale 2
ws_2 <- xwav[(index[1]+1):index[2]]
## Wavelet scale 3
ws_3 <- xwav[(index[2]+1):index[3]]
### upto Jmax
```

`DWTexact` *Exact discrete wavelet decomposition*

Description

Computes the discrete wavelet transform of the data using the pyramidal algorithm.

Usage

```
DWTexact(x, filter)
```

Arguments

`x`	vector of raw data
`filter`	Quadrature mirror filter (also called scaling filter, as returned by the `scaling_filter` function)

Value

`dwt`	computable Wavelet coefficients without taking into account the border effect.
`indmaxband`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\texttt{dwt}(k)$ for $k \in [\texttt{indmaxband}(j-1)+1, \texttt{indmaxband}(j)]$ and for $j = 1$, $\texttt{dwt}(k)$ for $k \in [1, \texttt{indmaxband}(1)]$.
`Jmax`	largest available scale index (=length of `indmaxband`).

Note

This function was rewritten from an original matlab version by Fay et al. (2009)

Author(s)

S. Achard and I. Gannaz

References

G. Fay, E. Moulines, F. Roueff, M. S. Taqqu (2009) Estimators of long-memory: Fourier versus wavelets. *Journal of Econometrics*, vol. 151, N. 2, pages 159-177.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`scaling_filter`

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
u <- rnorm(2^10,0,1)
x <- vfracdiff(u,d=0.2)

 resw <- DWTexact(x,filter)
  xwav <- resw$dwt
  index <- resw$indmaxband
  Jmax <- resw$Jmax

## Wavelet scale 1
ws_1 <- xwav[1:index[1]]
## Wavelet scale 2
ws_2 <- xwav[(index[1]+1):index[2]]
## Wavelet scale 3
ws_3 <- xwav[(index[2]+1):index[3]]
### upto Jmax
```

`fivarma` *simulation of FIVARMA process*

Description

Generates N observations of a realisation of a multivariate FIVARMA process X.

Usage

```
fivarma(N, d = 0, cov_matrix = diag(length(d)), VAR = NULL,
             VMA = NULL,skip = 2000)
```

Arguments

`N`	number of time points.
`d`	vector of parameters of long-memory.
`cov_matrix`	matrix of correlation between the innovations (optional, default is identity).
`VAR`	array of VAR coefficient matrices (optional).
`VMA`	array of VMA coefficient matrices (optional).
`skip`	number of initial observations omitted, after applying the ARMA operator and the fractional integration (optional, the default is 2000).

Details

Let $(e(t))_t$ be a multivariate gaussian process with a covariance matrix cov_matrix. The values of the process X are given by the equations:

$$VAR(L)U(t) = VMA(L)e(t),$$

and

$$diag((1-L)^d)X(t) = U(t)$$

where `L` is the lag-operator.

Value

`x`	vector containing the N observations of the vector ARFIMA(arlags, d, malags) process.
`long_run_cov`	matrix of covariance of the spectral density of x around the zero frequency.
`d`	vector of parameters of long-range dependence, modified in case of cointegration.

Author(s)

S. Achard and I. Gannaz

References

R. J. Sela and C. M. Hurvich (2009) Computationaly efficient methods for two multivariate fractionnaly integrated models. *Journal of Time Series Analysis*, Vol 30, N. 6, pages 631-651.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`varma`, `vfracdiff`

Examples

```
rho1 <- 0.3
rho2 <- 0.8
cov <- matrix(c(1,rho1,rho2,rho1,1,rho1,rho2,rho1,1),3,3)
d <- c(0.2,0.3,0.4)

J <- 9
N <- 2^J
VMA <- diag(c(0.4,0.1,0))
### or another example VAR <- array(c(0.8,0,0,0,0.6,0,0,0,0.2,0,0,0,0,0.4,0,0,0,0.5),dim=c(3,3,2))
VAR <- diag(c(0.8,0.6,0))
```

```
resp <- fivarma(N, d, cov_matrix=cov, VAR=VAR, VMA=VMA)
x <- resp$x
long_run_cov <- resp$long_run_cov
d <- resp$d
```

---

hwlet *Common-Factor wavelet filter coefficients*

Description

Provides the Hilbert transform pair of orthogonal wavelet bases given by Common factor construction of Selesnick (2001), with perfect reconstruction condition

Usage

```
hwlet(M,L,type)
```

Arguments

`M`	Number of vanishing moments.
`L`	Degree of fractional delay.
`type`	Type of factorization of the common factors. If 'mid' the factorisation of the Bezout solution is obtained with all roots of absolute magnitude less than 1. Three possible values are 'min', 'mid', and 'const'. If 'min' the factorization is given by 'min'-phase solutions (see Selesnisk (2001)). If 'const' the wavelet does not satisfy perfect reconstruction (see Achard and Gannaz 2024).

Value

`h`	Real part of the filter (up to a normalization)
`g`	Imaginary part of the filter (up to a normalization)
`tau`	Common-factor filter, defined by (h+i*g)/sqrt(2).

Author(s)

S. Achard and I. Gannaz

References

I.W. Selesnick (2001) Hilbert transform pairs of wavelet bases, *IEEE Signal Processing Letters*, Vol 8, N.6, pp 170-173.

I.W. Selesnick (2002) The design of approximate Hilbert transform pairs of wavelet bases, *IEEE Transactions on Signal Processing*, Vol 50, N.5, pp 1144-1152.

S. Achard, M. Clausel, I. Gannaz, F. Roueff (2020). New results on approximate Hilbert pairs of wavelet filters with common factor structure. *Applied and Computational Harmonic Analysis*, Vol 49, N.3, pp 1025-1045.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`DWTcomplex`

Examples

```
filter_complex <- hwlet(M=4,L=4,type='const')
```

`K_eval` *Evaluation of function K*

Description

Computes the function K as defined in (Achard and Gannaz 2014) and in (Achard and Gannaz 2024).

Usage

```
K_eval(psi_hat,u,d)
```

Arguments

`psi_hat`	Fourier transform of the wavelet mother at values `u`
`u`	grid for the approximation of the integral
`d`	vector of long-memory parameters.

Details

`K_eval` computes the matrix K with elements

$$K(d_l, d_m) = \int u^{(d_l+d_m)} |\texttt{psi\_hat}(u)|^2 du$$

Value

value of function K as a matrix.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`psi_hat_exact`

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha
res_psi <- psi_hat_exact(filter,J=10)
K_eval(res_psi$psih,res_psi$grid,d=c(0.2,0.2))
```

`leja` *Evaluation of the roots for spectral factorization*

Description

This program orders the values x_in (supposed to be the roots of a polynomial) in this way that computing the polynomial coefficients by using the function `poly` yields numerically accurate results.

Usage

```
leja(x_in)
```

Arguments

`x_in`	Roots of a polynomial

Value

Reordering of `x_in`

Author(s)

Matlab codes provided by Markus Lang : <lang@dsp.rice.edu> in 1993, Rice University. R code by Achard, Clausel, Gannaz, Roueff (2017).

References

I.W. Selesnick (2001) Hilbert transform pairs of wavelet bases, *IEEE Signal Processing Letters*, Vol 8, N.6, pp 170-173.

See Also

`seprts`, `sfact`,`hwlet`

Examples

```
z = exp(1i*(1:100)*2*pi/100)
p1 = signal::poly(z)
p2 = signal::poly(leja(z))
```

`mcw` *Multivariate complex (or real) wavelet Whittle estimation*

Description

Computes the multivariate complex (or real) wavelet Whittle estimation for the long-memory parameter vector `d` and the long-run covariance matrix, using `DWTcomplex` for the wavelet decomposition.

Usage

```
mcw(x, filter_complex, LU = NULL, J=10)
```

Arguments

`x`	data (matrix with time in rows and variables in columns).
`filter_complex`	wavelet filter as obtain with `scaling_filter` for a real wavelet and `hwlet` for a complex wavelet.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L`=3 if Jmax>3, and `U=Jmax`.)
`J`	2^J corresponds to the size of the grid for the discretisation of the wavelet. The default value is set to 10.

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

d	estimation of vector of long-memory parameters.
cov	estimation of long-run covariance matrix.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`mcw_wav`,`mcw_wav_eval`,`mcw_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(2,11)

res_mww <- mww(x,filter,LU)
```

`mcw_wav` *Multivariate complex (or real) wavelet Whittle estimation for data as wavelet coefficients*

Description

Computes the multivariate complex (or real) wavelet Whittle estimation of the long-memory parameter vector `d` and the long-run covariance matrix for the already wavelet decomposed data.

Usage

```
mcw_wav(xwav, index, psih, grid_K, LU = NULL)
```

Arguments

`xwav`	wavelet coefficients matrix (with scales in rows and variables in columns).
`index`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\texttt{dwt}(k)$ for $k \in [\texttt{indmaxband}(j-1)+1, \texttt{indmaxband}(j)]$ and for $j = 1$, $\texttt{dwt}(k)$ for $k \in [1, \texttt{indmaxband}(1)]$.
`psih`	the Fourier transform of the wavelet mother at values `grid_K`.
`grid_K`	the grid for the approximation of the integral in K.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L=1`, and `U=Jmax`.)

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

`d`	estimation of the vector of long-memory parameters.
`cov`	estimation of the long-run covariance matrix.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`mcw`,`mcw_wav_eval`,`mcw_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h

LU <- c(2,11)

### wavelet decomposition

if(is.matrix(x)){
     N <- dim(x)[1]
     k <- dim(x)[2]
}else{
     N <- length(x)
     k <- 1
}
x <- as.matrix(x,dim=c(N,k))

     ## Wavelet decomposition
     xwav <- matrix(0,N,k)
     for(j in 1:k){
          xx <- x[,j]

          resw <- DWTexact(xx,filter)
          xwav_temp <- resw$dwt
          index <- resw$indmaxband
          Jmax <- resw$Jmax
```

```
          xwav[1:index[Jmax],j] <- xwav_temp;
     }
     ## we free some memory
     new_xwav <- matrix(0,min(index[Jmax],N),k)
     if(index[Jmax]<N){
          new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
     }
     xwav <- new_xwav
     index <- c(0,index)

##### Compute the wavelet functions
res_psi <- psi_hat_exact(filter,10)
psih <- res_psi$psih
grid <- res_psi$grid

res_mww <- mww_wav(xwav,index, psih, grid,LU)
```

`mcw_wav_cov_eval` *Multivariate complex (or real) wavelet Whittle estimation of the long-run covariance matrix*

Description

Computes the multivariate complex (or real) wavelet Whittle estimation of the long-run covariance matrix given the long-memory parameter vector `d` for the already wavelet decomposed data.

Usage

```
mcw_wav_cov_eval(d, xwav, index, psih, grid_K, LU)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of xwav).
`xwav`	wavelet coefficients matrix (with scales in rows and variables in columns).
`index`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\mathtt{dwt}(k)$ for $k \in [\mathtt{indmaxband}(j-1)+1, \mathtt{indmaxband}(j)]$ and for $j = 1$, $\mathtt{dwt}(k)$ for $k \in [1, \mathtt{indmaxband}(1)]$.
`psih`	the Fourier transform of the wavelet mother at values `grid_K`
`grid_K`	the grid for the approximation of the integral in K
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition.

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

Long-run covariance matrix estimation.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`mcw`,`mcw_wav`,`mcw_wav_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho<-0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d<-c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(2,11)

### wavelet decomposition
```

```
if(is.matrix(x)){
     N <- dim(x)[1]
     k <- dim(x)[2]
}else{
     N <- length(x)
     k <- 1
}
x <- as.matrix(x,dim=c(N,k))

     ## Wavelet decomposition
     xwav <- matrix(0,N,k)
     for(j in 1:k){
          xx <- x[,j]

          resw <- DWTexact(xx,filter)
          xwav_temp <- resw$dwt
          index <- resw$indmaxband
          Jmax <- resw$Jmax
          xwav[1:index[Jmax],j] <- xwav_temp;
     }
     ## we free some memory
     new_xwav <- matrix(0,min(index[Jmax],N),k)
     if(index[Jmax]<N){
          new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
     }
     xwav <- new_xwav
     index <- c(0,index)

##### Compute the wavelet functions
res_psi <- psi_hat_exact(filter,10)
psih<-res_psi$psih
grid<-res_psi$grid

res_mww <- mww_wav_cov_eval(d,xwav,index, psih, grid,LU)
```

`mcw_wav_eval` *Multivariate real wavelet Whittle estimation for data as wavelet coefficients*

Description

Evaluates the multivariate complex (or real) wavelet Whittle objective function at a given long-memory parameter vector `d` for the already wavelet decomposed data.

Usage

```
mcw_wav_eval(d, xwav, index, LU = NULL)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of x).
`xwav`	wavelet coefficients matrix (with scales in rows and variables in columns).
`index`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\texttt{dwt}(k)$ for $k \in [\texttt{indmaxband}(j-1)+1, \texttt{indmaxband}(j)]$ and for $j = 1$, $\texttt{dwt}(k)$ for $k \in [1, \texttt{indmaxband}(1)]$.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L`=1, and `U=Jmax`.)

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

multivariate wavelet Whittle criterion.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`mcw`,`mcw_wav`,`mcw_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
```

```
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h

LU <- c(2,11)

### wavelet decomposition

if(is.matrix(x)){
     N <- dim(x)[1]
     k <- dim(x)[2]
}else{
     N <- length(x)
     k <- 1
}
x <- as.matrix(x,dim=c(N,k))

     ## Wavelet decomposition
     xwav <- matrix(0,N,k)
     for(j in 1:k){
          xx <- x[,j]

          resw <- DWTexact(xx,filter)
          xwav_temp <- resw$dwt
          index <- resw$indmaxband
          Jmax <- resw$Jmax
          xwav[1:index[Jmax],j] <- xwav_temp;
     }
     ## we free some memory
     new_xwav <- matrix(0,min(index[Jmax],N),k)
     if(index[Jmax]<N){
          new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
     }
     xwav <- new_xwav
     index <- c(0,index)

res_mww <- mww_wav_eval(d,xwav,index,LU)
res_d <- optim(rep(0,k),mww_wav_eval,xwav=xwav,index=index,LU=LU,
          method='Nelder-Mead',lower=-Inf,upper=Inf)$par
```

`mfw` *multivariate Fourier Whittle estimators*

Description

Computes the multivariate Fourier Whittle estimators of the long-memory parameters and the long-run covariance matrix also called fractal connectivity.

Usage

```
mfw(x, m)
```

Arguments

`x`	data (matrix with time in rows and variables in columns).
`m`	truncation number used for the estimation of the periodogram.

Details

The choice of m determines the range of frequencies used in the computation of the periodogram, $\lambda_j = 2\pi j/N$, j = 1,... , `m`. The optimal value depends on the spectral properties of the time series such as the presence of short range dependence. In Shimotsu (2007), `m` is chosen to be equal to $N^{0.65}$.

Value

`d`	estimation of the vector of long-memory parameters.
`cov`	estimation of the long-run covariance matrix.

Author(s)

S. Achard and I. Gannaz

References

K. Shimotsu (2007) Gaussian semiparametric estimation of multivariate fractionally integrated processes *Journal of Econometrics* Vol. 137, N. 2, pages 277-310.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mfw_eval`, `mfw_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov
```

```
m <- 57 ## default value of Shimotsu 2007
res_mfw <- mfw(x,m)
```

`mfw_cov_eval` *multivariate Fourier Whittle estimators*

Description

Computes the multivariate Fourier Whittle estimator of the long-run covariance matrix (also called fractal connectivity) for a given value of long-memory parameters `d`.

Usage

```
mfw_cov_eval(d, x, m)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of x)
`x`	data (matrix with time in rows and variables in columns)
`m`	truncation number used for the estimation of the periodogram

Details

The choice of m determines the range of frequencies used in the computation of the periodogram, $\lambda_j = 2\pi j/N$, j = 1,... , `m`. The optimal value depends on the spectral properties of the time series such as the presence of short range dependence. In Shimotsu (2007), `m` is chosen to be equal to $N^{0.65}$.

Value

long-run covariance matrix estimation.

Author(s)

S. Achard and I. Gannaz

References

K. Shimotsu (2007) Gaussian semiparametric estimation of multivariate fractionally integrated processes *Journal of Econometrics* Vol. 137, N. 2, pages 277-310.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mfw_eval`, `mfw`

Examples

```
### Simulation of ARFIMA(0,\code{d},0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

m <- 57 ## default value of Shimotsu
G <- mfw_cov_eval(d,x,m) # estimation of the covariance matrix when d is known
```

`mfw_eval` *evaluation of multivariate Fourier Whittle estimator*

Description

Evaluates the multivariate Fourier Whittle criterion at a given long-memory parameter value `d`.

Usage

```
mfw_eval(d, x, m)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of x).
`x`	data (matrix with time in rows and variables in columns).
`m`	truncation number used for the estimation of the periodogram.

Details

The choice of m determines the range of frequencies used in the computation of the periodogram, $\lambda_j = 2\pi j/N$, j = 1,... , `m`. The optimal value depends on the spectral properties of the time series such as the presence of short range dependence. In Shimotsu (2007), `m` is chosen to be equal to $N^{0.65}$.

Value

multivariate Fourier Whittle estimator computed at point `d`.

Author(s)

S. Achard and I. Gannaz

References

K. Shimotsu (2007) Gaussian semiparametric estimation of multivariate fractionally integrated processes *Journal of Econometrics* Vol. 137, N. 2, pages 277-310.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mfw_cov_eval`, `mfw`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

m <- 57 ## default value of Shimotsu
res_mfw <- mfw(x,m)
d <- res_mfw$d
G <- mfw_eval(d,x,m)
k <- length(d)
res_d <- optim(rep(0,k),mfw_eval,x=x,m=m,method='Nelder-Mead',lower=-Inf,upper=Inf)$par
```

`mww` *Multivariate real wavelet Whittle estimation*

Description

Computes the multivariate real wavelet Whittle estimation for the long-memory parameter vector `d` and the long-run covariance matrix, using `DWTexact` for the wavelet decomposition.

Usage

```
mww(x, filter, LU = NULL)
```

Arguments

`x`	data (matrix with time in rows and variables in columns).
`filter`	wavelet filter as obtain with `scaling_filter`.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L`=2 if Jmax>2, and `U=Jmax`.)

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

`d`	estimation of vector of long-memory parameters.
`cov`	estimation of long-run covariance matrix.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mww_eval`, `mww_cov_eval`,`mww_wav`,`mww_wav_eval`,`mww_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
```

```
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(2,11)

res_mww <- mww(x,filter,LU)
```

`mww_cov_eval` *Multivariate real wavelet Whittle estimation of the long-run covariance matrix*

Description

Computes the multivariate real wavelet Whittle estimation of the long-run covariance matrix given the long-memory parameter vector `d`, using `DWTexact` for the wavelet decomposition.

Usage

```
mww_cov_eval(d, x, filter, LU)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of x).
`x`	data (matrix with time in rows and variables in columns).
`filter`	wavelet filter as obtain with `scaling_filter`.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition.

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

long-run covariance matrix estimation.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mww`, `mww_eval`,`mww_wav`,`mww_wav_eval`,`mww_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(2,11)

res_mww <- mww_cov_eval(d,x,filter,LU)
```

`mww_eval` *Evaluation of multivariate real wavelet Whittle estimation*

Description

Evaluates the multivariate real wavelet Whittle criterion at a given long-memory parameter vector `d` using `DWTexact` for the wavelet decomposition.

Usage

```
mww_eval(d, x, filter, LU = NULL)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of x).
`x`	data (matrix with time in rows and variables in columns).
`filter`	wavelet filter as obtain with `scaling_filter`.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L`=2, and `U=Jmax`.)

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

multivariate wavelet Whittle criterion.

Author(s)

S. Achard and I. Gannaz

References

E. Moulines, F. Roueff, M. S. Taqqu (2009) A wavelet whittle estimator of the memory parameter of a nonstationary Gaussian time series. *Annals of statistics*, vol. 36, N. 4, pages 1925-1956

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mww`, `mww_cov_eval`,`mww_wav`,`mww_wav_eval`,`mww_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov
```

```
## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(2,11)

res_mww <- mww_eval(d,x,filter,LU)
k <- length(d)
res_d <- optim(rep(0,k),mww_eval,x=x,filter=filter,
            LU=LU,method='Nelder-Mead',lower=-Inf,upper=Inf)$par
```

`mww_wav` *Multivariate real wavelet Whittle estimation for data as wavelet coefficients*

Description

Computes the multivariate real wavelet Whittle estimation of the long-memory parameter vector `d` and the long-run covariance matrix for the already wavelet decomposed data.

Usage

```
mww_wav(xwav, index, psih, grid_K, LU = NULL)
```

Arguments

`xwav`	wavelet coefficients matrix (with scales in rows and variables in columns).
`index`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\mathtt{dwt}(k)$ for $k \in [\mathtt{indmaxband}(j-1)+1, \mathtt{indmaxband}(j)]$ and for $j = 1$, $\mathtt{dwt}(k)$ for $k \in [1, \mathtt{indmaxband}(1)]$.
`psih`	the Fourier transform of the wavelet mother at values `grid_K`.
`grid_K`	the grid for the approximation of the integral in K.
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L`=1, and `U=Jmax`.)

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

`d`	estimation of the vector of long-memory parameters.
`cov`	estimation of the long-run covariance matrix.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mww_eval`, `mww_cov_eval`,`mww`,`mww_wav_eval`,`mww_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h

LU <- c(2,11)

### wavelet decomposition

if(is.matrix(x)){
     N <- dim(x)[1]
     k <- dim(x)[2]
}else{
     N <- length(x)
     k <- 1
}
x <- as.matrix(x,dim=c(N,k))

     ## Wavelet decomposition
     xwav <- matrix(0,N,k)
```

```
    for(j in 1:k){
        xx <- x[,j]

        resw <- DWTexact(xx,filter)
        xwav_temp <- resw$dwt
        index <- resw$indmaxband
        Jmax <- resw$Jmax
        xwav[1:index[Jmax],j] <- xwav_temp;
    }
    ## we free some memory
    new_xwav <- matrix(0,min(index[Jmax],N),k)
    if(index[Jmax]<N){
        new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
    }
    xwav <- new_xwav
    index <- c(0,index)

##### Compute the wavelet functions
res_psi <- psi_hat_exact(filter,10)
psih <- res_psi$psih
grid <- res_psi$grid

res_mww <- mww_wav(xwav,index, psih, grid,LU)
```

`mww_wav_cov_eval` *Multivariate real wavelet Whittle estimation of the long-run covariance matrix*

Description

Computes the multivariate real wavelet Whittle estimation of the long-run covariance matrix given the long-memory parameter vector `d` for the already wavelet decomposed data.

Usage

```
mww_wav_cov_eval(d, xwav, index,psih,grid_K, LU)
```

Arguments

`d`	vector of long-memory parameters (dimension should match dimension of xwav).
`xwav`	wavelet coefficients matrix (with scales in rows and variables in columns).
`index`	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\texttt{dwt}(k)$ for $k \in [\texttt{indmaxband}(j-1)+1, \texttt{indmaxband}(j)]$ and for $j = 1$, $\texttt{dwt}(k)$ for $k \in [1, \texttt{indmaxband}(1)]$.
`psih`	the Fourier transform of the wavelet mother at values `grid_K`
`grid_K`	the grid for the approximation of the integral in K
`LU`	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition.

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

Long-run covariance matrix estimation.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mww`, `mww_eval`,`mww_wav`,`mww_wav_eval`,`mww_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho<-0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d<-c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha

LU <- c(2,11)

### wavelet decomposition

if(is.matrix(x)){
     N <- dim(x)[1]
```

```
     k <- dim(x)[2]
}else{
     N <- length(x)
     k <- 1
}
x <- as.matrix(x,dim=c(N,k))

     ## Wavelet decomposition
     xwav <- matrix(0,N,k)
     for(j in 1:k){
          xx <- x[,j]

          resw <- DWTexact(xx,filter)
          xwav_temp <- resw$dwt
          index <- resw$indmaxband
          Jmax <- resw$Jmax
          xwav[1:index[Jmax],j] <- xwav_temp;
     }
     ## we free some memory
     new_xwav <- matrix(0,min(index[Jmax],N),k)
     if(index[Jmax]<N){
          new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
     }
     xwav <- new_xwav
     index <- c(0,index)

##### Compute the wavelet functions
res_psi <- psi_hat_exact(filter,10)
psih<-res_psi$psih
grid<-res_psi$grid

res_mww <- mww_wav_cov_eval(d,xwav,index, psih, grid,LU)
```

`mww_wav_eval` *Multivariate real wavelet Whittle estimation for data as wavelet coefficients*

Description

Evaluates the multivariate real wavelet Whittle objective function at a given long-memory parameter vector `d` for the already wavelet decomposed data.

Usage

```
mww_wav_eval(d, xwav, index, LU = NULL)
```

Arguments

d	vector of long-memory parameters (dimension should match dimension of x).
xwav	wavelet coefficients matrix (with scales in rows and variables in columns).
index	vector containing the largest index of each band, i.e. for $j > 1$ the wavelet coefficients of scale j are $\texttt{dwt}(k)$ for $k \in [\texttt{indmaxband}(j-1)+1, \texttt{indmaxband}(j)]$ and for $j = 1$, $\texttt{dwt}(k)$ for $k \in [1, \texttt{indmaxband}(1)]$.
LU	bivariate vector (optional) containing `L`, the lowest resolution in wavelet decomposition `U`, the maximal resolution in wavelet decomposition. (Default values are set to `L`=1, and `U=Jmax`.)

Details

`L` is fixing the lower limit of wavelet scales. `L` can be increased to avoid finest frequencies that can be corrupted by the presence of high frequency phenomena.

`U` is fixing the upper limit of wavelet scales. `U` can be decreased when highest frequencies have to be discarded.

Value

multivariate wavelet Whittle criterion.

Author(s)

S. Achard and I. Gannaz

References

E. Moulines, F. Roueff, M. S. Taqqu (2009) A wavelet whittle estimator of the memory parameter of a nonstationary Gaussian time series. *Annals of statistics*, vol. 36, N. 4, pages 1925-1956

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`mww`, `mww_cov_eval`,`mww_wav`,`mww_eval`,`mww_wav_cov_eval`

Examples

```
### Simulation of ARFIMA(0,d,0)
rho <- 0.4
cov <- matrix(c(1,rho,rho,1),2,2)
d <- c(0.4,0.2)
J <- 9
N <- 2^J

resp <- fivarma(N, d, cov_matrix=cov)
```

```
x <- resp$x
long_run_cov <- resp$long_run_cov

## wavelet coefficients definition
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h

LU <- c(2,11)

### wavelet decomposition

if(is.matrix(x)){
     N <- dim(x)[1]
     k <- dim(x)[2]
}else{
     N <- length(x)
     k <- 1
}
x <- as.matrix(x,dim=c(N,k))

     ## Wavelet decomposition
     xwav <- matrix(0,N,k)
     for(j in 1:k){
          xx <- x[,j]

          resw <- DWTexact(xx,filter)
          xwav_temp <- resw$dwt
          index <- resw$indmaxband
          Jmax <- resw$Jmax
          xwav[1:index[Jmax],j] <- xwav_temp;
     }
     ## we free some memory
     new_xwav <- matrix(0,min(index[Jmax],N),k)
     if(index[Jmax]<N){
          new_xwav[(1:(index[Jmax])),] <- xwav[(1:(index[Jmax])),]
     }
     xwav <- new_xwav
     index <- c(0,index)

res_mww <- mww_wav_eval(d,xwav,index,LU)
res_d <- optim(rep(0,k),mww_wav_eval,xwav=xwav,index=index,LU=LU,
          method='Nelder-Mead',lower=-Inf,upper=Inf)$par
```

`psi_hat_exact` *Discrete Fourier transform of a real wavelet*

Description

Computes the discrete Fourier transform of the real wavelet associated to the given filter using `scaling_function`. The length of the Fourier transform is equal to the length of the grid where the wavelet is evaluated.

Usage

```
psi_hat_exact(filter,J=10)
```

Arguments

`filter`	wavelet filter as obtained with `scaling_filter`.
`J`	2^J corresponds to the size of the grid for the discretisation of the wavelet. The default value is set to 10.

Value

`phih`	Values of the discrete Fourier transform of the scaling wavelet.
`psih`	Values of the discrete Fourier transform of the mother wavelet.
`grid`	Frequencies where the Fourier transform is evaluated.

Author(s)

S. Achard and I. Gannaz

References

G. Fay, E. Moulines, F. Roueff, M. S. Taqqu (2009) Estimators of long-memory: Fourier versus wavelets. *Journal of Econometrics*, vol. 151, N. 2, pages 159-177.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`DWTexact`, `scaling_filter`

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
psi_hat_exact(filter,J=6)
```

`psi_hat_exact_complex` *Discrete Fourier transform of a complex Common-Factor wavelet*

Description

Computes the discrete Fourier transform of the complex Common-Factor wavelet associated to the given filters using `scaling_function`. The length of the Fourier transform is equal to the length of the grid where the wavelet is evaluated.

Usage

```
psi_hat_exact_complex(h,g,J=10)
```

Arguments

`h`	Real part of the filter (up to a normalization)
`g`	Imaginary part of the filter (up to a normalization)
`J`	2^J corresponds to the size of the grid for the discretisation of the wavelet. The default value is set to 10.

Value

`phih`	Values of the discrete Fourier transform of the scaling wavelet.
`psih`	Values of the discrete Fourier transform of the mother wavelet.
`grid`	Frequencies where the Fourier transform is evaluated.

Author(s)

S. Achard and I. Gannaz

References

I.W. Selesnick (2001) Hilbert transform pairs of wavelet bases, *IEEE Signal Processing Letters*, Vol 8, N.6, pp 170-173.

I.W. Selesnick (2002) The design of approximate Hilbert transform pairs of wavelet bases, *IEEE Transactions on Signal Processing*, Vol 50, N.5, pp 1144-1152.

S. Achard, M. Clausel, I. Gannaz, F. Roueff (2020). New results on approximate Hilbert pairs of wavelet filters with common factor structure. *Applied and Computational Harmonic Analysis*, Vol 49, N.3, pp 1025-1045.

S. Achard, I. Gannaz (2024). Local Whittle estimation with (quasi-)analytic wavelets. *Journal of Time Series Analysis*, Vol 45, pp 421-443.

See Also

`DWTexact`, `scaling_filter`

Examples

```
filter_complex <- hwlet(M=4,L=4,type='const')
psi_hat_exact_complex(filter_complex$h,filter_complex$g,J=6)
```

`scaling_filter` *wavelet scaling filter coefficients*

Description

Computes the filter coefficients of the Haar or Daubechies wavelet family with a specific order

Usage

```
scaling_filter(family, parameter)
```

Arguments

`family`	Wavelet family, `'Haar'` or `'Daubechies'`
`parameter`	Order of the Daubechies wavelet (equal to twice the number of vanishing moments). The value of `parameter` can be 2,4,8,10,12,14 and 16.

Value

`h`	Vector of scaling filter coefficients.
`M`	Number of vanishing moments.
`alpha`	Fourier decay exponent.

Author(s)

S. Achard and I. Gannaz

References

G. Fay, E. Moulines, F. Roueff, M. S. Taqqu (2009) Estimators of long-memory: Fourier versus wavelets. *Journal of Econometrics*, vol. 151, N. 2, pages 159-177.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`DWTexact`

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
M <- res_filter$M
alpha <- res_filter$alpha
```

`scaling_function` *scaling function and the wavelet function*

Description

Computes the scaling function and the wavelet function (for compactly supported wavelet) using the cascade algorithm on the grid of dyadic integer 2^{-J}

Usage

```
scaling_function(filter,J)
```

Arguments

`filter`	wavelet filter as obtained with `scaling_filter`.
`J`	value of the largest scale.

Value

`phi`	Scaling function.
`psi`	Wavelet function.

Note

This function was rewritten from an original matlab version by Fay et al. (2009)

Author(s)

S. Achard and I. Gannaz

References

G. Fay, E. Moulines, F. Roueff, M. S. Taqqu (2009) Estimators of long-memory: Fourier versus wavelets. *Journal of Econometrics*, vol. 151, N. 2, pages 159-177.

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

DWTexact, scaling_filter

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
scaling_function(filter,J=6)
```

---

seprts *Evaluation of the roots for spectral factorization*

Description

This program is for spectral factorization. The roots on the unit circle must have even degree. Roots with high multiplicity will cause problems, they should be handled by extracting them prior to using this program.

Usage

```
seprts(p, type='mid')
```

Arguments

`p`	A polynomial which admits a specral factorization.
`type`	If 'mid' the factorisation of the Bezout solution is obtained with all roots of absolute magnitude less than 1. If 'min' the factorization is given by 'min'-phase solutions (see Selesnisk (2001)).

Value

The roots of the polynomial `p`, separated depending of either they are inside the unit circle or on the unit circle. Technical function for the function `sfact`.

Author(s)

Matlab codes provided by Selesnick (2001). R code by Achard, Clausel, Gannaz, Roueff (2017).

References

I.W. Selesnick (2001) Hilbert transform pairs of wavelet bases, *IEEE Signal Processing Letters*, Vol 8, N.6, pp 170-173.

I.W. Selesnick (2002) The design of approximate Hilbert transform pairs of wavelet bases, *IEEE Transactions on Signal Processing*, Vol 50, N.5, pp 1144-1152.

See Also

sfact, leja,hwlet

`sfact` *Spectral factorization of a polynomial.*

Description

Spectral factorization of a polynomial h.

Usage

```
sfact(h, type='mid')
```

Arguments

`h`	polynomial
`type`	If 'mid' the factorisation of the Bezout solution is obtained with all roots of absolute magnitude less than 1. If 'min' the factorization is given by 'min'-phase solutions (see Selesnisk (2001)).

Value

`poly`	A new polynomial `b`, used in the construction of the common-factor filters, such that `h - conv(b,rev(b))` is equal to zeros.
`r`	Roots of the polunomial `b`

Author(s)

Matlab codes provided by Markus Lang : <lang@dsp.rice.edu> in 1993, Rice University. R code by Achard, Clausel, Gannaz, Roueff (2017).

References

I.W. Selesnick (2001) Hilbert transform pairs of wavelet bases, *IEEE Signal Processing Letters*, Vol 8, N.6, pp 170-173.

I.W. Selesnick (2002) The design of approximate Hilbert transform pairs of wavelet bases, *IEEE Transactions on Signal Processing*, Vol 50, N.5, pp 1144-1152.

S. Achard, M. Clausel, I. Gannaz, F. Roueff (2020). New results on approximate Hilbert pairs of wavelet filters with common factor structure. *Applied and Computational Harmonic Analysis*, Vol 49, N.3, pp 1025-1045.

See Also

`seprts`, `leja`,`hwlet`

Examples

```
g = runif(10)
h = conv(g,rev(g))
b = sfact(h)$poly
h - conv(b,rev(b)) ## should be zeros
```

`toeplitz_nonsym` *Transform a vector in a non symmetric Toeplitz matrix*

Description

Transform a vector in a non symmetric Toeplitz matrix

Usage

```
toeplitz_nonsym(vec)
```

Arguments

`vec`	input vector.

Value

the corresponding matrix.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`scaling_function`

Examples

```
res_filter <- scaling_filter('Daubechies',8);
filter <- res_filter$h
Htmp <- toeplitz_nonsym(filter)
```

`varma` *simulation of multivariate ARMA process*

Description

generates N observations of a k-vector ARMA process

Usage

```
varma(N, k = 1, VAR = NULL, VMA = NULL, cov_matrix = diag(k), innov=NULL)
```

Arguments

`N`	number of time points.
`k`	dimension of the vector ARMA (optional, default is univariate)
`VAR`	array of VAR coefficient matrices (optional).
`VMA`	array of VMA coefficient matrices (optional).
`cov_matrix`	matrix of correlation between the innovations (optional, default is identity).
`innov`	matrix of the innovations (optional, default is a gaussian process).

Value

vector containing the N observations of the k-vector ARMA process.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`fivarma`, `vfracdiff`

Examples

```
rho1 <- 0.3
rho2 <- 0.8
cov <- matrix(c(1,rho1,rho2,rho1,1,rho1,rho2,rho1,1),3,3)

J <- 9
N <- 2^J
VMA <- diag(c(0.4,0.1,0))
### or another example VAR <- array(c(0.8,0,0,0,0.6,0,0,0,0.2,0,0,0,0,0.4,0,0,0,0.5),dim=c(3,3,2))
VAR <- diag(c(0.8,0.6,0))
x <- varma(N, k=3, cov_matrix=cov, VAR=VAR, VMA=VMA)
```

`vfracdiff` *simulation of vector fractional differencing process*

Description

Given a vector process `x` and a vector of long memory parameters `d`, this function is producing the corresponding fractional differencing process.

Usage

```
vfracdiff(x, d)
```

Arguments

`x`	initial process.
`d`	vector of long-memory parameters

Details

Given a process `x`, this function applied a fractional difference procedure using the formula:

$$diag((1-L)^d)x,$$

where `L` is the lag operator.

Value

vector fractional differencing of `x`.

Author(s)

S. Achard and I. Gannaz

References

S. Achard, I. Gannaz (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. `http://arxiv.org/abs/1412.0391`.

K. Shimotsu (2007) Gaussian semiparametric estimation of multivariate fractionally integrated processes *Journal of Econometrics* Vol. 137, N. 2, pages 277-310.

S. Achard, I Gannaz (2019) Wavelet-Based and Fourier-Based Multivariate Whittle Estimation: multiwave. *Journal of Statistical Software*, Vol 89, N. 6, pages 1-31.

See Also

`varma`, `fivarma`

Examples

```
rho1 <- 0.3
rho2 <- 0.8
cov <- matrix(c(1,rho1,rho2,rho1,1,rho1,rho2,rho1,1),3,3)
d <- c(0.2,0.3,0.4)


J <- 9
N <- 2^J
VMA <- diag(c(0.4,0.1,0))
### or another example VAR <- array(c(0.8,0,0,0,0.6,0,0,0,0.2,0,0,0,0,0.4,0,0,0,0.5),dim=c(3,3,2))
VAR <- diag(c(0.8,0.6,0))
x <- varma(N, k=3, cov_matrix=cov, VAR=VAR, VMA=VMA)
vx<-vfracdiff(x,d)
```

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