Package: meteorits (via r-universe)

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```
Version 0.1.1
Description Provides a unified mixture-of-experts (ME) modeling and
      estimation framework with several original and flexible ME
      models to model, cluster and classify heterogeneous data in
      many complex situations where the data are distributed
      according to non-normal, possibly skewed distributions, and
      when they might be corrupted by atypical observations.
      Mixtures-of-Experts models for complex and non-normal
      distributions ('meteorits') are originally introduced and
      written in 'Matlab' by Faicel Chamroukhi. The references are
      mainly the following ones. The references are mainly the
      following ones. Chamroukhi F., Same A., Govaert, G. and Aknin
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URL https://github.com/fchamroukhi/MEteorits
BugReports https://github.com/fchamroukhi/MEteorits/issues
License GPL (>= 3)
Depends R (>= 2.10)
Imports pracma, methods, stats, MASS, Rcpp
Suggests knitr, rmarkdown
LinkingTo Rcpp, RcppArmadillo
Collate meteorits-package.R RcppExports.R logsumexp.R utils.R
      sampleUnivNMoE.R sampleUnivSNMoE.R sampleUnivStMoE.R
```

sampleUnivTMoE.R ParamSNMoE.R ParamStMoE.R ParamTMoE.R ParamNMoE.R StatSNMoE.R StatStMoE.R StatTMoE.R StatNMoE.R

Title Mixture-of-Experts Modeling for Complex Non-Normal Distributions

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meteorits-package	MEteorits: Mixtures-of-ExperTs modEling for cOmplex and non-noRmal dIsTributions

Description

meteorits is a package containing several original and flexible mixtures-of-experts models to model, cluster and classify heteregenous data in many complex situations where the data are distributed according to non-normal and possibly skewed distributions, and when they might be corrupted by atypical observations. The toolbox also contains sparse mixture-of-experts models for high-dimensional data.

meteorits contains the following Mixture-of-Experts models:

- NMoE (Normal Mixtures-of-Experts) provides a flexible framework for heterogenous data with Normal expert regressors network;
- SNMoE (Skew-Normal Mixtures-of-Experts) provides a flexible modeling framework for heterogenous data with possibly skewed distributions to generalize the standard Normal mixture of expert model;
- tMoE (t Mixtures-of-Experts) provides a flexible and robust modeling framework for heterogenous data with possibly heavy-tailed distributions and corrupted by atypical observations;
- StMoE (Skew t Mixtures-of-Experts) provides a flexible and robust modeling framework for heterogenous data with possibly skewed, heavy-tailed distributions and corrupted by atypical observations.

For the advantages/differences of each of them, the user is referred to our mentioned paper references.

To learn more about meteorits, start with the vignettes: browseVignettes(package = "meteorits")

Author(s)

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- Marius Bartcus <marius.bartcus@gmail.com> (R port) [translator]

References

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Chamroukhi, F., A. Same, G. Govaert, and P. Aknin. 2009. *Time Series Modeling by a Regression Approach Based on a Latent Process*. Neural Networks 22 (5-6): 593–602. https://chamroukhi.com/papers/Chamroukhi_Neural_Networks_2009.pdf.

See Also

Useful links:

- https://github.com/fchamroukhi/MEteorits
- Report bugs at https://github.com/fchamroukhi/MEteorits/issues

emNMoE implements the EM algorithm to fit a Normal Mixture of Experts (NMoE).

Description

emNMoE implements the maximum-likelihood parameter estimation of a Normal Mixture of Experts (NMoE) model by the Expectation-Maximization (EM) algorithm.

Usage

```
emNMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
    threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

Χ	Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
Υ	Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
K	The number of experts.
p	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the EM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the EM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the EM as stopping criteria.

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verbose Optional. A logical value indicating whether or not values of the log-likelihood should be printed during EM iterations.

verbose_IRLS Optional. A logical value indicating whether or not values of the criterion opti-

mized by IRLS should be printed at each step of the EM algorithm.

Details

emNMoE function implements the EM algorithm for the NMoE model. This function starts with an initialization of the parameters done by the method initParam of the class ParamNMoE, then it alternates between the E-Step (method of the class StatNMoE) and the M-Step (method of the class ParamNMoE) until convergence (until the relative variation of log-likelihood between two steps of the EM algorithm is less than the threshold parameter).

Value

EM returns an object of class ModelNMoE.

See Also

ModelNMoE, ParamNMoE, StatNMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly
nmoe <- emNMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)
nmoe$summary()
nmoe$plot()</pre>
```

emSNMoE

emSNMoE implements the ECM algorithm to fit a Skew-Normal Mixture of Experts (SNMoE).

Description

emSNMoE implements the maximum-likelihood parameter estimation of a Skew-Normal Mixture of Experts (SNMoE) model by the Expectation Conditional Maximization (ECM) algorithm.

Usage

```
emSNMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

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Χ	Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
Υ	Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
K	The number of experts.
р	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the ECM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the ECM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the ECM as stopping criteria.
verbose	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during ECM iterations.
verbose_IRLS	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the ECM algorithm.

Details

emSNMoE function implements the ECM algorithm for the SNMoE model. This function starts with an initialization of the parameters done by the method initParam of the class ParamSNMoE, then it alternates between the E-Step (method of the class StatSNMoE) and the M-Step (method of the class ParamSNMoE) until convergence (until the relative variation of log-likelihood between two steps of the ECM algorithm is less than the threshold parameter).

Value

ECM returns an object of class ModelSNMoE.

See Also

ModelSNMoE, ParamSNMoE, StatSNMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly
snmoe <- emSNMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)
snmoe$summary()
snmoe$plot()</pre>
```

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emStMoE	emStMoE implements the ECM algorithm to fit a Skew-t Mixture of Experts (StMoE).
	The state of the s

Description

emStMoE implements the maximum-likelihood parameter estimation of a Skew-t Mixture of Experts (StMoE) model by the Expectation Conditional Maximization (ECM) algorithm.

Usage

```
emStMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

Χ	Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
Υ	Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
K	The number of experts.
р	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the ECM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the ECM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the ECM as stopping criteria.
verbose	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during ECM iterations.
verbose_IRLS	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the ECM algorithm.

Details

emStMoE function implements the ECM algorithm for the StMoE model. This function starts with an initialization of the parameters done by the method initParam of the class ParamStMoE, then it alternates between the E-Step (method of the class StatStMoE) and the M-Step (method of the class ParamStMoE) until convergence (until the relative variation of log-likelihood between two steps of the ECM algorithm is less than the threshold parameter).

Value

ECM returns an object of class ModelStMoE.

See Also

ModelStMoE, ParamStMoE, StatStMoE

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Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly
stmoe <- emStMoE(X = x, Y = y, K = 2, p = 1, threshold = 1e-4, verbose = TRUE)
stmoe$summary()
stmoe$plot()</pre>
```

emTMoE

emTMoE implements the ECM algorithm to fit a t Mixture of Experts (TMoE).

Description

emTMoE implements the maximum-likelihood parameter estimation of a Student Mixture of Experts (TMoE) model by the Conditional Expectation Maximization (ECM) algorithm.

Usage

```
emTMoE(X, Y, K, p = 3, q = 1, n_tries = 1, max_iter = 1500,
    threshold = 1e-06, verbose = FALSE, verbose_IRLS = FALSE)
```

Arguments

Χ	Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
Υ	Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
K	The number of experts.
р	Optional. The order of the polynomial regression for the experts.
q	Optional. The order of the logistic regression for the gating network.
n_tries	Optional. Number of runs of the ECM algorithm. The solution providing the highest log-likelihood will be returned.
max_iter	Optional. The maximum number of iterations for the ECM algorithm.
threshold	Optional. A numeric value specifying the threshold for the relative difference of log-likelihood between two steps of the ECM as stopping criteria.
verbose	Optional. A logical value indicating whether or not values of the log-likelihood should be printed during ECM iterations.
verbose_IRLS	Optional. A logical value indicating whether or not values of the criterion optimized by IRLS should be printed at each step of the ECM algorithm.

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Details

emTMoE function implements the ECM algorithm for the TMoE model. This function starts with an initialization of the parameters done by the method initParam of the class ParamTMoE, then it alternates between the E-Step (method of the class StatTMoE) and the M-Step (method of the class ParamTMoE) until convergence (until the relative variation of log-likelihood between two steps of the ECM algorithm is less than the threshold parameter).

Value

ECM returns an object of class ModelTMoE.

See Also

ModelTMoE, ParamTMoE, StatTMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

tmoe <- emTMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

tmoe$summary()

tmoe$plot()</pre>
```

ModelNMoE-class

A Reference Class which represents a fitted NMoE model.

Description

ModelNMoE represents an estimated NMoE model.

Fields

```
param A ParamNMoE object. It contains the estimated values of the parameters. stat A StatNMoE object. It contains all the statistics associated to the NMoE model.
```

Methods

```
plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...) Plot method.
    what The type of graph requested:
```

- "meancurve" = Estimated mean and estimated experts means given the input X (fields Ey and Ey_k of class StatNMoE).
- "confregions" = Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field Vary of class StatNMoE).

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- "clusters" = Estimated experts means (field Ey_k) and hard partition (field klas of class StatNMoE).
- "loglikelihood" = Value of the log-likelihood for each iteration (field stored_loglik of class StatNMoE).

... Other graphics parameters.

By default, all the graphs mentioned above are produced.

```
summary(digits = getOption("digits")) Summary method.
```

digits The number of significant digits to use when printing.

See Also

ParamNMoE, StatNMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

nmoe <- emNMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

# nmoe is a ModelNMoE object. It contains some methods such as 'summary' and 'plot'
nmoe$summary()
nmoe$plot()

# nmoe has also two fields, stat and param which are reference classes as well

# Log-likelihood:
nmoe$stat$loglik

# Parameters of the polynomial regressions:
nmoe$param$beta</pre>
```

ModelSNMoE-class

A Reference Class which represents a fitted SNMoE model.

Description

ModelSNMoE represents an estimated SNMoE model.

Fields

```
param A ParamSNMoE object. It contains the estimated values of the parameters.

stat A StatSNMoE object. It contains all the statistics associated to the SNMoE model.
```

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Methods

```
plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...) Plot method.
    what The type of graph requested:
```

- "meancurve" = Estimated mean and estimated experts means given the input X (fields Ey and Ey_k of class StatSNMoE).
- "confregions" = Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field Vary of class StatSNMoE).
- "clusters" = Estimated experts means (field Ey_k) and hard partition (field klas of class StatSNMoE).
- "loglikelihood" = Value of the log-likelihood for each iteration (field stored_loglik of class StatSNMoE).

... Other graphics parameters.

By default, all the graphs mentioned above are produced.

```
summary(digits = getOption("digits")) Summary method.
```

digits The number of significant digits to use when printing.

See Also

ParamSNMoE, StatSNMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly
snmoe <- emSNMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)
# snmoe is a ModelSNMoE object. It contains some methods such as 'summary' and 'plot'
snmoe$summary()
snmoe$plot()
# snmoe has also two fields, stat and param which are reference classes as well
# Log-likelihood:
snmoe$stat$loglik
# Parameters of the polynomial regressions:
snmoe$param$beta</pre>
```

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ModelStMoE-class

A Reference Class which represents a fitted StMoE model.

Description

ModelStMoE represents an estimated StMoE model.

Fields

```
param A ParamStMoE object. It contains the estimated values of the parameters. stat A StatStMoE object. It contains all the statistics associated to the StMoE model.
```

Methods

```
plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...) Plot method.
    what The type of graph requested:
```

- "meancurve" = Estimated mean and estimated experts means given the input X (fields Ey and Ey_k of class StatStMoE).
- "confregions" = Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field Vary of class StatStMoE).
- "clusters" = Estimated experts means (field Ey_k) and hard partition (field klas of class StatStMoE).
- "loglikelihood" = Value of the log-likelihood for each iteration (field stored_loglik of class StatStMoE).

... Other graphics parameters.

By default, all the graphs mentioned above are produced.

```
summary(digits = getOption("digits")) Summary method.
```

digits The number of significant digits to use when printing.

See Also

ParamStMoE, StatStMoE

Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

stmoe <- emStMoE(X = x, Y = y, K = 2, p = 1, threshold = 1e-4, verbose = TRUE)

# stmoe is a ModelSTMoE object. It contains some methods such as 'summary' and 'plot'
stmoe$summary()
stmoe$plot()</pre>
```

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```
# stmoe has also two fields, stat and param which are reference classes as well
# Log-likelihood:
stmoe$stat$loglik
# Parameters of the polynomial regressions:
stmoe$param$beta
```

ModelTMoE-class

A Reference Class which represents a fitted TMoE model.

Description

ModelTMoE represents an estimated TMoE model.

Fields

param A ParamTMoE object. It contains the estimated values of the parameters. stat A StatTMoE object. It contains all the statistics associated to the TMoE model.

Methods

```
plot(what = c("meancurve", "confregions", "clusters", "loglikelihood"), ...) Plot method.
    what The type of graph requested:
```

- "meancurve" = Estimated mean and estimated experts means given the input X (fields Ey and Ey_k of class StatTMoE).
- "confregions" = Estimated mean and confidence regions. Confidence regions are computed as plus and minus twice the estimated standard deviation (the squarre root of the field Vary of class StatTMoE).
- "clusters" = Estimated experts means (field Ey_k) and hard partition (field klas of class StatTMoE).
- "loglikelihood" = Value of the log-likelihood for each iteration (field stored_loglik of class StatTMoE).

... Other graphics parameters.

By default, all the graphs mentioned above are produced.

```
summary(digits = getOption("digits")) Summary method.
```

digits The number of significant digits to use when printing.

See Also

ParamTMoE, StatTMoE

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Examples

```
data(tempanomalies)
x <- tempanomalies$Year
y <- tempanomalies$AnnualAnomaly

tmoe <- emTMoE(X = x, Y = y, K = 2, p = 1, verbose = TRUE)

# tmoe is a ModelTMoE object. It contains some methods such as 'summary' and 'plot'
tmoe$summary()
tmoe$plot()

# tmoe has also two fields, stat and param which are reference classes as well

# Log-likelihood:
tmoe$stat$loglik

# Parameters of the polynomial regressions:
tmoe$param$beta</pre>
```

ParamNMoE-class

A Reference Class which contains parameters of a NMoE model.

Description

ParamNMoE contains all the parameters of a NMoE model.

Fields

- X Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
- Y Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
- n Numeric. Length of the response/output vector Y.
- K The number of experts.
- p The order of the polynomial regression for the experts.
- q The order of the logistic regression for the gating network.
- alpha Parameters of the gating network. $\alpha = (\alpha_1, \dots, \alpha_{K-1})$ is a matrix of dimension (q + 1, K 1), with q the order of the logistic regression for the gating network. q is fixed to 1 by default.
- beta Polynomial regressions coefficients for each expert. $\beta = (\beta_1, \dots, \beta_K)$ is a matrix of dimension (p+1, K), with p the order of the polynomial regression. p is fixed to 3 by default.
- sigma2 The variances for the K mixture components (matrix of size (1, K)).
- df The degree of freedom of the NMoE model representing the complexity of the model.

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Methods

initParam(segmental = FALSE) Method to initialize parameters alpha, beta and sigma2.

If segmental = TRUE then alpha, beta and sigma2 are initialized by clustering the response Y uniformly into K contiguous segments. Otherwise, alpha, beta and sigma2 are initialized by clustering randomly the response Y into K segments.

ParamSNMoE-class

A Reference Class which contains parameters of a SNMoE model.

Description

ParamSNMoE contains all the parameters of a SNMoE model.

Fields

- X Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
- Y Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
- n Numeric. Length of the response/output vector Y.
- K The number of experts.
- p The order of the polynomial regression for the experts.
- q The order of the logistic regression for the gating network.
- alpha Parameters of the gating network. $\alpha = (\alpha_1, \dots, \alpha_{K-1})$ is a matrix of dimension (q + 1, K 1), with q the order of the logistic regression for the gating network. q is fixed to 1 by default.
- beta Polynomial regressions coefficients for each expert. $\beta = (\beta_1, \dots, \beta_K)$ is a matrix of dimension (p+1, K), with p the order of the polynomial regression. p is fixed to 3 by default.

sigma2 The variances for the K mixture components (matrix of size (1, K)).

lambda The skewness parameters for each experts (matrix of size (1, K)).

delta delta is equal to $\delta = \frac{\lambda}{\sqrt{1+\lambda^2}}$.

df The degree of freedom of the SNMoE model representing the complexity of the model.

Methods

initParam(segmental = FALSE) Method to initialize parameters alpha, beta and sigma2.

If segmental = TRUE then alpha, beta and sigma2 are initialized by clustering the response Y uniformly into K contiguous segments. Otherwise, alpha, beta and sigma2 are initialized by clustering randomly the response Y into K segments.

MStep(statSNMoE, verbose_IRLS) Method which implements the M-step of the EM algorithm to learn the parameters of the SNMoE model based on statistics provided by the object statSNMoE of class StatSNMoE (which contains the E-step).

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ParamStMoE-class

A Reference Class which contains parameters of a StMoE model.

Description

ParamStMoE contains all the parameters of a StMoE model.

Fields

- X Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
- Y Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
- n Numeric. Length of the response/output vector Y.
- K The number of experts.
- p The order of the polynomial regression for the experts.
- q The order of the logistic regression for the gating network.
- alpha Parameters of the gating network. $\alpha = (\alpha_1, \dots, \alpha_{K-1})$ is a matrix of dimension (q+1, K-1), with q the order of the logistic regression for the gating network. q is fixed to 1 by default.
- beta Polynomial regressions coefficients for each expert. $\beta = (\beta_1, \dots, \beta_K)$ is a matrix of dimension (p+1, K), with p the order of the polynomial regression. p is fixed to 3 by default.
- sigma2 The variances for the K mixture components (matrix of size (1, K)).
- lambda The skewness parameters for each experts (matrix of size (1, K)).
- delta delta is equal to $\delta = \frac{\lambda}{\sqrt{1+\lambda^2}}$.
- nu The degree of freedom for the Student distribution for each experts (matrix of size (1, K)).
- df The degree of freedom of the StMoE model representing the complexity of the model.

Methods

initParam(segmental = FALSE) Method to initialize parameters alpha, beta and sigma2.

If segmental = TRUE then alpha, beta and sigma2 are initialized by clustering the response Y uniformly into K contiguous segments. Otherwise, alpha, beta and sigma2 are initialized by clustering randomly the response Y into K segments.

MStep(statStMoE, calcAlpha = FALSE, calcBeta = FALSE, calcSigma2 = FALSE, calcLambda = FALSE, calcNu = FALSE Method which implements the M-step of the EM algorithm to learn the parameters of the StMoE model based on statistics provided by the object statStMoE of class StatStMoE (which contains the E-step).

ParamTMoE-class 17

ParamTMoE-class

A Reference Class which contains parameters of a TMoE model.

Description

ParamTMoE contains all the parameters of a TMoE model.

Fields

- X Numeric vector of length n representing the covariates/inputs x_1, \ldots, x_n .
- Y Numeric vector of length n representing the observed response/output y_1, \ldots, y_n .
- n Numeric. Length of the response/output vector Y.
- K The number of experts.
- p The order of the polynomial regression for the experts.
- q The order of the logistic regression for the gating network.
- alpha Parameters of the gating network. $\alpha = (\alpha_1, \dots, \alpha_{K-1})$ is a matrix of dimension (q + 1, K 1), with q the order of the logistic regression for the gating network. q is fixed to 1 by default.
- beta Polynomial regressions coefficients for each expert. $\beta = (\beta_1, \dots, \beta_K)$ is a matrix of dimension (p+1, K), with p the order of the polynomial regression. p is fixed to 3 by default.
- sigma2 The variances for the K mixture components (matrix of size (1, K)).
- nu The degree of freedom for the Student distribution for each experts (matrix of size (1, K)).
- df The degree of freedom of the TMoE model representing the complexity of the model.

Methods

- initParam(segmental = FALSE) Method to initialize parameters alpha, beta and sigma2.
 - If segmental = TRUE then alpha, beta and sigma2 are initialized by clustering the response Y uniformly into K contiguous segments. Otherwise, alpha, beta and sigma2 are initialized by clustering randomly the response Y into K segments.
- MStep(statTMoE, verbose_IRLS) Method which implements the M-step of the EM algorithm to learn the parameters of the TMoE model based on statistics provided by the object statTMoE of class StatTMoE (which contains the E-step).

18 sampleUnivNMoE

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Draw a sample from a normal mixture of linear experts model.

Description

Draw a sample from a normal mixture of linear experts model.

Usage

```
sampleUnivNMoE(alphak, betak, sigmak, x)
```

Arguments

alphak	The parameters of the gating network. alphak is a matrix of size $(q + 1, K - 1)$, with $K - 1$, the number of regressors (experts) and q the order of the logistic regression
betak	Matrix of size $(p + 1, K)$ representing the regression coefficients of the experts network.
sigmak	Vector of length K giving the standard deviations of the experts network.
x	A vector og length n representing the inputs (predictors).

Value

A list with the output variable y and statistics.

- y Vector of length *n* giving the output variable.
- zi A vector of size n giving the hidden label of the expert component generating the i-th observation. Its elements are zi[i] = k, if the i-th observation has been generated by the k-th expert.
- z A matrix of size (n, K) giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- stats A list whose elements are:
 - Ey_k Matrix of size (n, K) giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation zi = k, and the value of predictor X = xi.
 - Ey Vector of length n giving the conditional expectation of Yi given the value of predictor
 X = xi.
 - Vary_k Vector of length k representing the conditional variance of Yi given zi = k, and X = xi.
 - Vary Vector of length n giving the conditional expectation of Yi given X = xi.

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Examples

```
n <- 500 # Size of the sample
alphak <- matrix(c(0, 8), ncol = 1) # Parameters of the gating network
betak <- matrix(c(0, -2.5, 0, 2.5), ncol = 2) # Regression coefficients of the experts
sigmak <- c(1, 1) # Standard deviations of the experts
x <- seq.int(from = -1, to = 1, length.out = n) # Inputs (predictors)

# Generate sample of size n
sample <- sampleUnivNMoE(alphak = alphak, betak = betak, sigmak = sigmak, x = x)

# Plot points and estimated means
plot(x, sample$y, pch = 4)
lines(x, sample$stats$Ey_k[, 1], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey_k[, 2], col = "blue", lty = "dotted", lwd = 1.5)
lines(x, sample$stats$Ey, col = "red", lwd = 1.5)</pre>
```

sampleUnivSNMoE

Draw a sample from a skew-normal mixture of linear experts model.

Description

Draw a sample from a skew-normal mixture of linear experts model.

Usage

```
sampleUnivSNMoE(alphak, betak, sigmak, lambdak, x)
```

Arguments

alphak	The parameters of the gating network. alphak is a matrix of size $(q + 1, K - 1)$, with $K - 1$, the number of regressors (experts) and q the order of the logistic regression
betak	Matrix of size $(p + 1, K)$ representing the regression coefficients of the experts network.
sigmak	Vector of length K giving the standard deviations of the experts network.
lambdak	Vector of length K giving the skewness parameter of each experts.
X	A vector og length <i>n</i> representing the inputs (predictors).

Value

A list with the output variable y and statistics.

- y Vector of length *n* giving the output variable.
- zi A vector of size n giving the hidden label of the expert component generating the i-th observation. Its elements are zi[i] = k, if the i-th observation has been generated by the k-th expert.

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• z A matrix of size (n, K) giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.

- stats A list whose elements are:
 - Ey_k Matrix of size (n, K) giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation zi = k, and the value of predictor X = xi.
 - Ey Vector of length n giving the conditional expectation of Yi given the value of predictor
 X = xi.
 - Vary_k Vector of length k representing the conditional variance of Yi given zi = k, and X = xi.
 - Vary Vector of length n giving the conditional expectation of Yi given X = xi.

Examples

sampleUnivStMoE

Draw a sample from a univariate skew-t mixture.

Description

Draw a sample from a univariate skew-t mixture.

Usage

```
sampleUnivStMoE(alphak, betak, sigmak, lambdak, nuk, x)
```

Arguments

alphak

The parameters of the gating network. alphak is a matrix of size (q + 1, K - 1), with K - 1, the number of regressors (experts) and q the order of the logistic regression

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betak	Matrix of size $(p + 1, K)$ representing the regression coefficients of the experts network.
sigmak	Vector of length K giving the standard deviations of the experts network.
lambdak	Vector of length K giving the skewness parameter of each experts.
nuk	Vector of length K giving the degrees of freedom of the experts network t densities.
X	A vector og length <i>n</i> representing the inputs (predictors).

Value

A list with the output variable y and statistics.

- y Vector of length *n* giving the output variable.
- zi A vector of size n giving the hidden label of the expert component generating the i-th observation. Its elements are zi[i] = k, if the i-th observation has been generated by the k-th expert.
- z A matrix of size (n, K) giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- stats A list whose elements are:
 - Ey_k Matrix of size (n, K) giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation zi = k, and the value of predictor X = xi.
 - Ey Vector of length n giving the conditional expectation of Yi given the value of predictor
 X = xi.
 - Vary_k Vector of length k representing the conditional variance of Yi given zi = k, and X = xi.
 - Vary Vector of length n giving the conditional expectation of Yi given X = xi.

Examples

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sampleUnivTMoE	Draw a sample from a univariate t mixture of experts (TMoE).	

Description

Draw a sample from a univariate t mixture of experts (TMoE).

Usage

```
sampleUnivTMoE(alphak, betak, sigmak, nuk, x)
```

Arguments

alphak	The parameters of the gating network. alphak is a matrix of size $(q + 1, K - 1)$, with $K - 1$, the number of regressors (experts) and q the order of the logistic regression
betak	Matrix of size $(p + I, K)$ representing the regression coefficients of the experts network.
sigmak	Vector of length K giving the standard deviations of the experts network.
nuk	Vector of length K giving the degrees of freedom of the experts network t densities.
X	A vector of length <i>n</i> representing the inputs (predictors).

Value

A list with the output variable y and statistics.

- y Vector of length *n* giving the output variable.
- zi A vector of size n giving the hidden label of the expert component generating the i-th observation. Its elements are zi[i] = k, if the i-th observation has been generated by the k-th expert.
- z A matrix of size (n, K) giving the values of the binary latent component indicators Z_{ik} such that $Z_{ik} = 1$ iff $Z_i = k$.
- stats A list whose elements are:
 - Ey_k Matrix of size (n, K) giving the conditional expectation of Yi the output variable given the value of the hidden label of the expert component generating the ith observation zi = k, and the value of predictor X = xi.
 - Ey Vector of length n giving the conditional expectation of Yi given the value of predictor
 X = xi.
 - Vary_k Vector of length k representing the conditional variance of Yi given zi = k, and X = xi.
 - Vary Vector of length n giving the conditional expectation of Yi given X = xi.

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Examples

StatNMoE-class

A Reference Class which contains statistics of a NMoE model.

Description

StatNMoE contains all the statistics associated to a NMoE model. It mainly includes the E-Step of the EM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelhood.

Fields

- piik Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \Psi) = P(z_i = k | x; \Psi)$ of the latent variable $z_i, i = 1, ..., n$.
- z_ik Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z_i = 1$ if $z_i = 1$ if
- klas Column matrix of the labels issued from z_ik. Its elements are $klas(i) = k, k = 1, \dots, K$.
- tik Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.
- Ey_k Matrix of dimension (n, K) giving the estimated means of the experts.
- Ey Column matrix of dimension *n* giving the estimated mean of the NMoE.
- Var_yk Column matrix of dimension K giving the estimated means of the experts.
- Vary Column matrix of dimension n giving the estimated variance of the response.
- loglik Numeric. Observed-data log-likelihood of the NMoE model.
- com_loglik Numeric. Complete-data log-likelihood of the NMoE model.
- stored_loglik Numeric vector. Stored values of the log-likelihood at each EM iteration.
- BIC Numeric. Value of BIC (Bayesian Information Criterion).

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- ICL Numeric. Value of ICL (Integrated Completed Likelihood).
- AIC Numeric. Value of AIC (Akaike Information Criterion).
- log_piik_fik Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i, z_i = k | \mathbf{x}, \mathbf{\Psi}), i = 1, ..., n$.

log_sum_piik_fik Column matrix of size m giving the values of $\log \sum_{k=1}^K P(y_i, z_i = k | x, \Psi)$, $i = 1, \ldots, n$.

Methods

- computeLikelihood(reg_irls) Method to compute the log-likelihood. reg_irls is the value of the regularization part in the IRLS algorithm.
- computeStats(paramNMoE) Method used in the EM algorithm to compute statistics based on parameters provided by the object paramNMoE of class ParamNMoE.
- EStep(paramNMoE) Method used in the EM algorithm to update statistics based on parameters provided by the object paramNMoE of class ParamNMoE (prior and posterior probabilities).
- MAP() MAP calculates values of the fields z_ik and klas by applying the Maximum A Posteriori Bayes allocation rule.
 - $z_{ik} = 1$ if $k = \arg \max_s \tau_{is}$; 0 otherwise

See Also

ParamNMoE

StatSNMoE-class

A Reference Class which contains statistics of a SNMoE model.

Description

StatSNMoE contains all the statistics associated to a SNMoE model. It mainly includes the E-Step of the ECM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelhood.

Fields

- piik Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \Psi) = P(z_i = k | \boldsymbol{x}; \Psi)$ of the latent variable $z_i, i = 1, ..., n$.
- z_ik Hard segmentation logical matrix of dimension (n,K) obtained by the Maximum a posteriori (MAP) rule: $z_ik=1$ if $z_ik=1$ arg max $_s$ τ_{is} ; 0 otherwise, $k=1,\ldots,K$.
- klas Column matrix of the labels issued from z_ik. Its elements are $klas(i) = k, k = 1, \dots, K$.
- tik Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.
- Ey_k Matrix of dimension (n, K) giving the estimated means of the experts.
- Ey Column matrix of dimension n giving the estimated mean of the SNMoE.

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Var_yk Column matrix of dimension *K* giving the estimated means of the experts.

Vary Column matrix of dimension n giving the estimated variance of the response.

loglik Numeric. Observed-data log-likelihood of the SNMoE model.

com_loglik Numeric. Complete-data log-likelihood of the SNMoE model.

stored_loglik Numeric vector. Stored values of the log-likelihood at each ECM iteration.

BIC Numeric. Value of BIC (Bayesian Information Criterion).

ICL Numeric. Value of ICL (Integrated Completed Likelihood).

AIC Numeric. Value of AIC (Akaike Information Criterion).

log_piik_fik Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i, z_i = k | \boldsymbol{x}, \boldsymbol{\Psi}), i = 1, ..., n.$

log_sum_piik_fik Column matrix of size m giving the values of $\log \sum_{k=1}^K P(y_i, z_i = k | \boldsymbol{x}, \boldsymbol{\Psi}),$ $i = 1, \dots, n.$

Elik Conditional expectations of U_i (Matrix of size (n, K)).

E2ik Conditional expectations of U_i^2 (Matrix of size (n, K)).

Methods

computeLikelihood(reg_irls) Method to compute the log-likelihood. reg_irls is the value of the regularization part in the IRLS algorithm.

computeStats(paramSNMoE) Method used in the ECM algorithm to compute statistics based on parameters provided by the object paramSNMoE of class ParamSNMoE.

EStep(paramSNMoE) Method used in the ECM algorithm to update statistics based on parameters provided by the object paramSNMoE of class ParamSNMoE (prior and posterior probabilities).

MAP() MAP calculates values of the fields z_ik and klas by applying the Maximum A Posteriori Bayes allocation rule.

```
z_{ik} = 1 if k = \arg \max_s \tau_{is}; 0 otherwise
```

See Also

ParamSNMoE

StatStMoE-class

A Reference Class which contains statistics of a StMoE model.

Description

StatStMoE contains all the statistics associated to a StMoE model. It mainly includes the E-Step of the ECM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelhood.

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Fields

- piik Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \Psi) = P(z_i = k | \boldsymbol{x}; \Psi)$ of the latent variable $z_i, i = 1, ..., n$.
- z_ik Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z_i = 1$ if $z_i = 1$ if
- klas Column matrix of the labels issued from z_ik. Its elements are $klas(i) = k, k = 1, \dots, K$.
- tik Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.
- Ey_k Matrix of dimension (n, K) giving the estimated means of the experts.
- Ey Column matrix of dimension n giving the estimated mean of the StMoE.
- Var_yk Column matrix of dimension K giving the estimated means of the experts.
- Vary Column matrix of dimension n giving the estimated variance of the response.
- loglik Numeric. Observed-data log-likelihood of the StMoE model.
- com_loglik Numeric. Complete-data log-likelihood of the StMoE model.
- stored_loglik Numeric vector. Stored values of the log-likelihood at each ECM iteration.
- BIC Numeric. Value of BIC (Bayesian Information Criterion).
- ICL Numeric. Value of ICL (Integrated Completed Likelihood).
- AIC Numeric. Value of AIC (Akaike Information Criterion).
- log_piik_fik Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i, z_i = k | \boldsymbol{x}, \boldsymbol{\Psi}), i = 1, \dots, n.$
- $\log_{\min_i} P(y_i, z_i = k | x, \Psi),$ i = 1, ..., n.
- dik It represents the value of d_{ik} .
- wik Conditional expectations w_{ik} .
- E1ik Conditional expectations $e_{1,ik}$.
- E2ik Conditional expectations $e_{2,ik}$.
- E3ik Conditional expectations $e_{3,ik}$.
- stme_pdf Skew-t mixture of experts density.

Methods

- computeLikelihood(reg_irls) Method to compute the log-likelihood. reg_irls is the value of the regularization part in the IRLS algorithm.
- computeStats(paramStMoE) Method used in the ECM algorithm to compute statistics based on parameters provided by the object paramStMoE of class ParamStMoE.
- EStep(paramStMoE, calcTau = FALSE, calcE1 = FALSE, calcE2 = FALSE, calcE3 = FALSE) Method used in the ECM algorithm to update statistics based on parameters provided by the object paramStMoE of class ParamStMoE (prior and posterior probabilities).
- MAP() MAP calculates values of the fields z_ik and klas by applying the Maximum A Posteriori Bayes allocation rule.

```
z_{ik} = 1 if k = \arg\max_{s} \tau_{is}; 0 otherwise
```

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See Also

ParamStMoE

StatTMoE-class

A Reference Class which contains statistics of a TMoE model.

Description

StatTMoE contains all the statistics associated to a TMoE model. It mainly includes the E-Step of the ECM algorithm calculating the posterior distribution of the hidden variables, as well as the calculation of the log-likelhood.

Fields

- piik Matrix of size (n, K) representing the probabilities $\pi_k(x_i; \Psi) = P(z_i = k | x; \Psi)$ of the latent variable $z_i, i = 1, ..., n$.
- z_ik Hard segmentation logical matrix of dimension (n, K) obtained by the Maximum a posteriori (MAP) rule: $z_i = 1$ if $z_i = 1$ if
- klas Column matrix of the labels issued from z_ik. Its elements are $klas(i) = k, k = 1, \dots, K$.
- tik Matrix of size (n, K) giving the posterior probability τ_{ik} that the observation y_i originates from the k-th expert.
- Ey_k Matrix of dimension (n, K) giving the estimated means of the experts.
- Ey Column matrix of dimension n giving the estimated mean of the TMoE.
- Var_yk Column matrix of dimension K giving the estimated means of the experts.
- Vary Column matrix of dimension *n* giving the estimated variance of the response.
- loglik Numeric. Observed-data log-likelihood of the TMoE model.
- com_loglik Numeric. Complete-data log-likelihood of the TMoE model.
- stored_loglik Numeric vector. Stored values of the log-likelihood at each ECM iteration.
- BIC Numeric. Value of BIC (Bayesian Information Criterion).
- ICL Numeric. Value of ICL (Integrated Completed Likelihood).
- AIC Numeric. Value of AIC (Akaike Information Criterion).
- log_piik_fik Matrix of size (n, K) giving the values of the logarithm of the joint probability $P(y_i, z_i = k | \boldsymbol{x}, \boldsymbol{\Psi}), i = 1, ..., n.$
- $\log_{\min_i} P(y_i, z_i = k | x, \Psi),$ $i = 1, \dots, n.$
- Wik Conditional expectations w_{ik} .

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Methods

computeLikelihood(reg_irls) Method to compute the log-likelihood. reg_irls is the value of the regularization part in the IRLS algorithm.

computeStats(paramTMoE) Method used in the ECM algorithm to compute statistics based on parameters provided by the object paramTMoE of class ParamTMoE.

EStep(paramTMoE) Method used in the ECM algorithm to update statistics based on parameters provided by the object paramTMoE of class ParamTMoE (prior and posterior probabilities).

MAP() MAP calculates values of the fields z_ik and klas by applying the Maximum A Posteriori Bayes allocation rule.

```
z_{ik} = 1 if k = \arg \max_s \tau_{is}; 0 otherwise
```

See Also

ParamTMoE

tempanomalies

Global Annual Temperature Anomalies (Land Meteorological Stations) (1880-2015)

Description

This dataset is from https://cdiac.ess-dive.lbl.gov/ftp/trends/temp/hansen/gl_land.txt

Usage

tempanomalies

Format

A data frame with 136 rows and 3 columns:

Year Year of observation.

AnnualAnomaly Value in degrees C of the global annual temperature anomaly.

5-YearMean 5-Year mean of temperature anomalies.

Details

Global annual temperature anomalies (degrees C) computed using data from land meteorological stations, 1880-2015. Anomalies are relative to the 1951-1980 base period means.

Non-computed values are indicated by "-99.99".

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