Package: mhn (via r-universe)

May 27, 2026

Type Package

Title The Modified Half-Normal Distribution

Version 0.1.0

Description Provides density, distribution, quantile, and random generation functions for the Modified Half-Normal (MHN) distribution, along with moments, mode, and the Fox-Wright Psi function used as the normalizing constant. The MHN distribution arises as a conditional posterior in Bayesian MCMC and generalizes the half-normal, truncated normal, and square-root gamma distributions. Implements efficient sampling via the Sun, Kong & Pal (2023) <doi:10.1080/03610926.2021.1934700> algorithms and the Gao & Wang (2025) <doi:10.1080/03610918.2025.2524551> RTDR method.

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Encoding UTF-8

RoxygenNote 7.3.3

Depends R (>= 4.0.0)

Imports stats, Rcpp (>= 1.0.7)

LinkingTo Rcpp, BH (>= 1.78.0-0)

SystemRequirements C++17

Suggests testthat (>= 3.0.0), knitr, rmarkdown, bench, Rmpfr

Config/testthat/edition 3

VignetteBuilder knitr

URL `https://github.com/t-momozaki/mhn`, `https://t-momozaki.github.io/mhn/`

BugReports `https://github.com/t-momozaki/mhn/issues`

NeedsCompilation yes

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Repository https://cran.r-universe.dev

Date/Publication 2026-05-27 08:30:02 UTC

RemoteUrl https://github.com/cran/mhn

RemoteRef HEAD

RemoteSha 042611f1184215d89e52ef100f24be7683c0f704

Contents

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dmhn *Density of the Modified Half-Normal Distribution*

Description

Computes the probability density function (or log-density) of the Modified Half-Normal (MHN) distribution with parameters alpha, beta, and gamma.

Usage

```
dmhn(x, alpha = 1, beta = 1, gamma = 0, log = FALSE)
```

Arguments

x	Numeric vector of evaluation points.
alpha	Shape parameter ($\alpha > 0$). Scalar or numeric vector. Default: 1.
beta	Scale parameter ($\beta > 0$). Scalar or numeric vector. Default: 1.
gamma	Location parameter ($\gamma \in R$). Scalar or numeric vector. Default: 0.
log	Logical; if TRUE, log-density is returned. Default: FALSE.

Details

The MHN density is

$$f(x \mid \alpha, \beta, \gamma) = \frac{2\beta^{\alpha/2} x^{\alpha-1} \exp(-\beta x^2 + \gamma x)}{\Psi[\alpha/2, \gamma/\sqrt{\beta}]} \quad (x > 0)$$

where $\Psi[a, z]$ is the Fox-Wright Psi function (Sun et al., 2023, Lemma 1a).

The default parameters `alpha = 1, beta = 1, gamma = 0` correspond to the half-normal distribution $\mathrm{HN}(1/\sqrt{2})$.

Special cases are detected and dispatched to closed-form solutions:

- $\gamma = 0$: sqrt-Gamma distribution
- $\alpha = 1$: truncated normal distribution

Computation is performed in log-space to avoid numerical underflow/overflow.

When any of `alpha`, `beta`, `gamma` is a vector, the density is evaluated element-wise. The Fox-Wright Ψ normalizing constant is recomputed only when consecutive elements present a different (α, β, γ) triple, so passing grouped parameters is significantly faster than calling `dmhn` inside an R loop.

Value

A numeric vector. The output length equals `max(length(x), length(alpha), length(beta), length(gamma))`; each input is recycled to that length following standard R recycling rules. For `x < 0`, the density is 0 (`-Inf` if `log = TRUE`).

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

See Also

`mhn_mean`, `mhn_var`, `mhn_mode`

Examples

```
x <- seq(0, 5, length.out = 100)
plot(x, dmhn(x, alpha = 2, beta = 1, gamma = 1), type = "l")

# Log-density
dmhn(1, alpha = 2, beta = 1, gamma = 1, log = TRUE)
```

mhn_kurtosis *Excess Kurtosis of the Modified Half-Normal Distribution*

Description

Computes the excess kurtosis $\gamma_2 = E[(X-\mu)^4]/\sigma^4 - 3$ for $X \sim \mathrm{MHN}(\alpha, \beta, \gamma)$.

Usage

```
mhn_kurtosis(alpha, beta, gamma)
```

Arguments

alpha	Shape parameter ($\alpha > 0$).
beta	Scale parameter ($\beta > 0$).
gamma	Location parameter ($\gamma \in R$).

Details

Uses the moment recurrence (Sun et al., 2023, Lemma 2b) to compute raw moments up to fourth order, then converts to central moments.

Value

A numeric scalar.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2b)

See Also

mhn_skewness, mhn_mean

Examples

```
mhn_kurtosis(alpha = 2, beta = 1, gamma = 0)
```

`mhn_mean` *Mean of the Modified Half-Normal Distribution*

Description

Computes $E(X)$ for $X \sim \text{MHN}(\alpha, \beta, \gamma)$.

Usage

```
mhn_mean(alpha, beta, gamma)
```

Arguments

`alpha`	Shape parameter ($\alpha > 0$).
`beta`	Scale parameter ($\beta > 0$).
`gamma`	Location parameter ($\gamma \in R$).

Details

The mean is computed as a ratio of Fox-Wright Psi functions:

$$E(X) = \frac{\Psi[(\alpha+1)/2,\ \gamma/\sqrt{\beta}]}{\sqrt{\beta}\,\Psi[\alpha/2,\ \gamma/\sqrt{\beta}]}$$

Value

A numeric scalar.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2a)

See Also

`mhn_var`, `dmhn`

Examples

```
mhn_mean(alpha = 2, beta = 1, gamma = 0)
```

mhn_mode *Mode of the Modified Half-Normal Distribution*

Description

Computes the mode (most probable value) of the MHN distribution.

Usage

```
mhn_mode(alpha, beta, gamma)
```

Arguments

alpha	Shape parameter ($\alpha > 0$).
beta	Scale parameter ($\beta > 0$).
gamma	Location parameter ($\gamma \in R$).

Details

The mode depends on α:

$\alpha > 1$ $(\gamma + \sqrt{\gamma^2 + 8\beta(\alpha - 1)})/(4\beta)$ (Sun et al., 2023, Lemma 3b).

$\alpha = 1$ $\max(0, \gamma/(2\beta))$, obtained as the mode of the truncated normal $\mathrm{TN}(\gamma/(2\beta), 1/\sqrt{2\beta}, 0, \infty)$ that the MHN reduces to in this case (Sun et al., 2023, Lemma 6b).

$0 < \alpha < 1$ An interior mode exists only when $\gamma > 0$ and $\alpha \geq 1 - \gamma^2/(8\beta)$ (Sun et al., 2023, Lemma 3c); otherwise the density is monotonically decreasing (Sun et al., 2023, Lemma 3d) and `NA` is returned.

Value

A numeric scalar. Returns `NA` when no interior mode exists (density is monotonically decreasing on $(0, \infty)$).

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 3b–d, Lemma 6b)

See Also

`dmhn`, `mhn_mean`

Examples

```
mhn_mode(alpha = 2, beta = 1, gamma = 1)
mhn_mode(alpha = 1, beta = 1, gamma = 2)
mhn_mode(alpha = 0.5, beta = 1, gamma = -1)  # NA
```

`mhn_skewness` *Skewness of the Modified Half-Normal Distribution*

Description

Computes the skewness $\gamma_1 = E[(X - \mu)^3]/\sigma^3$ for $X \sim \text{MHN}(\alpha, \beta, \gamma)$.

Usage

```
mhn_skewness(alpha, beta, gamma)
```

Arguments

`alpha`	Shape parameter ($\alpha > 0$).
`beta`	Scale parameter ($\beta > 0$).
`gamma`	Location parameter ($\gamma \in R$).

Details

Uses the moment recurrence (Sun et al., 2023, Lemma 2b) to compute raw moments up to third order, then converts to central moments.

Value

A numeric scalar.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2b)

See Also

`mhn_kurtosis`, `mhn_mean`

Examples

```
mhn_skewness(alpha = 2, beta = 1, gamma = 0)
```

mhn_var	*Variance of the Modified Half-Normal Distribution*

Description

Computes $\mathrm{Var}(X)$ for $X \sim \mathrm{MHN}(\alpha, \beta, \gamma)$.

Usage

```
mhn_var(alpha, beta, gamma)
```

Arguments

`alpha`	Shape parameter ($\alpha > 0$).
`beta`	Scale parameter ($\beta > 0$).
`gamma`	Location parameter ($\gamma \in R$).

Details

Uses the formula (Sun et al., 2023, Lemma 2c):

$$\mathrm{Var}(X) = \frac{\alpha}{2\beta} + E(X)\left(\frac{\gamma}{2\beta} - E(X)\right)$$

For $\alpha \geq 1$, the variance satisfies $\mathrm{Var}(X) \leq 1/(2\beta)$ (Sun et al., 2023, Lemma 4c).

Value

A numeric scalar.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2c)

See Also

`mhn_mean`, `dmhn`

Examples

```
mhn_var(alpha = 2, beta = 1, gamma = 0)
```

pmhn *Distribution Function of the Modified Half-Normal Distribution*

Description

Computes the cumulative distribution function (CDF) of the Modified Half-Normal (MHN) distribution with parameters `alpha`, `beta`, and `gamma`.

Usage

```
pmhn(q, alpha = 1, beta = 1, gamma = 0, lower.tail = TRUE, log.p = FALSE)
```

Arguments

`q`	Numeric vector of quantiles.
`alpha`	Shape parameter ($\alpha > 0$). Scalar or numeric vector. Default: 1.
`beta`	Scale parameter ($\beta > 0$). Scalar or numeric vector. Default: 1.
`gamma`	Location parameter ($\gamma \in R$). Scalar or numeric vector. Default: 0.
`lower.tail`	Logical; if `TRUE` (default), probabilities are $P(X \leq q)$, otherwise $P(X > q)$.
`log.p`	Logical; if `TRUE`, probabilities are returned on the log scale. Default: `FALSE`.

Details

The CDF is computed via the series representation

$$F(x \mid \alpha, \beta, \gamma) = \frac{1}{\Psi[\alpha/2, \gamma/\sqrt{\beta}]} \sum_{i=0}^{\infty} \frac{z^i}{i!} \Gamma(s_i) \, P(s_i, \beta x^2)$$

where $z = \gamma/\sqrt{\beta}$, $s_i = (\alpha + i)/2$, and $P(s, y)$ is the regularized lower incomplete gamma function (Sun et al., 2023, Lemma 1b; equivalent to the paper's form via the identity $\Gamma(s) \, P(s, y) = \gamma(s, y)$, where $\gamma(s, y)$ is the lower incomplete gamma function used in the paper). The infinite sum is truncated at the constructive bound $K = \max\{K_1, K_2\}$ from Sun et al. (2023), Supplementary Lemma 10(d), which makes the truncation residual bounded by the user's tolerance divided by Ψ. When double-precision cancellation in the alternating-sign accumulator for $\gamma < 0$ would exceed that tolerance, the series is replaced by a Gauss-Kronrod (or tanh-sinh for $\alpha < 1$) numerical integration of the density on $[0, q]$.

Special cases are detected and dispatched to standard R primitives:

- $\gamma = 0$: `pgamma(q^2, alpha/2, scale = 1/beta)`
- $\alpha = 1$: truncated-normal CDF via `pnorm`

When any of `alpha`, `beta`, `gamma` is a vector, the CDF is evaluated element-wise. The Fox-Wright Ψ normalizing constant is recomputed only when consecutive elements present a different (α, β, γ) triple, so passing grouped parameters is significantly faster than calling `pmhn` inside an R loop.

Value

A numeric vector. The output length equals max(length(q), length(alpha), length(beta), length(gamma)); each input is recycled to that length following standard R recycling rules. For q <= 0 the CDF is 0; for q = Inf it is 1.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

See Also

dmhn, qmhn, rmhn

Examples

```
# Basic evaluation
pmhn(c(0.5, 1, 1.5), alpha = 2, beta = 1, gamma = 1)

# Tail / log forms
pmhn(2, alpha = 2, beta = 1, gamma = 1, lower.tail = FALSE)
pmhn(2, alpha = 2, beta = 1, gamma = 1, log.p = TRUE)

# Special case: gamma = 0 reduces to sqrt-Gamma
all.equal(pmhn(1.5, alpha = 2, beta = 1, gamma = 0),
          pgamma(1.5^2, shape = 1, rate = 1))
```

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qmhn *Quantile Function of the Modified Half-Normal Distribution*

Description

Computes the quantile (inverse cumulative) function of the Modified Half-Normal (MHN) distribution with parameters alpha, beta, and gamma.

Usage

```
qmhn(p, alpha = 1, beta = 1, gamma = 0, lower.tail = TRUE, log.p = FALSE)
```

Arguments

p	Numeric vector of probabilities.
alpha	Shape parameter ($\alpha > 0$). Scalar or numeric vector. Default: 1.
beta	Scale parameter ($\beta > 0$). Scalar or numeric vector. Default: 1.
gamma	Location parameter ($\gamma \in R$). Scalar or numeric vector. Default: 0.
lower.tail	Logical; if TRUE (default), probabilities are $P(X \leq q)$, otherwise $P(X > q)$.
log.p	Logical; if TRUE, probabilities are provided on the log scale. Default: FALSE.

Details

For the general case, $q = F^{-1}(p)$ is obtained by a TOMS 748 root-finder applied to the series CDF (Sun et al., 2023, Lemma 1b). The initial bracket is $[\sqrt{\epsilon},\ E(X) + 8\sqrt{\mathrm{Var}(X)}]$ and is doubled on the right (up to 30 times) until it brackets the target probability.

Special cases are detected and dispatched to standard R primitives:

- $\gamma = 0$: `sqrt(qgamma(p, alpha/2, scale = 1/beta))`
- $\alpha = 1$: truncated-normal inverse via `qnorm`

When any of `alpha`, `beta`, `gamma` is a vector, the quantile is evaluated element-wise. The Fox-Wright Ψ normalizing constant and moments $E(X)$, $\mathrm{Var}(X)$ (used to size the root-finder bracket) are recomputed only when consecutive elements present a different (α, β, γ) triple.

Value

A numeric vector. The output length equals `max(length(p), length(alpha), length(beta), length(gamma))`; each input is recycled to that length following standard R recycling rules. `qmhn(0) = 0` and `qmhn(1) = Inf`. Probabilities outside $[0, 1]$ yield `NaN`.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

See Also

`dmhn`, `pmhn`, `rmhn`

Examples

```
# Basic evaluation
qmhn(c(0.1, 0.5, 0.9), alpha = 2, beta = 1, gamma = 1)

# Round-trip: F(F^-1(p)) ~ p
p <- c(0.05, 0.25, 0.5, 0.75, 0.95)
all.equal(pmhn(qmhn(p, alpha = 2, beta = 1, gamma = 1),
               alpha = 2, beta = 1, gamma = 1),
          p, tolerance = 1e-6)

# Tail / log forms
qmhn(0.95, alpha = 2, beta = 1, gamma = 1, lower.tail = FALSE)
qmhn(log(0.05), alpha = 2, beta = 1, gamma = 1, log.p = TRUE)
```

rmhn *Random Generation from the Modified Half-Normal Distribution*

Description

Draws random variates from the Modified Half-Normal (MHN) distribution with parameters `alpha`, `beta`, and `gamma`.

Usage

```
rmhn(n, alpha = 1, beta = 1, gamma = 0, method = c("auto", "rtdr", "sun"))
```

Arguments

`n`	Non-negative integer giving the number of variates to draw. `n = 0` returns `numeric(0)`.
`alpha`	Shape parameter ($\alpha > 0$). Scalar or numeric vector. Default: 1.
`beta`	Scale parameter ($\beta > 0$). Scalar or numeric vector. Default: 1.
`gamma`	Location parameter ($\gamma \in R$). Scalar or numeric vector. Default: 0.
`method`	Sampling algorithm. One of `"auto"` (default), `"rtdr"`, or `"sun"`. See Details.

Details

The MHN density is

$$f(x \mid \alpha, \beta, \gamma) = \frac{2\beta^{\alpha/2} x^{\alpha-1} \exp(-\beta x^2 + \gamma x)}{\Psi[\alpha/2, \gamma/\sqrt{\beta}]} \quad (x > 0)$$

where $\Psi[a, z]$ is the Fox-Wright Psi function. `rmhn` does not evaluate Ψ; the rejection-sampling kernels cancel it out.

The default parameters `alpha = 1, beta = 1, gamma = 0` correspond to the half-normal distribution $\mathrm{HN}(1/\sqrt{2})$.

The `method` argument selects the rejection sampler:

- `"auto"`: Special-case shortcuts when applicable ($\gamma \approx 0$ -> sqrt-Gamma, $\alpha \approx 1$ -> truncated normal). Otherwise dispatches to RTDR (Gao & Wang, 2025).
- `"rtdr"`: Force the Relaxed Transformed Density Rejection method of Gao & Wang (2025). The acceptance probability is bounded below by $1/e \approx 0.368$ uniformly over the parameter space. Note: Gao & Wang (2025) use the parameterization (λ, α, β) with density proportional to $x^{\lambda-1} \exp(-\alpha x^2 - \beta x)$; the mapping to the Sun et al. parameterization used here is $\lambda \leftrightarrow \alpha$, $\alpha \leftrightarrow \beta$, $\beta \leftrightarrow -\gamma$ (sign flip on the linear term).
- `"sun"`: Force the Sun et al. (2023) algorithms. Algorithm 1 is used when $\gamma > 0$ and $\alpha > 1$; Algorithm 3 is used when $\gamma \leq 0$. The combination $\alpha < 1$ with $\gamma > 0$ is unsupported and triggers an error.

Vector parameters are recycled to length `n` following standard R rules. Trailing parameter elements beyond index `n - 1` are silently ignored, matching the convention of `rnorm`.

Internally the setup state of the chosen sampler is reused as long as consecutive (α, β, γ) triples are equal, so passing parameters grouped by triple is faster than calling `rmhn` inside an R loop.

Value

A numeric vector of length `n`. If any of `alpha`, `beta`, `gamma` (after recycling to length `n`) is `NA` or non-finite (`Inf`, `-Inf`, `NaN`), the corresponding output element is `NA`.

References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

Gao, F. & Wang, H.-B. (2025). Generating modified-half-normal random variates by a relaxed transformed density rejection method. *Communications in Statistics - Simulation and Computation*.

Robert, C. P. (1995). Simulation of truncated normal variables. *Statistics and Computing*, 5(2), 121–125.

See Also

`dmhn`, `mhn_mean`, `mhn_var`

Examples

```
set.seed(1)
rmhn(10, alpha = 2, beta = 1, gamma = 0.5)

# Vector parameters are recycled to length n.
set.seed(1)
rmhn(5, alpha = c(1, 2, 3, 4, 5))
```

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