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`aggregateConsequents` *Aggregation of fired consequents into a resulting fuzzy set*

Description

Take a character vector of consequent names, a numeric vector representing the degree of consequents' firing and a matrix that models fuzzy sets corresponding to the consequent names, and perform an aggregation of the consequents into a resulting fuzzy set.

Usage

```
aggregateConsequents(
  conseq,
  degrees,
  partition,
  firing = lukas.residuum,
  aggreg = pgoedel.tnorm
)
```

Arguments

`conseq`	A character vector of consequents. Each value in the vector must correspond to a name of some column of the `partition` matrix. The length of this vector must be the same as of the `degrees` argument.
`degrees`	A numeric vector of membership degrees at which the corresponding consequents (see the `conseq` argument) are fired.
`partition`	A matrix of membership degrees that describes the meaning of the consequents in vector `conseq`: each column of the matrix corresponds to a fuzzy set that models a single consequent (of a name given by column names of the matrix), each row corresponds to a single crisp value (which is not important for this function), hence each cell corresponds to a membership degree in which the

crisp value is a member of a fuzzy set modeling the consequent. Each consequent in `conseq` must correspond to some column of this matrix. Such matrix may be created e.g., by using `fcut()` or `lcut()` functions.

`firing` A two-argument function used to compute the resulting truth value of the consequent. Function is evaluated for each consequent in `conseq`, with corresponding `degrees` value as the first argument and corresponding truth-value of the consequent (from `partition`) as the second argument. In default, the Lukasiewicz residuum (`lukas.residuum()`) is evaluated that way.

`aggreg` An aggregation function to be used to combine fuzzy sets resulting from firing the consequents with the `firing` function. The function should accept multiple numeric vectors of membership degrees as its arguments. In default, the Goedel t-norm (`pgoedel.tnorm()`) is evaluated.

Details

This function is typically used within an inference mechanism, after a set of firing rules is determined and membership degrees of their antecedents are computed, to combine the consequents of the firing rules into a resulting fuzzy set. The result of this function is then typically defuzzified (see `defuzz()`) to obtain a crisp result of the inference.

Function assumes a set of rules with antecedents firing at degrees given in `degrees` and with consequents in `conseq`. The meaning of the consequents is modeled with fuzzy sets whose membership degree values are captured in the `partition` matrix.

With default values of `firing` and `aggreg` parameters, the function computes a fuzzy set that results from a conjunction (Goedel minimum t-norm) of all provided implicative (Lukasiewicz residuum) rules.

In detail, the function first computes the fuzzy set of each fired consequent by calling `part\[i\] <- firing(degrees\[i\],` for each `i`-th consequent and the results are aggregated using the `aggreg` parameter: `aggreg(part\[1\], part\[2\], ...)`. In order to aggregate consequents in a Mamdani-Assilian's fashion, set `firing` to `pgoedel.tnorm()` and `aggreg` to `pgoedel.tconorm()`.

Value

A vector of membership degrees of fuzzy set elements that correspond to rows in the `partition` matrix. If empty vector of consequents is provided, vector of 1's is returned. The length of the resulting vector equals to the number of rows of the `partition` matrix.

Author(s)

Michal Burda

See Also

`fire()`, `perceive()`, `pbld()`, `infer()`, `defuzz()`, `fcut()`, `lcut()`

Examples

```
# create a partition matrix
partition <- matrix(c(0:10/10, 10:0/10, rep(0, 5),
```

```
                        rep(0, 5), 0:10/10, 10:0/10,
                        0:12/12, 1, 12:0/12),
                      byrow=FALSE,
                      ncol=3)
colnames(partition) <- c('a', 'b', 'c')

# the result of aggregation is equal to:
# pmin(1, partition[, 1] + (1 - 0.5), partition[, 2] + (1 - 0.8))
aggregateConsequents(c('a', 'b'), c(0.5, 0.8), partition)
```

---

algebra *Algebra for Fuzzy Sets*

Description

Compute triangular norms (t-norms), triangular conorms (t-conorms), residua, bi-residua, and negations.

Usage

```
algebra(name, stdneg = FALSE, ...)

is.algebra(a)

goedel.tnorm(...)

lukas.tnorm(...)

goguen.tnorm(...)

pgoedel.tnorm(...)

plukas.tnorm(...)

pgoguen.tnorm(...)

goedel.tconorm(...)

lukas.tconorm(...)

goguen.tconorm(...)

pgoedel.tconorm(...)

plukas.tconorm(...)

pgoguen.tconorm(...)
```

```
goedel.residuum(x, y)

lukas.residuum(x, y)

goguen.residuum(x, y)

goedel.biresiduum(x, y)

lukas.biresiduum(x, y)

goguen.biresiduum(x, y)

invol.neg(x)

strict.neg(x)
```

Arguments

name	The name of the algebra to be created. Must be one of: "goedel", "lukasiewicz", "goguen" (or an unambiguous abbreviation).
stdneg	(Deprecated.) TRUE to force the use of a "standard" negation (i.e., involutive negation). Otherwise, the appropriate negation is used in the algebra (e.g., strict negation in Goedel and Goguen algebras and involutive negation in Lukasiewicz algebra).
...	For t-norms and t-conorms, these arguments are numeric vectors of values to compute t-norms or t-conorms from. Values outside the $[0, 1]$ interval cause an error. NA values are also permitted. For the algebra() function, these arguments are passed to the factory functions that create the algebra. (Currently unused.)
a	An object to be checked if it is a valid algebra (i.e., a list returned by the algebra function).
x	Numeric vector of values to compute a residuum or bi-residuum from. Values outside the $[0, 1]$ interval cause an error. NA values are also permitted.
y	Numeric vector of values to compute a residuum or bi-residuum from. Values outside the $[0, 1]$ interval cause an error. NA values are also permitted.

Details

goedel.tnorm, lukas.tnorm, and goguen.tnorm compute the Goedel, Lukasiewicz, and Goguen triangular norm (t-norm) from all values in the arguments. If the arguments are vectors they are combined together firstly so that a numeric vector of length 1 is returned.

pgoedel.tnorm, plukas.tnorm, and pgoguen.tnorm compute the same t-norms, but in an element-wise manner. I.e. the values with indices 1 of all arguments are used to compute the t-norm, then the second values (while recycling the vectors if they do not have the same size) so that the result is a vector of values.

`goedel.tconorm`, `lukas.tconorm`, `goguen.tconorm`, are similar to the previously mentioned functions, except that they compute triangular conorms (t-conorms). `pgoedel.tconorm`, `plukas.tconorm`, and `pgoguen.tconorm` are their element-wise alternatives.

`goedel.residuum`, `lukas.residuum`, and `goguen.residuum` compute residua (i.e., implications) and `goedel.biresiduum`, `lukas.biresiduum`, and `goguen.biresiduum` compute bi-residua. Residua and bi-residua are computed in an element-wise manner, for each corresponding pair of values in `x` and `y` arguments.

`invol.neg` and `strict.neg` compute the involutive and strict negation, respectively.

Let a, b be values from the interval $[0, 1]$. The realized functions can be defined as follows:

- Goedel t-norm: $mina, b$;
- Goguen t-norm: ab;
- Lukasiewicz t-norm: $max0, a + b - 1$;
- Goedel t-conorm: $maxa, b$;
- Goguen t-conorm: $a + b - ab$;
- Lukasiewicz t-conorm: $min1, a + b$;
- Goedel residuum (standard Goedel implication): 1 if $a \leq b$ and b otherwise;
- Goguen residuum (implication): 1 if $a \leq b$ and b/a otherwise;
- Lukasiewicz residuum (standard Lukasiewicz implication): 1 if $a \leq b$ and $1 - a + b$ otherwise;
- Involutive negation: $1 - x$;
- Strict negation: 1 if $x = 0$ and 0 otherwise.

Bi-residuum B is derived from residuum R as follows:

$$B(a, b) = inf(R(a, b), R(b, a)),$$

where inf is the operation of infimum, which for all three algebras corresponds to the min operation.

The arguments have to be numbers from the interval $[0, 1]$. Values outside that range cause an error. NaN values are treated as NAs.

If some argument is NA or NaN, the result is NA. For other handling of missing values, see algebraNA.

Selection of a t-norm may serve as a basis for definition of other operations. From the t-norm, the operation of a residual implication may be defined, which in turn allows the definition of a residual negation. If the residual negation is not involutive, the involutive negation is often added as a new operation and together with the t-norm can be used to define the t-conorm. Therefore, the `algebra` function returns a named list of operations derived from the selected Goedel, Goguen, or Lukasiewicz t-norm. Concretely:

- `algebra("goedel")`: returns the strict negation as the residual negation, the involutive negation, and also the Goedel t-norm, t-conorm, residuum, and bi-residuum;
- `algebra("goguen")`: returns the strict negation as the residual negation, the involutive negation, and also the Goguen t-norm, t-conorm, residuum, and bi-residuum;
- `algebra("lukasiewicz")`: returns involutive negation as both residual and involutive negation, and also the Lukasiewicz t-norm, t-conorm, residuum, and bi-residuum.

Moreover, algebra returns the supremum and infimum functions computed as maximum and minimum, respectively.

is.algebra tests whether the given a argument is a valid algebra, i.e., a list returned by the algebra function.

Value

Functions for t-norms and t-conorms (such as goedel.tnorm) return a numeric vector of size 1 that is the result of the appropriate t-norm or t-conorm applied on all values of all arguments.

Element-wise versions of t-norms and t-conorms (such as pgoedel.tnorm) return a vector of results after applying the appropriate t-norm or t-conorm on argument in an element-wise (i.e., by indices) way. The resulting vector is of length of the longest argument (shorter arguments are recycled).

Residua and bi-residua functions return a numeric vector of length of the longest argument (shorter argument is recycled).

strict.neg and invol.neg compute negations and return a numeric vector of the same size as the argument x.

algebra returns a list of functions of the requested algebra: "n" (residual negation), "ni" (involutive negation), "t" (t-norm), "pt" (element-wise t-norm), "c" (t-conorm), "pc" (element-wise t-conorm), "r" (residuum), "b" (bi-residuum), "s" (supremum), "ps" (element-wise supremum), "i" (infimum), and "pi" (element-wise infimum).

For Lukasiewicz algebra, the elements "n" and "ni" are the same, i.e. the invol.neg function. For Goedel and Goguen algebra, "n" (the residual negation) equals strict.neg and "ni" (the involutive negation) equals invol.neg.

"s", "ps", "i", "pi" are the same for each type of algebra: goedel.conorm, pgoedel.conorm, goedel.tnorm, and pgoedel.tnorm.

Author(s)

Michal Burda

Examples

```
# direct and element-wise version of functions
goedel.tnorm(c(0.3, 0.2, 0.5), c(0.8, 0.1, 0.5))  # 0.1
pgoedel.tnorm(c(0.3, 0.2, 0.5), c(0.8, 0.1, 0.5)) # c(0.3, 0.1, 0.5)

# algebras
x <- runif(10)
y <- runif(10)
a <- algebra('goedel')
a$n(x)     # residual negation
a$ni(x)    # involutive negation
a$t(x, y)  # t-norm
a$pt(x, y) # element-wise t-norm
a$c(x, y)  # t-conorm
a$pc(x, y) # element-wise t-conorm
a$r(x, y)  # residuum
a$b(x, y)  # bi-residuum
```

```
a$s(x, y)  # supremum
a$ps(x, y) # element-wise supremum
a$i(x, y)  # infimum
a$pi(x, y) # element-wise infimum

is.algebra(a) # TRUE
```

`antecedents` *Extract antecedent-part (left-hand side) of rules in a list*

Description

Given a list of rules or an instance of the S3 `farules()` class, the function returns a list of their antecedents (i.e. left-hand side of rules).

Usage

```
antecedents(rules)
```

Arguments

`rules`	Either a list of character vectors or an object of class `farules()`.

Details

This function assumes `rules` to be a valid `farules()` object or a list of character vectors where the first element of each vector is a consequent part and the rest is an antecedent part of rules. Function returns a list of antecedents.

Value

A list of character vectors.

Author(s)

Michal Burda

See Also

`consequents()`, `farules()`, `searchrules()`

Examples

```
rules <- list(c('a', 'b', 'c'), c('d'), c('a', 'e'))
antecedents(rules)
```

`as.data.frame.farules` *Convert the instance of the* `farules()` *S3 class into a data frame. Empty* `farules()` *object is converted into an empty* `data.frame()`.

Description

Convert the instance of the `farules()` S3 class into a data frame. Empty `farules()` object is converted into an empty `data.frame()`.

Usage

```
## S3 method for class 'farules'
as.data.frame(x, ...)
```

Arguments

`x`	An instance of class `farules()` to be transformed.
`...`	Unused.

Value

A data frame of statistics of the rules that are stored in the given `farules()` object. Row names of the resulting data frame are in the form: `A1 & A2 & ... & An => C`, where `Ai` are antecedent predicates and `C` is a consequent. An empty `farules()` object is converted into an empty `data.frame()` object.

Author(s)

Michal Burda

See Also

`farules()`, `searchrules()`

`as.data.frame.fsets` *Convert an object of* `fsets` *class into a matrix or data frame This function converts an instance of S3 class fsets into a matrix or a data frame. The* `vars()` *and* `specs()` *attributes of the original object are deleted.*

Description

Convert an object of `fsets` class into a matrix or data frame This function converts an instance of S3 class fsets into a matrix or a data frame. The `vars()` and `specs()` attributes of the original object are deleted.

Usage

```
## S3 method for class 'fsets'
as.data.frame(x, ...)

## S3 method for class 'fsets'
as.matrix(x, ...)
```

Arguments

x	An instance of class fsets to be converted
...	arguments further passed to as.data.frame after converting to matrix in as.data.frame.fsets. Unused in as.matrix.fsets.

Value

A numeric matrix or data frame of membership degrees.

Author(s)

Michal Burda

See Also

fsets(), fcut(), lcut()

---

c.farules *Take a sequence of instances of S3 class farules() and combine them into a single object. An error is thrown if some argument does not inherit from the farules() class.*

Description

Take a sequence of instances of S3 class farules() and combine them into a single object. An error is thrown if some argument does not inherit from the farules() class.

Usage

```
## S3 method for class 'farules'
c(..., recursive = FALSE)
```

Arguments

...	A sequence of objects of class farules() to be concatenated.
recursive	This argument has currently no function and is added here only for compatibility with generic c function.

Value

An object of class farules() that is created by merging the arguments together, i.e. by concatenating the rules and row-binding the statistics of given objects.

Author(s)

Michal Burda

See Also

farules(), searchrules()

Examples

```
ori1 <- farules(rules=list(letters[1:3],
                           letters[2:5]),
                statistics=matrix(runif(16), nrow=2))
ori2 <- farules(rules=list(letters[4],
                           letters[3:8]),
                statistics=matrix(runif(16), nrow=2))
res <- c(ori1, ori2)
print(res)
```

---

cbind.fsets *Combine several 'fsets' objects into a single one*

Description

Take a sequence of objects of class 'fsets' and combine them by columns. This version of cbind takes care of the vars() and specs() attributes of the arguments and merges them to the result. If some argument does not inherit from the 'fsets' class, an error is thrown.

Usage

```
## S3 method for class 'fsets'
cbind(..., deparse.level = 1, warn = TRUE)
```

Arguments

...	A sequence of objects of class 'fsets' to be merged by columns.
deparse.level	This argument has currently no function and is added here only for compatibility with generic cbind() function.
warn	Whether to issue warning when combining two fsets having the same vars about the fact that specs may not be accurate

Details

The vars() attribute is merged by concatenating the vars() attributes of each argument. Also the specs() attributes of the arguments are merged together.

Value

An object of class 'fsets' that is created by merging the arguments by columns. Also the arguments' attributes vars() and specs() are merged together.

Author(s)

Michal Burda

See Also

vars(), specs(), fcut(), lcut()

Examples

```
d1 <- fcut(CO2[, 1:2])
d2 <- fcut(CO2[, 3:4], breaks=list(conc=1:4*1000/4))
r <- cbind(d1, d2)

print(colnames(d1))
print(colnames(d2))
print(colnames(r))

print(vars(d1))
print(vars(d2))
print(vars(r))

print(specs(d1))
print(specs(d2))
print(specs(r))
```

---

compose *Composition of Fuzzy Relations*

Description

Composition of Fuzzy Relations

Usage

```
compose(
  x,
  y,
  e = NULL,
  alg = c("goedel", "goguen", "lukasiewicz"),
  type = c("basic", "sub", "super", "square"),
  quantifier = NULL,
  sorting = sort
)
```

Arguments

x	A first fuzzy relation to be composed. It must be a numeric matrix with values within the $[0, 1]$ interval. The number of columns must match with the number of rows of the y matrix.
y	A second fuzzy relation to be composed. It must be a numeric matrix with values within the $[0, 1]$ interval. The number of columns must match with the number of rows of the x matrix.
e	Deprecated. An excluding fuzzy relation. If not NULL, it must be a numeric matrix with dimensions equal to the y matrix.
alg	An algebra to be used for composition. It must be one of 'goedel' (default), 'goguen', or 'lukasiewicz', or an instance of class algebra (see algebra()).
type	A type of a composition to be performed. It must be one of 'basic' (default), 'sub', 'super', or 'square'.
quantifier	Deprecated. If not NULL, it must be a function taking a single argument, a vector of relative cardinalities, that would be translated into membership degrees. A result of the lingexpr() function is a good candidate for that. Note that the vector of relative cardinalities contains also two attributes, x and y, which carry the original R's data row (in x) and S's feature column (in y). These attributes are accessible using the standard base::attr() function. Find examples below that define some quantifiers.
sorting	Deprecated. Sorting function used within quantifier application. The given function must sort the membership degrees and allow the decreasing argument as in base::sort(). This function have to be explicitly specified typically if performing compositions that handle NA values.

Details

Function composes a fuzzy relation x (i.e., a numeric matrix of size (u, v)) with a fuzzy relation y (i.e., a numeric matrix of size (v, w)) and possibly with the deprecated use of an exclusion fuzzy relation e (i.e., a numeric matrix of size (v, w)).

The style of composition is determined by the algebra alg, the composition type type, and possibly also by a deprecated quantifier.

This function performs four main composition types, the basic composition (also known as direct product), the Bandler-Kohout subproduct (also subdirect product), the Bandler-Kohout superproduct (also supdirect product), and finally, the Bandler-Kohout square product. More complicated

composition operations may be performed by using the `mult()` function and/or by combining multiple composition results with the `algebra()` operations.

Value

A matrix with v rows and w columns, where v is the number of rows of `x` and w is the number of columns of `y`.

Author(s)

Michal Burda

See Also

[algebra(), `mult()`, `lingexpr()`

Examples

```
R <- matrix(c(0.1, 0.6, 1, 0, 0, 0,
              0, 0.3, 0.7, 0.9, 1, 1,
              0, 0, 0.6, 0.8, 1, 0,
              0, 1, 0.5, 0, 0, 0,
              0, 0, 1, 1, 0, 0), byrow=TRUE, nrow=5)

S <- matrix(c(0.9, 1, 0.9, 1,
              1, 1, 1, 1,
              0.1, 0.2, 0, 0.2,
              0, 0, 0, 0,
              0.7, 0.6, 0.5, 0.4,
              1, 0.9, 0.7, 0.6), byrow=TRUE, nrow=6)

RS <- matrix(c(0.6, 0.6, 0.6, 0.6,
               1, 0.9, 0.7, 0.6,
               0.7, 0.6, 0.5, 0.4,
               1, 1, 1, 1,
               0.1, 0.2, 0, 0.2), byrow=TRUE, nrow=5)

compose(R, S, alg='goedel', type='basic') # should be equal to RS
```

`consequents` *Extract consequent-part (right-hand side) of rules in a list*

Description

Given a list of rules or an instance of the S3 `farules()` class, the function returns a list of their consequents (i.e. right-hand side of rules).

Usage

```
consequents(rules)
```

Arguments

`rules`	Either a list of character vectors or an object of class `farules()`.

Details

This function assumes `rules` to be a valid `farules()` object or a list of character vectors where the first element of each vector is a consequent part and the rest is an antecedent part of rules. Function returns a list of consequents.

Value

A list of character vectors.

Author(s)

Michal Burda

See Also

`antecedents()`, `farules()`, `searchrules()`

Examples

```
rules <- list(c('a', 'b', 'c'), c('d'), c('a', 'e'))
consequents(rules)
unlist(consequents(rules))   # as vector
```

---

ctx *Context for linguistic expressions*

Description

A context describes a range of allowed values for a data column.

Usage

```
ctx3(
  low = 0,
  center = low + (high - low) * relCenter,
  high = 1,
  relCenter = 0.5
)

ctx3bilat(
  negMax = -1,
  negCenter = origin + (negMax - origin) * relCenter,
  origin = 0,
  center = origin + (max - origin) * relCenter,
```

```
  max = 1,
  relCenter = 0.5
)

ctx5(
  low = 0,
  lowerCenter = mean(c(low, center)),
  center = low + (high - low) * relCenter,
  upperCenter = mean(c(center, high)),
  high = 1,
  relCenter = 0.5
)

ctx5bilat(
  negMax = -1,
  negUpperCenter = mean(c(negCenter, negMax)),
  negCenter = origin + (negMax - origin) * relCenter,
  negLowerCenter = mean(c(origin, negCenter)),
  origin = 0,
  lowerCenter = mean(c(origin, center)),
  center = origin + (max - origin) * relCenter,
  upperCenter = mean(c(center, max)),
  max = 1,
  relCenter = 0.5
)

as.ctx3(x)

## S3 method for class 'ctx3'
as.ctx3(x)

## S3 method for class 'ctx3bilat'
as.ctx3(x)

## S3 method for class 'ctx5'
as.ctx3(x)

## S3 method for class 'ctx5bilat'
as.ctx3(x)

## Default S3 method:
as.ctx3(x)

as.ctx3bilat(x)

## S3 method for class 'ctx3bilat'
as.ctx3bilat(x)
```

```
## S3 method for class 'ctx3'
as.ctx3bilat(x)

## S3 method for class 'ctx5'
as.ctx3bilat(x)

## S3 method for class 'ctx5bilat'
as.ctx3bilat(x)

## Default S3 method:
as.ctx3bilat(x)

as.ctx5(x)

## S3 method for class 'ctx5'
as.ctx5(x)

## S3 method for class 'ctx3'
as.ctx5(x)

## S3 method for class 'ctx3bilat'
as.ctx5(x)

## S3 method for class 'ctx5bilat'
as.ctx5(x)

## Default S3 method:
as.ctx5(x)

as.ctx5bilat(x)

## S3 method for class 'ctx5bilat'
as.ctx5bilat(x)

## S3 method for class 'ctx3'
as.ctx5bilat(x)

## S3 method for class 'ctx3bilat'
as.ctx5bilat(x)

## S3 method for class 'ctx5'
as.ctx5bilat(x)

## Default S3 method:
as.ctx5bilat(x)

is.ctx3(x)
```

```
is.ctx3bilat(x)

is.ctx5(x)

is.ctx5bilat(x)
```

Arguments

low	Lowest value of an unilateral context.
center	A positive middle value of a bilateral context, or simply a middle value of an unilateral context.
high	Highest value of an unilateral context.
relCenter	A relative quantity used to compute the negCenter and/or center, if they are not specified explicitly. The sensible value is 0.5 for context symmetric around center, or 0.42 as proposed by Novak.
negMax	Lowest negative value of a bilateral context.
negCenter	A negative middle value.
origin	Origin, i.e., the initial point of the bilateral context. It is typically a value of zero.
max	Highest value of a bilateral context.
lowerCenter	A typical positive value between origin and center.
upperCenter	A typical positive value between center and maximum.
negUpperCenter	A typical negative value between negMax and negCenter.
negLowerCenter	A typical negative value between negCenter and negOrigin.
x	A value to be examined or converted. For as.ctx*, it can be an instance of any ctx* class or a numeric vector of size equal to the number of points required for the given context type.

Details

A context describes a range of allowed values for a data column. For that, only the borders of the interval, i.e., minimum and maximum, are usually needed, but we use contexts to hold more additional information that is crucial for the construction of linguistic expressions.

Currently, four different contexts are supported that determine the types of possible linguistic expressions, as constructed with lingexpr(). Unilateral or bilateral context is allowed in the variants of trichotomy or pentachotomy. Trichotomy distinguishes three points in the interval: the lowest value, highest value, and center. Pentachotomy adds lower center and upper center to them. As opposite to unilateral, the bilateral context handles explicitly the negative values. That is, bilateral context expects some middle point, the origin (usually 0), around which the positive and negative values are placed.

Concretely, the type of the context determines the allowed atomic expressions as follows:

- ctx3: trichotomy (low, center, high) enables atomic expressions: small, medium, big;
- ctx5: pentachotomy (low, lowerCenter, center, upperCenter, high) enables atomic expressions: small, lower medium, medium, upper medium, big;

- `ctx3bilat`: bilateral trichotomy (negMax, negCenter, origin, center, max) enables atomic expressions: negative big, negative medium, negative small, zero, small, medium, big;
- `ctx5bilat`: bilateral pentachotomy (negMax, negCenter, origin, center, max) enables atomic expressions: negative big, negative medium, negative small, zero, small, medium, big.

The `as.ctx*` functions return instance of the appropriate class. The functions perform the conversion so that missing points of the new context are computed from the old context that is being transformed. In the subsequent table, rows represent compatible values of different context types:

ctx3	ctx5	ctx3bilat	ctx5bilat
		negMax	negMax
			negUpperCenter
		negCenter	negCenter
			negLowerCenter
low	low	origin	origin
	lowerCenter		lowerCenter
center	center	center	center
	upperCenter		upperCenter
high	high	max	max

The `as.ctx*` conversion is performed by replacing values by rows, as indicated in the table above. When converting from a context with less points to a context with more points (e.g., from unilateral to bilateral, or from trichotomy to pentachotomy), missing points are computed as follows:

- `center` is computed as a mean of `origin` (or `low`) and `max` (or `high`).
- `lowerCenter` is computed as a mean of `origin` (or `low`) and `center`.
- `upperCenter` is computed as a mean of `mas` (or `high`) and `center`.
- negative points (such as `negMax`, `negCenter` etc.) are computed symmetrically around `origin` to the corresponding positive points.

The code `as.ctx*` functions allow the parameter to be also a numeric vector of size equal to the number of points required for the given context type, i.e. 3 (`ctx3`), 5 (`ctx3bilat`, `ctx5`), or 9 (`ctx5bilat`).

Value

`ctx*` and `as.ctx*` return an instance of the appropriate class. `is.ctx*` returns `TRUE` or `FALSE`.

Author(s)

Michal Burda

See Also

`minmax()`, `lingexpr()`, `horizon()`, `hedge()`, `fcut()`, `lcut()`

Examples

```
ctx3(low=0, high=10)
as.ctx3bilat(ctx3(low=0, high=10))
```

defaultHedgeParams *A list of the parameters that define the shape of the hedges.*

Description

A list of the parameters that define the shape of the hedges.

Usage

```
defaultHedgeParams
```

defuzz *Convert fuzzy set into a crisp numeric value*

Description

Take a discretized fuzzy set (i.e., a vector of membership degrees and a vector of numeric values that correspond to that degrees) and perform a selected type of defuzzification, i.e., conversion of the fuzzy set into a single crisp value.

Usage

```
defuzz(
  degrees,
  values,
  type = c("mom", "fom", "lom", "dee", "cog", "expun", "expw1", "expw2")
)
```

Arguments

degrees	A fuzzy set in the form of a numeric vector of membership degrees of values provided as the values argument.
values	A universe for the fuzzy set.
type	Type of the requested defuzzification method. The possibilities are: • 'mom': Mean of Maxima - maximum membership degrees are found and a mean of values that correspond to that degrees is returned; • 'fom': First of Maxima - first value with maximum membership degree is returned; • 'lom': Last of Maxima - last value with maximum membership degree is returned; • 'dee': Defuzzification of Evaluative Expressions - method used by the pbld() inference mechanism that combines the former three approaches according to the shape of the degrees vector: If degrees is non-increasing then 'lom' type is used, if it is non-decreasing then 'fom' is applied, else 'mom' is selected;

- 'cog': Center of Gravity - the result is a mean of values weighted by degrees;
- 'exp1': Experimental 1.

Details

Function converts input fuzzy set into a crisp value. The definition of input fuzzy set is provided by the arguments degrees and values. These arguments should be numeric vectors of the same length, the former containing membership degrees in the interval $[0, 1]$ and the latter containing the corresponding crisp values: i.e., values[i] has a membership degree degrees[i].

Value

A defuzzified value.

Author(s)

Michal Burda

See Also

fire(), aggregateConsequents(), perceive(), pbld(), infer(), fcut(), lcut()

Examples

```
# returns mean of maxima, i.e., mean of 6, 7, 8
defuzz(c(0, 0, 0, 0.1, 0.3, 0.9, 0.9, 0.9, 0.2, 0),
       1:10,
       type='mom')
```

---

equidist *Return equidistant breaks*

Description

If both left and right equal to "none", the function returns a vector of n values from x that divide the range of values in x into n - 1 equidistant intervals.

Usage

```
equidist(
  x,
  n,
  left = c("infinity", "same", "none"),
  right = c("infinity", "same", "none")
)
```

Arguments

x	A numeric vector of input values
n	The number of breaks of x to find (n must be at least 2)
left	The left border of the returned vector of breaks: "infinity", "same" or "none" (see the description below)
right	The right border of the returned vector of breaks: "infinity", "same" or "none" (see the description below)

Details

If the left (resp. right) argument equals to "infinity", -Inf (resp. Inf) is prepended (resp. appended) to the result. If it equals to "same", the first (resp. last) value is doubled. Such functionality is beneficial if using the result of this function with e.g., the fcut() function: Inf values at the beginning (resp. at the end) of the vector of breaks means that the fuzzy set partition starts with a fuzzy set with kernel going to negative (resp. positive) infinity; the doubled value at the beginning (resp. end) results in half-cut (trimmed) fuzzy set.

Value

A vector of equidistant breaks, which can be used e.g., in fcut()

Author(s)

Michal Burda

See Also

equifreq(), fcut()

---

equifreq *Return equifrequent breaks*

Description

If both left and right equal to "none", the function returns a vector of n values from x that divide the range of values in x into n - 1 equifrequent intervals.

Usage

```
equifreq(
  x,
  n,
  left = c("infinity", "same", "none"),
  right = c("infinity", "same", "none")
)
```

Arguments

x	A numeric vector of input values
n	The number of breaks of x to find (n must be at least 2)
left	The left border of the returned vector of breaks: "infinity", "same" or "none" (see the description below)
right	The right border of the returned vector of breaks: "infinity", "same" or "none" (see the description below)

Details

If the left (resp. right) argument equals to "infinity", -Inf (resp. Inf) is prepended (resp. appended) to the result. If it equals to "same", the first (resp. last) value is doubled. Such functionality is beneficial if using the result of this function with e.g., the fcut() function: Inf values at the beginning (resp. at the end) of the vector of breaks means that the fuzzy set partition starts with a fuzzy set with kernel going to negative (resp. positive) infinity; the doubled value at the beginning (resp. end) results in half-cut (trimmed) fuzzy set.

Value

A vector of equifrequent breaks

Author(s)

Michal Burda

See Also

equidist(), fcut()

---

evalfrbe *Evaluate the performance of the FRBE forecast*

Description

Take a FRBE forecast and compare it with real values using arbitrary error function.

Usage

```
evalfrbe(fit, real, error = c("smape", "mase", "rmse"))
```

Arguments

`fit`	A FRBE model of class `frbe` as returned by the `frbe()` function.
`real`	A numeric vector of real (known) values. The vector must correspond to the values being forecasted, i.e., the length must be the same as the horizon forecasted by `frbe()`.
`error`	Error measure to be computed. It can be either Symmetric Mean Absolute Percentage Error (`smape`), Mean Absolute Scaled Error (`mase`), or Root Mean Squared Error (`rmse`). See also `smape()`, `mase()`, and `rmse()` for more details.

Details

Take a FRBE forecast and compare it with real values by evaluating a given error measure. FRBE forecast should be made for a horizon of the same value as length of the vector of real values.

Value

Function returns a data.frame with single row and columns corresponding to the error of the individual forecasting methods that the FRBE is computed from. Additionally to this, a column "avg" is added with error of simple average of the individual forecasting methods and a column "frbe" with error of the FRBE forecasts.

Author(s)

Michal Burda

References

Štěpnička, M., Burda, M., Štěpničková, L. Fuzzy Rule Base Ensemble Generated from Data by Linguistic Associations Mining. FUZZY SET SYST. 2015.

See Also

`frbe()`, `smape()`, `mase()`, `rmse()`

Examples

```
# prepare data (from the forecast package)
library(forecast)
horizon <- 10
train <- wineind[-1 * (length(wineind)-horizon+1):length(wineind)]
test <- wineind[(length(wineind)-horizon+1):length(wineind)]
f <- frbe(ts(train, frequency=frequency(wineind)), h=horizon)
evalfrbe(f, test)
```

`farules` *Create an instance of S3 class* `farules` *which represents a set of fuzzy association rules and their statistical characteristics.*

Description

This function is a constructor that returns an instance of the `farules` S3 class. To search for fuzzy association rules, refer to the `searchrules()` function.

Usage

```
farules(rules, statistics)
```

Arguments

`rules`	A list of character vectors, where each vector represents a rule and each value of the vector represents a predicate. The first value of the vector is assumed to be a rule's consequent, the rest is a rule's antecedent.
`statistics`	A numeric matrix of various statistical characteristics of the rules. Each column of that matrix corresponds to some statistic (such as support, confidence, etc.) and each row corresponds to a rule in the list of `rules`.

Value

Returns an object of class `farules`.

Author(s)

Michal Burda

See Also

`searchrules()`

`fcut` *Transform data into a* `fsets` *S3 class using shapes derived from triangles or raised cosines*

Description

This function creates a set of fuzzy attributes from crisp data. Factors, numeric vectors, matrix or data frame columns are transformed into a set of fuzzy attributes, i.e., columns with membership degrees. Unlike `lcut()`, the linguistic approach is not used for transformation, but rather partitioning using regular shapes of the fuzzy sets (such as triangles or raised cosines).

Usage

```
fcut(x, ...)

## Default S3 method:
fcut(x, ...)

## S3 method for class 'factor'
fcut(x, name = deparse(substitute(x)), ...)

## S3 method for class 'logical'
fcut(x, name = deparse(substitute(x)), ...)

## S3 method for class 'numeric'
fcut(
  x,
  breaks,
  name = deparse(substitute(x)),
  type = c("triangle", "raisedcos"),
  merge = 1,
  parallel = FALSE,
  ...
)

## S3 method for class 'data.frame'
fcut(
  x,
  breaks = NULL,
  name = NULL,
  type = c("triangle", "raisedcos"),
  merge = 1,
  parallel = FALSE,
  ...
)

## S3 method for class 'matrix'
fcut(x, ...)
```

Arguments

x	Data to be transformed: a vector, matrix, or data frame. Non-numeric data are allowed.
...	Other parameters to some methods.
name	A name to be added as a suffix to the created fuzzy attribute names. This parameter can be used only if x is a vector. If x is a matrix or data frame, name should be NULL because the fuzzy attribute names are taken from column names of the argument x.
breaks	This argument determines the break-points of the positions of the fuzzy sets (see also equidist(). It should be an ordered vector of numbers such that the i-th

index specifies the beginning, $(i+1)$-th the center, and $(i+2)$-th the ending of the i-th fuzzy set.

I.e. the minimum number of breaks-points is 3; $n-2$ elementary fuzzy sets would be created for n break-points.

If considering an i-th fuzzy set (of `type='triangle'`), `x` values lower than i-th break (and greater than $(i+2)$-th break) would result in zero membership degree, values equal to $(i+1)$-th break would have membership degree equal 1 and values between them the appropriate membership degree between 0 and 1.

The resulting fuzzy sets would be named after the original data by adding dot (".") and a number i of fuzzy set.

Unlike `base::cut()`, `x` values, that are lower or greater than the given break-points, will have all membership degrees equal to zero.

For non-numeric data, this argument is ignored. For `x` being a numeric vector, it must be a vector of numeric values. For `x` being a numeric matrix or data frame, it must be a named list containing a numeric vector for each column - if not, the values are repeated for each column.

`type` The type of fuzzy sets to create. Currently, `'triangle'` or `'raisedcos'` may be used. The `type` argument may be also a function with 3 or 4 arguments:

- if `type` is a 4-argument function, it is assumed that that it computes membership degrees from values of the first argument while considering the boundaries given by the next 3 arguments;
- if `type` is a 3-argument function, it is assumed that it is a factory function similar to `triangular()` or `raisedcosine()`, which, from given three boundaries, creates a function that computes membership degrees.

`merge` This argument determines whether to derive additional fuzzy sets by merging the elementary fuzzy sets (whose position is determined with the `breaks` argument) into super-sets. The argument is ignored for non-numeric data in `x`.

`merge` may contain any integer number from `1` to `length(breaks) - 2`. Value `1` means that the elementary fuzzy sets should be present in the output. Value `2` means that the two consecutive elementary fuzzy sets should be combined by using the Lukasiewic t-conorm, value `3` causes combining three consecutive elementary fuzzy sets etc.

The names of the derived (merged) fuzzy sets is derived from the names of the original elementary fuzzy sets by concatenating them with the "|" (pipe) separator.

`parallel` Whether the processing should be run in parallel or not. Parallelization is implemented using the `foreach::foreach()` function. The parallel environment must be set properly in advance, e.g., with the `doMC::registerDoMC()` function. Currently this argument is applied only if `x` is a matrix or data frame.

Details

The aim of this function is to transform numeric data into a set of fuzzy attributes. The result is in the form of an object of class "fsets", i.e., a numeric matrix whose columns represent fuzzy sets (fuzzy attributes) with values being the membership degrees.

The function behaves differently depending on the type of input `x`.

If x is a factor or a logical vector (or other non-numeric data) then for each distinct value of an input, a fuzzy set is created, and data would be transformed into crisp membership degrees 0 or 1 only.

If x is a numeric vector then fuzzy sets are created according to break-points specified in the breaks argument with 1st, 2nd and 3rd break-point specifying the first fuzzy set, 2nd, 3rd and 4th break-point specifying the second fuzzy set etc. The shape of the fuzzy set is determined by the type argument that may be equal either to a string 'triangle' or 'raisedcos' or it could be a function that computes the membership degrees itself (see triangular() or raisedcosine() functions for details). Additionally, super-sets of these elementary sets may be created by specifying the merge argument. Values of this argument specify how many consecutive fuzzy sets should be combined (by using Lukasiewicz's t-conorm) to produce super-sets - see the description of merge above.

If a matrix (resp. data frame) is provided to this function instead of a single vector, all columns are processed separately as described above and the result is combined with the cbind.fsets() function.

The function sets up properly the vars() and specs() properties of the result.

Value

An object of class "fsets" is returned, which is a numeric matrix with columns representing the fuzzy attributes. Each source column of the x argument corresponds to multiple columns in the resulting matrix. Columns have names that indicate the name of the source as well as a index i of fuzzy set(s) – see the description of arguments breaks and merge above.

The resulting object would also have set the vars() and specs() properties with the former being created from original column names (if x is a matrix or data frame) or the name argument (if x is a numeric vector). The specs() incidency matrix would be created to reflect the superset-hood of the merged fuzzy sets.

Author(s)

Michal Burda

See Also

lcut(), equidist(), farules(), pbld(), vars(), specs(), cbind.fsets()

Examples

```
# fcut on non-numeric data
ff <- factor(substring("statistics", 1:10, 1:10), levels = letters)
fcut(ff)

# transform a single vector into a single fuzzy set
x <- runif(10)
fcut(x, breaks=c(0, 0.5, 1), name='age')

# transform single vector into a partition of the interval 0-1
# (the boundary triangles are right-angled)
fcut(x, breaks=c(0, 0, 0.5, 1, 1), name='age')
```

```
# also create supersets
fcut(x, breaks=c(0, 0, 0.5, 1, 1), name='age', merge=c(1, 2))

# transform all columns of a data frame
# with different breakpoints
data <- CO2[, c('conc', 'uptake')]
fcut(data, breaks=list(conc=c(95, 95, 350, 1000, 1000),
                       uptake=c(7, 7, 28.3, 46, 46)))

# using a custom 3-argument function (a function factory):
f <- function(a, b, c) {
  return(function(x) ifelse(a <= x & x <= b, 1, 0))
}
fcut(x, breaks=c(0, 0.5, 1), name='age', type=f)

# using a custom 4-argument function:
f <- function(x, a, b, c) {
  return(ifelse(a <= x & x <= b, 1, 0))
}
fcut(x, breaks=c(0, 0.5, 1), name='age', type=f)
```

`fire` *Evaluate rules and obtain truth-degrees*

Description

Given truth degrees of predicates, compute the truth value of given list of rules.

Usage

```
fire(
  x,
  rules,
  tnorm = c("goedel", "goguen", "lukasiewicz"),
  onlyAnte = TRUE,
  parallel = FALSE
)
```

Arguments

`x`	Truth degrees of predicates. `x` could be either a numeric matrix or a numeric vector. If vector is given then each named element represents a truth value of a predicate. If matrix is given then each row of the matrix is evaluated sequentially as a vector. The values must be in the interval $[0, 1]$.
`rules`	Either an object of S3 class `farules()` or a list of character vectors where each vector is a rule in a conjunctive form. Elements of these character vectors (i.e., predicate names) must correspond to the `x`'s names (of elements resp. columns if `x` is a vector resp. matrix).

tnorm	A character string representing a triangular norm to be used (either "goedel", "goguen", or "lukasiewicz") or an arbitrary function that performs element-wise computation on arbitrary number of vector parameters similarly as e.g. pgoedel.tnorm(), pgoguen.tnorm() or plukas.tnorm().
onlyAnte	TRUE is useful if rules store both the antecedent and consequent and if only the antecedent-part of a rule should be included into the evaluated conjunction. Antecedent-part of a rule are all predicates in the vector starting from the 2nd position. TRUE value in this parameter causes the first element of each rule to be ignored. If FALSE, all predicates in a rule will be included in the conjunction.
parallel	Deprecated parameter. Computation is done sequentially.

Details

The aim of this function is to compute the truth value of each rule in a rules list by assigning truth values to rule's predicates given by data x.

x is a numeric vector or numeric matrix of truth values of predicates. If x is vector then names(x) must correspond to the predicate names in rules. If x is a matrix then each column should represent a predicate and thus colnames(x) must correspond to predicate names in rules. Values of x are interpreted as truth values, i.e., they must be from the interval $[0, 1]$. If matrix is given, the resulting truth values are computed row-wisely.

rules may be a list of character vectors or an instance of the S3 class farules(). The character vectors in the rules list represent formulae in conjunctive form. If onlyAnte=FALSE, fire() treats the rule as a conjunction of all predicates, i.e., a conjunction of all predicates is computed. If onlyAnte=TRUE, the first element of each rule is removed prior evaluation, i.e., a conjunction of all predicates except the first are computed: this is useful if rules is a farules() object, since farules() objects save a rule's consequent as the first element (see also antecedents() and consequents() functions).

The type of conjunction to be computed can be specified with the tnorm parameter.

Value

If x is a matrix then the result of this function is a list of numeric vectors with truth values of each rule, i.e., each element of the resulting list corresponds to a rule and each value of the vector in the resulting list corresponds to a row of the original data matrix x.

x as a vector is treated as a single-row matrix.

Author(s)

Michal Burda

See Also

aggregateConsequents(), defuzz(), perceive(), pbld(), infer(), fcut(), lcut(), farules()

Examples

```
# fire whole rules on a vector
x <- 1:10 / 10
names(x) <- letters[1:10]
rules <- list(c('a', 'c', 'e'),
              c('b'),
              c('d', 'a'),
              c('c', 'a', 'b'))
fire(x, rules, tnorm='goguen', onlyAnte=FALSE)

# fire antecedents of the rules on a matrix
x <- matrix(1:20 / 20, nrow=2)
colnames(x) <- letters[1:10]
rules <- list(c('a', 'c', 'e'),
              c('b'),
              c('d', 'a'),
              c('c', 'a', 'b'))
fire(x, rules, tnorm='goedel', onlyAnte=TRUE)

# the former command should be equal to
fire(x, antecedents(rules), tnorm='goedel', onlyAnte=FALSE)
```

---

frbe *Fuzzy Rule-Based Ensemble (FRBE) of time-series forecasts*

Description

This function computes the fuzzy rule-based ensemble of time-series forecasts. Several forecasting methods are used to predict future values of a given time-series and a weighted sum is computed from them with weights being determined from a fuzzy rule base.

Usage

```
frbe(d, h = 10)
```

Arguments

d	A source time-series in the ts time-series format. Note that the frequency of the time-series must be set properly.
h	A forecasting horizon, i.e., the number of values to forecast.

Details

This function computes the fuzzy rule-based ensemble of time-series forecasts. The evaluation comprises the following steps:

1. Several features are extracted from the given time-series d:

- length of the time-series;
- strength of trend;
- strength of seasonality;
- skewness;
- kurtosis;
- variation coefficient;
- stationarity;
- frequency. These features are used later to infer weights of the forecasting methods.

2. Several forecasting methods are applied on the given time-series `d` to obtain forecasts. Actually, the following methods are used:
   - ARIMA - by calling `forecast::auto.arima()`;
   - Exponential Smoothing - by calling `forecast::ets()`;
   - Random Walk with Drift - by calling `forecast::rwf()`;
   - Theta - by calling `forecast::thetaf()`.
3. Computed features are input to the fuzzy rule-based inference mechanism which yields weights of the forecasting methods. The fuzzy rule base is hardwired in this package and it was obtained by performing data mining with the use of the `farules()` function.
4. A weighted sum of forecasts is computed and returned as a result.

Value

Result is a list of class `frbe` with the following elements:

- `features` - a data frame with computed features of the given time-series;
- `forecasts` - a data frame with forecasts to be ensembled;
- `weights` - weights of the forecasting methods as inferred from the features and the hard-wired fuzzy rule base;
- `mean` - the resulting ensembled forecast (computed as a weighted sum of forecasts).

Author(s)

Michal Burda

References

Štěpnička, M., Burda, M., Štěpničková, L. Fuzzy Rule Base Ensemble Generated from Data by Linguistic Associations Mining. FUZZY SET SYST. 2015.

See Also

`evalfrbe()`

Examples

```
# prepare data (from the forecast package)
library(forecast)
horizon <- 10
train <- wineind[-1 * (length(wineind)-horizon+1):length(wineind)]
test <- wineind[(length(wineind)-horizon+1):length(wineind)]

# perform FRBE
f <- frbe(ts(train, frequency=frequency(wineind)), h=horizon)

# evaluate FRBE forecasts
evalfrbe(f, test)

# display forecast results
f$mean
```

`fsets` *S3 class representing a set of fuzzy sets on the fixed universe*

Description

The aim of the `fsets` S3 class is to store several fuzzy sets in the form of numeric matrix where columns represent fuzzy sets, rows are elements from the universe, and therefore a value of `i`-th row and `j`-th column is a membership degree of `i`-th element of the universe to `j`-th fuzzy set. The `fsets` object also stores the information about the origin of the fuzzy sets as well as a relation of specificity among them.

Usage

```
fsets(
  x,
  vars = rep(deparse(substitute(x)), ncol(x)),
  specs = matrix(0, nrow = ncol(x), ncol = ncol(x))
)

vars(f)

vars(f) <- value

specs(f)

specs(f) <- value
```

Arguments

`x`	A matrix of membership degrees. Columns of the matrix represent fuzzy sets, colnames are names of the fuzzy sets (and must not be NULL). Rows of the matrix represent elements of the universe.

vars	A character vector that must correspond to the columns of x. It is a vector of names of original variables that the fuzzy sets were created from. In other words, the vars vector should contain the same value for each x's column that corresponds to the same variable. Names of the vars vector are ignored. For instance, an fcut() function can transform a single numeric vector into several different fuzzy sets. To indicate that all of them in fact describe the same original variable, the same name is stored on appropriate positions of the vars vector.
specs	A square numeric matrix containing values from {0, 1}. It is a specificity matrix, for which both rows and columns correspond to x's columns and where specs[i][j] == 1 if and only if i-th fuzzy set (i.e. x[, i]) is more specific (is a subset or equal to) than j-th fuzzy set (i.e. x[, j]).
f	An instance of S3 class fsets.
value	Attribute values to be set to the object.

Details

The fsets() function is a constructor of an object of type fsets. Each object stores two attributes: vars and specs. The functions vars() and specs()). can be used to access these attributes.

It is assumed that the fuzzy sets are derived from some raw variables, e.g., numeric vectors or factors. vars attribute is a character vector of names of raw variables with size equal to the number of fuzzy sets in fsets object. It is then assumed that two fuzzy sets with the same name in vars() attribute are derived from the same variable.

specs attribute gives a square numeric matrix of size equal to the number of fuzzy sets in fsets. specs[i][j] == 1 if and only if the i-th fuzzy set is more specific than j-th fuzzy set. Specificity of fuzzy sets means the nestedness of fuzzy set: for instance, very small is more specific than small; similarly, extremely big is more specific than very big; on the other hand, very big and extremely small are incomparable. A necessary condition for specificity is subsethood.

Value

fsets() returns an object of S3 class fsets. vars() returns a vector of original variable names of the fsets object. specs returns the specificity matrix.

Author(s)

Michal Burda

See Also

fcut(), lcut(), is.specific()

Examples

```
# create a matrix of random membership degrees
m <- matrix(runif(30), ncol=5)
colnames(m) <- c('a1', 'a2', 'a12', 'b1', 'b2')

# create vars - first three (a1, a2, a3) and next two (b1, b2)
# fuzzy sets originate from the same variable
```

```
v <- c('a', 'a', 'a', 'b', 'b')
names(v) <- colnames(m)

# create specificity matrix - a1 and a2 are more specific than a12,
# the rest is incomparable
s <- matrix(c(0, 0, 1, 0, 0,
              0, 0, 1, 0, 0,
              0, 0, 0, 0, 0,
              0, 0, 0, 0, 0,
              0, 0, 0, 0, 0), byrow=TRUE, ncol=5)
colnames(s) <- colnames(m)
rownames(s) <- colnames(m)

# create a valid instance of the fsets class
o <- fsets(m, v, s)
```

`ft` *Fuzzy transform*

Description

Compute a fuzzy tranform of the given input matrix `x`.

Usage

```
ft(x, xmemb, y, order = 1)
```

Arguments

`x`	the numeric matrix of input values
`xmemb`	the partitioning of input values, i.e., a `fsets` object with membership degrees (see `fcut()`)
`y`	the numeric vector of target values
`order`	the order of the fuzzy transform (0, 1, 2, ...)

Value

the instance of the S3 class `ft`

Author(s)

Michal Burda

References

Perfilieva I. Fuzzy transforms: Theory and applications. FUZZY SET SYST, volume 157, issue 8, p. 993-1023. 2006.

See Also

ftinv(), is.ft()

Examples

```
# create the fuzzy transform object
y <- (1:30)^2
x <- as.matrix(data.frame(a = 1:30, b = 30:1))
xmemb <- fcut(x,
              breaks = list(a = equidist(x[, 'a'], 3),
                            b = equidist(x[, 'b'], 3)))
fit <- ft(x, xmemb, y, order = 1)

# obtain function values
x2 <- as.matrix(data.frame(a = 10:20, b = 20:10))
xmemb2 <- fcut(x2,
               breaks = list(a = equidist(x[, 'a'], 3),
                             b = equidist(x[, 'b'], 3)))
y2 <- ftinv(fit, x2, xmemb2)
print(y2)

# compare original values with those obtained by the fuzzy transform
y[10:20] - y2
```

---

ftinv *Inverse of the fuzzy transform*

Description

Compute an inverse of fuzzy transform `fit` for values `x` with corresponding membership degrees `xmemb`.

Usage

```
ftinv(fit, x, xmemb)
```

Arguments

`fit`	The fuzzy transform object as the instance of the `ft` S3 class
`x`	The numeric matrix of input values, for which the inverse fuzzy transform has to be computed
`xmemb`	the partitioning of input values, i.e., a `fsets` object with membership degrees (see `fcut()`). Such partitioning must correspond to the `xmemb` partitioning used to create `fit` using the `ft()` function.

Value

The inverse of the fuzzy transform `fit`, i.e., the approximated values of the original function that was the subject of the fuzzy transform

Author(s)

Michal Burda

References

Perfilieva I. Fuzzy transforms: Theory and applications. FUZZY SET SYST, volume 157, issue 8, p. 993-1023. 2006.

See Also

`ft()`, `is.ft()`

Examples

```
# create the fuzzy transform object
y <- (1:30)^2
x <- as.matrix(data.frame(a = 1:30, b = 30:1))
xmemb <- fcut(x,
              breaks = list(a = equidist(x[, 'a'], 3),
                            b = equidist(x[, 'b'], 3)))
fit <- ft(x, xmemb, y, order = 1)

# obtain function values
x2 <- as.matrix(data.frame(a = 10:20, b = 20:10))
xmemb2 <- fcut(x2,
               breaks = list(a = equidist(x[, 'a'], 3),
                             b = equidist(x[, 'b'], 3)))
y2 <- ftinv(fit, x2, xmemb2)
print(y2)

# compare original values with those obtained by the fuzzy transform
y - y2
```

---

hedge *Linguistic hedges*

Description

Returns a function that realizes linguistic hedging - i.e., transformation of linguistic horizon (see `horizon()`) into a linguistic expression.

Usage

```
hedge(
  type = c("ex", "si", "ve", "ty", "-", "ml", "ro", "qr", "vr"),
  hedgeParams = defaultHedgeParams
)
```

Arguments

`type`	The type of the required linguistic hedge
`hedgeParams`	Parameters that determine the shape of the hedges

Details

`hedge()` returns a function that realizes the selected linguistic hedge on its parameter:

- `ex`: extremely,
- `si`: significantly,
- `ve`: very,
- `ty`: typically,
- `-`: empty hedge (no hedging),
- `ml`: more or less,
- `ro`: roughly,
- `qr`: quite roughly,
- `vr`: very roughly.

This function is quite low-level. Perhaps a more convenient way to create linguistic expressions is to use the `lingexpr()` function.

Value

Returns a function with a single argument, which has to be a numeric vector.

Author(s)

Michal Burda

See Also

`horizon()`, `lingexpr()`, `fcut()`, `lcut()`, `ctx()`

Examples

```
a <- horizon(ctx3(), 'sm')
plot(a)
h <- hedge('ve')
plot(h)
verySmall <- function(x) h(a(x))
plot(verySmall)
```

```
# the last plot should be equal to:
plot(lingexpr(ctx3(), atomic='sm', hedge='ve'))
```

`horizon` *Create a function that computes linguistic horizons*

Description

Based on given `context` and `atomic` expression, this function returns a function that computes a linguistic horizon, i.e., a triangular function representing basic limits of what humans treat as "small", "medium", "big" etc. within given `context`. Linguistic horizon stands as a base for creation of linguistic expressions. A linguistic expression is created by applying a `hedge()` on horizon. (Atomic linguistic expression is created from horizon by applying an empty (`-`) hedge).

Usage

```
horizon(
  context,
 atomic = c("sm", "me", "bi", "lm", "um", "ze", "neg.sm", "neg.me", "neg.bi", "neg.lm",
    "neg.um")
)
```

Arguments

`context`	A context of linguistic expressions (see `ctx3()`, `ctx5()`, `ctx3bilat()` or `ctx5bilat()`)
`atomic`	An atomic expression whose horizon we would like to obtain

Details

The values of the `atomic` parameter have the following meaning (in ascending order):

- `neg.bi`: big negative (far from zero)
- `neg.um`: upper medium negative (between medium negative and big negative)
- `neg.me`: medium negative
- `neg.lm`: lower medium negative (between medium negative and small negative)
- `neg.sm`: small negative (close to zero)
- `ze`: zero
- `sm`: small
- `lm`: lower medium
- `me`: medium
- `um`: upper medium
- `bi`: big

Based on the context type, the following atomic expressions are allowed:

- `ctx3()` (trichotomy): small, medium, big;
- `ctx5()` (pentachotomy): small, lower medium, medium, upper medium, big;
- `ctx3bilat()` (bilateral trichotomy): negative big, negative medium, negative small, zero, small, medium, big;
- `ctx5bilat()` (bilateral pentachotomy): negative big, negative medium, negative small, zero, small, medium, big.

This function is quite low-level. Perhaps a more convenient way to create linguistic expressions is to use the `lingexpr()` function.

Value

A function of single argument that must be a numeric vector

Author(s)

Michal Burda

See Also

`ctx3()`, `ctx5()`, `ctx3bilat()`, `ctx5bilat()`, `hedge()`, `fcut()`, `lcut()`

Examples

```
plot(horizon(ctx3(), 'sm'), from=-1, to=2)
plot(horizon(ctx3(), 'me'), from=-1, to=2)
plot(horizon(ctx3(), 'bi'), from=-1, to=2)

a <- horizon(ctx3(), 'sm')
plot(a)
h <- hedge('ve')
plot(h)
verySmall <- function(x) h(a(x))
plot(verySmall)
```

---

infer *Perform Mamdani or implicative fuzzy inference with given rule-base on given dataset*

Description

Take a set of rules (a rule-base) and perform Mamdani or implicative inference on each row of a given matrix (or the `fsets()` object) of truth values.

Usage

```
infer(
  x,
  rules,
  partition,
  alg = c("goedel", "goguen", "lukasiewicz"),
  type = c("mamdani", "implicative"),
  parallel = FALSE
)
```

Arguments

x	Input to the inference. It must be a numeric matrix with columns representing fuzzy sets (truth values of predicates). Each row represents a single case of inference. Columns should be named after predicates in rules' antecedents. The values must be in the interval $[0, 1]$. x can also be an object of class fsets() (e.g., created by using the fcut() or lcut() functions).
rules	A rule-base (a.k.a. linguistic description) either in the form of the farules() object or as a list of character vectors where each element is a fuzzy set name (a predicate) and thus each such vector forms a rule. The first element of each rule is the consequent predicate, and the remaining elements are antecedent predicates.
partition	A numeric matrix (or a fsets() object) with columns that are consequents in rules. The column names of partition must correspond to consequent predicate names used in rules. Each row represents a point in the output domain.
alg	The algebra to use for the inference. It can be either a character string ("goedel", "goguen", or "lukasiewicz") or an instance of the algebra() class.
type	The type of inference to use. It can be either "mamdani" or "implicative".
parallel	Whether the processing should be run in parallel or not. Parallelization is implemented using the foreach::foreach() package. The parallel environment must be set properly in advance, e.g., with the doMC::registerDoMC() function.

Details

Perform Mamdani or implicative fuzzy inference with given rule-base rules on each row of input x. Columns of x are truth values of predicates that appear in the antecedent part of rules. The partition matrix determines the shape of predicates in consequents: each row represents a point in the output fuzzy set for which membership degrees are computed.

For both inference types, the predicates in the antecedent part of rules are combined using a t-norm (alg$pt) of the given algebra alg (see also the fire() function).

For the Mamdani inference (type = "mamdani"), the antecedent firing degrees are combined with the consequent partition using a t-norm (alg$pt) of the selected algebra alg, and the resulting fuzzy sets are aggregated with a t-conorm (alg$ps).

For the implicative inference (type = "implicative"), the antecedent firing degrees are combined with the consequent partition using a residuum (alg$r) of the selected algebra alg, and the resulting fuzzy sets are aggregated with a t-norm (alg$pi).

Value

A numeric matrix of inferred membership degrees. The rows of the result correspond to rows of the `x` argument, the columns of the result correspond to rows of the `partition` argument.

Author(s)

Michal Burda

See Also

`pbld()`, `fire()`, `aggregateConsequents()`, `defuzz()`, `algebra()`, `farules()`, `lcut()`

Examples

```
# Create rules: first element is the consequent, the rest are antecedents
rules <- list(
    c("High",   "temp_high",   "pressure_high"),
    c("Medium", "temp_medium", "pressure_medium"),
    c("Low",    "temp_low")
)

# Create input data (fuzzy set membership degrees)
# (Alternatively, fcut() or lcut() functions can be used)
x <- matrix(
    c(0.9, 0.8, 0.2, 0.3, 0.1,
      0.1, 0.2, 0.4, 0.5, 0.8),
    nrow = 2,
    byrow = TRUE
)
colnames(x) <- c("temp_high", "pressure_high",
                 "temp_medium", "pressure_medium", "temp_low")

# Create output partition
# (Alternatively, fcut() or lcut() functions can be used)
partition <- matrix(
    c(1.0, 0.2, 0.0,
      0.8, 0.5, 0.1,
      0.4, 1.0, 0.4,
      0.1, 0.5, 0.8,
      0.0, 0.1, 1.0),
    nrow = 5,
    byrow = TRUE
)
colnames(partition) <- c("Low", "Medium", "High")

# Mamdani inference with Goedel algebra
infer(x, rules, partition, alg = "goedel", type = "mamdani")

# Implicative inference with Lukasiewicz algebra
infer(x, rules, partition, alg = "lukasiewicz", type = "implicative")
```

is.farules *Test whether* x *inherits from the S3* farules *class.*

Description

Test whether x inherits from the S3 farules class.

Usage

```
is.farules(x)
```

Arguments

x	An object being tested.

Value

TRUE if x is a valid farules() object and FALSE otherwise.

Author(s)

Michal Burda

See Also

farules

is.frbe *Test whether* x *is a valid object of the S3* frbe *class*

Description

Test whether x has a valid format for the objects of the S3 frbe class.

Usage

```
is.frbe(x)
```

Arguments

x	An object being tested.

Details

This function tests whether x inherits from frbe i.e., whether it is a list with the following elements: forecasts data frame, features data frame, weights vector, and mean vector. Instances of the S3 class frbe are usually created by the frbe() function.

Value

TRUE if x is a valid frbe object and FALSE otherwise.

Author(s)

Michal Burda

References

Štěpnička, M., Burda, M., Štěpničková, L. Fuzzy Rule Base Ensemble Generated from Data by Linguistic Associations Mining. FUZZY SET SYST. 2015.

See Also

frbe()

---

is.fsets *Test whether* x *is a valid object of the S3* fsets *class*

Description

This function tests whether x inherits from S3 fsets class.

Usage

```
is.fsets(x)
```

Arguments

x	An object being tested.

Value

TRUE if x is a valid fsets object and FALSE otherwise.

Author(s)

Michal Burda

See Also

fsets()

is.ft — *Test whether* x *is a valid object of the S3* ft *class*

Description

Test whether x has a valid format for objects of the S3 ft class that represents the Fuzzy Transform.

Usage

```
is.ft(x)
```

Arguments

x	An object to be tested

Details

This function tests whether x is an instance of the ft class and whether it is a list with the following elements: inputs character vector, partitions list, order number, antecedents matrix and consequents matrix.

Value

TRUE if x is a valid ft object and FALSE otherwise.

Author(s)

Michal Burda

See Also

ft(), ftinv()

is.specific — *Determine whether the first set* x *of predicates is more specific (or equal) than* y *with respect to* vars *and* specs.

Description

The function takes two character vectors of predicates and determines whether x is more specific (or equal w.r.t. the specificity) than y. The specificity relation is fully determined with the values of the vars() vector and the specs() incidence matrix that is encapsulated in the given fsets object.

Usage

```
is.specific(x, y, fsets, vars = NULL, specs = NULL)
```

Arguments

x	The first character vector of predicates.
y	The second character vector of predicates.
fsets	A valid instance of the fsets() class such that all values in x and y can be found in colnames(fsets)
vars	Deprecated parameter must be NULL.
specs	Deprecated parameter must be NULL.

Details

Let x_i and y_j represent some predicates of vectors x and y, respectively. Function assumes that each vector x and y does not contain two or more predicates with the same value of vars().

This function returns TRUE iff all of the following conditions hold:

- for any y_j there exists x_i such that $vars[y_j] = vars[x_i]$;
- for any x_i there either does not exist y_j such that $vars[x_i] = vars[y_j]$, or $x_i = y_j$, or $specs[x_i, y_j] = 1$.

Value

TRUE or FALSE (see description).

Author(s)

Michal Burda

See Also

perceive(), pbld(), fsets(), vars(), specs()

Examples

```
# prepare fsets object
v <- c(rep('a', 3), rep('b', 3), rep('c', 3), rep('d', 3))
s <- matrix(c(0,1,0, 0,0,0, 0,0,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,

              0,0,0, 0,1,0, 0,0,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,

              0,0,0, 0,0,0, 0,1,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,

              0,0,0, 0,0,0, 0,0,0, 0,1,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0,
              0,0,0, 0,0,0, 0,0,0, 0,0,0),
```

```
            byrow=TRUE,
            ncol=12)
m <- matrix(0, nrow=1, ncol=12)
colnames(m) <- paste(rep(c('VeSm', 'Sm', 'Bi'), times=4),
                     rep(c('a', 'b', 'c', 'd'), each=3),
                     sep='.')
f <- fsets(m, v, s)


# returns TRUE
is.specific(c('VeSm.a', 'Bi.c'),
            c('VeSm.a', 'Bi.c'),
            f)

# returns TRUE (x and y swapped return FALSE)
is.specific(c('VeSm.a', 'Bi.c', 'Sm.d'),
            c('Sm.a', 'Bi.c', 'Sm.d'),
            f)

# returns TRUE (x and y swapped return FALSE)
is.specific(c('VeSm.a', 'Bi.c', 'Sm.d'),
            c('VeSm.a', 'Bi.c'),
            f)

# returns TRUE (x and y swapped return FALSE)
is.specific(c('VeSm.a', 'Bi.c', 'Sm.d'),
            character(),
            f)

# returns FALSE
is.specific(c('Sm.a'), c('Bi.c'), f)

# returns FALSE
is.specific(c('VeSm.a', 'Sm.c'),
            c('Sm.a', 'Bi.c'),
            f)
```

`lcut` *Transform data into a* `fsets` *S3 class of linguistic fuzzy attributes*

Description

This function creates a set of linguistic fuzzy attributes from crisp data. Numeric vectors, matrix or data frame columns are transformed into a set of fuzzy attributes, i.e., columns with membership degrees. Factors and other data types are transformed to fuzzy attributes by calling the `fcut()` function.

Usage

```
lcut(x, ...)
```

```
## Default S3 method:
lcut(x, ...)

## S3 method for class 'factor'
lcut(x, name = deparse(substitute(x)), ...)

## S3 method for class 'logical'
lcut(x, name = deparse(substitute(x)), ...)

## S3 method for class 'numeric'
lcut(
  x,
  context = minmax,
 atomic = c("sm", "me", "bi", "lm", "um", "ze", "neg.sm", "neg.me", "neg.bi", "neg.lm",
    "neg.um"),
  hedges = c("ex", "si", "ve", "ty", "-", "ml", "ro", "qr", "vr"),
  name = deparse(substitute(x)),
  hedgeParams = defaultHedgeParams,
  ...
)

## S3 method for class 'data.frame'
lcut(
  x,
  context = minmax,
 atomic = c("sm", "me", "bi", "lm", "um", "ze", "neg.sm", "neg.me", "neg.bi", "neg.lm",
    "neg.um"),
  hedges = c("ex", "si", "ve", "ty", "-", "ml", "ro", "qr", "vr"),
  ...
)

## S3 method for class 'matrix'
lcut(x, ...)
```

Arguments

x	Data to be transformed: if it is a numeric vector, matrix, or data frame, then the creation of linguistic fuzzy attributes takes place. For other data types the fcut() function is called implicitly.
...	Other parameters to some methods.
name	A name to be added as a suffix to the created fuzzy attribute names. This parameter can be used only if x is a numeric or logical vector or a factor. If x is a matrix or data frame, name should be NULL because the fuzzy attribute names are taken from column names of parameter x. The name is also used as a value for the vars attribute of the resulting fsets() instance.
context	A definition of context of a numeric attribute. It must be an instance of an S3 class ctx3(), ctx5(), ctx3bilat() or ctx5bilat().

	If `x` is a matrix or data frame then `context` should be a named list of contexts for each `x`'s column.
`atomic`	A vector of atomic linguistic expressions to be used for creation of fuzzy attributes.
`hedges`	A vector of linguistic hedges to be used for creation of fuzzy attributes.
`hedgeParams`	Parameters that determine the shape of the hedges

Details

The aim of this function is to transform numeric data into a set of fuzzy attributes. The resulting fuzzy attributes have direct linguistic interpretation. This is a unique variant of fuzzification that is suitable for the inference mechanism based on Perception-based Linguistic Description (PbLD) – see `pbld()`.

A numeric vector is transformed into a set of fuzzy attributes according to the following scheme:

$< hedge >< atomicexpression >$

where $< atomicexpression >$ is an atomic linguistic expression, a value from the following possibilities (note that the allowance of atomic expressions is influenced with `context` being used - see ctx for details):

- `neg.bi`: big negative (far from zero)
- `neg.um`: upper medium negative (between medium negative and big negative)
- `neg.me`: medium negative
- `neg.lm`: lower medium negative (between medium negative and small negative)
- `neg.sm`: small negative (close to zero)
- `ze`: zero
- `sm`: small
- `lm`: lower medium
- `me`: medium
- `um`: upper medium
- `bi`: big

A $< hedge >$ is a modifier that further concretizes the atomic expression (note that not each combination of hedge and atomic expression is allowed - see allowed.lingexpr for more details):

- `ex`: extremely,
- `si`: significantly,
- `ve`: very,
- `ty`: typically,
- `-`: empty hedge (no hedging),
- `ml`: more or less,
- `ro`: roughly,
- `qr`: quite roughly,

- vr: very roughly.

Accordingly to the theory developed by Novak (2008), not every hedge is suitable with each atomic expression (see the description of the hedges argument). The hedges to be used can be selected with the hedges argument. Function takes care of not to use hedge together with an unapplicable atomic expression by itself.

Obviously, distinct data have different meaning of what is "small", "medium", or "big" etc. Therefore, a context has to be set that specifies sensible values for these linguistic expressions.

If a matrix (resp. data frame) is provided to this function instead of a single vector, all columns are processed the same way.

The function also sets up properly the vars() and specs() properties of the result.

Value

An object of S3 class fsets is returned, which is a numeric matrix with columns representing the fuzzy attributes. Each source column of the x argument corresponds to multiple columns in the resulting matrix. Columns will have names derived from used $hedges$, atomic expression, and $name$ specified as the optional parameter.

The resulting object would also have set the vars() and specs() properties with the former being created from original column names (if x is a matrix or data frame) or the name argument (if x is a numeric vector). The specs() incidency matrix would be created to reflect the following order of the hedges: $"ex" < "si" < "ve" < " - " < "ml" < "ro" < "qr" < "vr"$ and $"ty" < "" < "ml" < "ro" < "qr" < "vr"$. Fuzzy attributes created from the same source numeric vector (or column) would be ordered that way, with other fuzzy attributes (from the other source) being incomparable.

Author(s)

Michal Burda

References

V. Novak, A comprehensive theory of trichotomous evaluative linguistic expressions, Fuzzy Sets and Systems 159 (22) (2008) 2939–2969.

See Also

fcut(), fsets(), vars(), specs()

Examples

```
# transform a single vector
x <- runif(10)
lcut(x, name='age')

# transform single vector with a custom context
lcut(x, context=ctx5(0, 0.2, 0.5, 0.7, 1), name='age')

# transform all columns of a data frame
```

```
# and do not use any hedges
data <- CO2[, c('conc', 'uptake')]
lcut(data)


# definition of custom contexts for different columns
# of a data frame while selecting only "ve" and "ro" hedges.
lcut(data,
     context=list(conc=minmax,
                  uptake=ctx3(0, 25, 50)),
     hedges=c('ve', 'ro'))


# lcut on non-numeric data is the same as fcut()
ff <- factor(substring("statistics", 1:10, 1:10), levels = letters)
lcut(ff)
```

`lingexpr` *Creator of functions representing linguistic expressions*

Description

A linguistic expression represents vague human terms such as "very small", "extremely big" etc. Such notions are always reasoned within a given context. `lingexpr` returns a function that models a selected linguistic expression. Accordingly to the given `context`, `atomic` expression (such as "small", "big") and a linguistic `hedge` (such as `very`, `extremely`), the returned function transforms numeric values into degrees (from `[0, 1]` interval), at which the values correspond to the expression.

Usage

```
lingexpr(
  context,
  atomic = c("sm", "me", "bi", "lm", "um", "ze", "neg.sm", "neg.me", "neg.bi", "neg.lm",
    "neg.um"),
  hedge = c("ex", "si", "ve", "ty", "-", "ml", "ro", "qr", "vr"),
  negated = FALSE,
  hedgeParams = defaultHedgeParams
)

allowed.lingexpr
```

Arguments

`context`	A context of linguistic expressions (see `ctx3()`, `ctx5()`, `ctx3bilat()` or `ctx5bilat()`)
`atomic`	An atomic expression whose horizon we would like to obtain
`hedge`	The type of the required linguistic hedge ('-' for no hedging)

`negated`	Negate the expression? (For instance, "not very small".) Negation is done using the `invol.neg()` function.
`hedgeParams`	Parameters that determine the shape of the hedges

Details

Based on the context type, the following atomic expressions are allowed:

- `ctx3()` (trichotomy): small, medium, big;
- `ctx5()` (pentachotomy): small, lower medium, medium, upper medium, big;
- `ctx3bilat()` (bilateral trichotomy): negative big, negative medium, negative small, zero, small, medium, big;
- `ctx5bilat()` (bilateral pentachotomy): negative big, negative medium, negative small, zero, small, medium, big.

The values of the `atomic` parameter have the following meaning (in ascending order):

- `neg.bi`: big negative (far from zero)
- `neg.um`: upper medium negative (between medium negative and big negative)
- `neg.me`: medium negative
- `neg.lm`: lower medium negative (between medium negative and small negative)
- `neg.sm`: small negative (close to zero)
- `ze`: zero
- `sm`: small
- `lm`: lower medium
- `me`: medium
- `um`: upper medium
- `bi`: big

`hedge` parameter has the following meaning:

- `ex`: extremely,
- `si`: significantly,
- `ve`: very,
- `ty`: typically,
- `-`: empty hedge,
- `ml`: more or less,
- `ro`: roughly,
- `qr`: quite roughly,
- `vr`: very roughly.

Accordingly to the theory of linguistic expressions by Novak, not every hedge is applicable to each atomic expression. The combinations of allowed pairs can be found in allowed.lingexpr. Trying to create forbidden combination results in error.

Value

Returns a function with a single argument, which has to be a numeric vector.

Author(s)

Michal Burda

See Also

horizon(), hedge(), fcut(), lcut(), ctx()

Examples

```
small <- lingexpr(ctx3(0, 0.5, 1), atomic='sm', hedge='-')
small(0)   # 1
small(0.8) # 0
plot(small)

verySmall <- lingexpr(ctx3(0, 0.5, 1), atomic='sm', hedge='ve')
plot(verySmall)
```

---

mase *Compute Mean Absolute Scaled Error (MASE)*

Description

MASE is computed as $sum(abs(validation - forecast))/sum(abs(validation[-1] - validation[-n]))/(n/(n-1))$.

Usage

```
mase(forecast, validation)
```

Arguments

forecast	A numeric vector of forecasted values
validation	A numeric vector of actual (real) values

Value

A Mean Absolute Scaled Error (MASE)

Author(s)

Michal Burda

See Also

rmse(), smape(), frbe()

`minmax` *Creating linguistic context directly from values*

Description

This function creates a context (i.e., an instance of S3 class `ctx3()`, `ctx3bilat()`, `ctx5()`, or `ctx5bilat()`) based on values of the numeric vector `x`. In default, the context is based on minimum and maximum value of `x` in the following way:

- `ctx3`, `ctx5`: low = minimum, high = maximum value of `x`;
- `ctx3bilat`, `ctx5bilat`: negMax = minimum, max = maximum value of `x`, origin = mean of minimum and maximum.

Usage

```
minmax(x, type = c("ctx3", "ctx5", "ctx3bilat", "ctx5bilat"), ...)
```

Arguments

`x`	A numeric vector to compute the context from
`type`	A type of the context to be returned. Must be one of: `ctx3`, `ctx5`, `ctx3bilat` or `ctx5bilat`
`...`	other parameters to be passed to the appropriate constructor (`ctx3()`, `ctx3bilat()`, `ctx5()`, and `ctx5bilat()`) that is called internally. These values overwrite the defaults computed by `minmax` – see the examples.

Details

Other values are computed according to the defaults as defined in the constructors `ctx3()`, `ctx3bilat()`, `ctx5()`, and `ctx5bilat()`).

Examples

```
minmax(0:100)                # returns ctx3: 0, 50, 100
minmax(0:100, high=80)       # returns ctx3: 0, 40, 80
minmax(0:100, relCenter=0.4) # returns ctx3: 0, 40, 100
minmax(0:100, type='ctx5')   # returns ctx5: 0, 25, 50, 75, 100
```

`mult` *Callback-based Multiplication of Matrices*

Description

Perform a custom multiplication of the matrices `x` and `y` by using the callback function `f`.

Usage

```
mult(x, y, f, ...)
```

Arguments

`x`	A first matrix. The number of columns must match with the number of rows of the `y` matrix.
`y`	A second matrix. The number of rows must match with the number of columns of the `x` matrix.
`f`	A function to be applied to the matrices in order to compute the multiplication. It must accept at least two arguments.
`...`	Additional arguments that are passed to the function `f`.

Details

For a matrix `x` of size (u, v) and a matrix `y` of size (v, w), `mult` calls the function `f` uw-times to create a resulting matrix of size (u, w). Each (i, j)-th element of the resulting matrix is obtained from a call of the function `f` with `x`'s i-th row and `y`'s j-th column passed as its arguments.

Value

A matrix with v rows and w columns, where v is the number of rows of `x` and w is the number of columns of `y`.

Author(s)

Michal Burda

See Also

`compose()`

Examples

```
x <- matrix(runif(24, -100, 100), ncol=6)
y <- matrix(runif(18, -100, 100), nrow=6)

mult(x, y, function(xx, yy) sum(xx * yy)) # the same as "x %*% y"
```

pbld *Perform a Perception-based Logical Deduction (PbLD) with given rule-base on given dataset*

Description

Take a set of rules (a rule-base) and perform a Perception-based Logical Deduction (PbLD) on each row of a given `fsets()` object.

Usage

```
pbld(
  x,
  rules,
  partition,
  values,
  type = c("global", "local"),
  parallel = FALSE
)
```

Arguments

`x`	Input to the inference. It should be an object of class `fsets()` (e.g., created by using the `fcut()` or `lcut()` functions). It is basically a matrix with columns representing fuzzy sets. Each row represents a single case of inference. Columns should be named after predicates in rules' antecedents.
`rules`	A rule-base (a.k.a. linguistic description) either in the form of the `farules()` object or as a list of character vectors where each element is a fuzzy set name (a predicate) and thus each such vector forms a rule.
`partition`	A `fsets()` object with columns that are consequents in `rules`. These membership degrees must correspond to `values`.
`values`	Crisp values that correspond to rows of membership degrees in the `partition` matrix. Function assumes that the values are sorted in the ascending order.
`type`	The type of inference to use. It can be either `"local"` or `"global"` (default).
`parallel`	Whether the processing should be run in parallel or not. Parallelization is implemented using the `foreach::foreach()` package. The parallel environment must be set properly in advance, e.g., with the `doMC::registerDoMC()` function.

Details

Perform a Perception-based Logical Deduction (PbLD) with given rule-base `rules` on each row of input `x`. Columns of `x` are truth values of predicates that appear in the antecedent part of `rules`, `partition` together with `values` determine the shape of predicates in consequents: each element in `values` corresponds to a row of membership degrees in `partition`.

Value

A vector of inferred defuzzified values. The number of resulting values corresponds to the number of rows of the x argument.

Author(s)

Michal Burda

References

A. Dvořák, M. Štěpnička, On perception-based logical deduction and its variants, in: Proc. 16th World Congress of the International Fuzzy Systems Association and 9th Conference of the European Society for Fuzzy Logic and Technology (IFSA-EUSFLAT 2015), Advances in Intelligent Systems Research, Atlantic Press, Gijon, 2015.

See Also

infer(), lcut(), searchrules(), fire(), aggregateConsequents(), defuzz()

Examples

```
# --- TRAINING PART ---
# custom context of the RHS variable
uptakeContext <- ctx3(7, 28.3, 46)

# convert data into fuzzy sets
d <- lcut(CO2, context=list(uptake=uptakeContext))

# split data into the training and testing set
testingIndices <- 1:5
trainingIndices <- setdiff(seq_len(nrow(CO2)), testingIndices)
training <- d[trainingIndices, ]
testing <- d[testingIndices, ]

# search for rules
r <- searchrules(training, lhs=1:38, rhs=39:58, minConfidence=0.5)

# --- TESTING PART ---
# prepare values and partition
v <- seq(uptakeContext[1], uptakeContext[3], length.out=1000)
p <- lcut(v, name='uptake', context=uptakeContext)

# do the inference
pbld(testing, r, p, v)
```

perceive *From a set of rules, remove each rule for which another rule exists that is more specific.*

Description

Examine rules in a list and remove all of them for which other more specific rules are present in the list. The specificity is determined by calling the `is.specific()` function. This operation is a part of the `pbld()` inference mechanism.

Usage

```
perceive(
  rules,
  fsets,
  type = c("global", "local"),
  fired = NULL,
  vars = NULL,
  specs = NULL
)
```

Arguments

`rules`	A list of character vectors where each element is a fuzzy set name (a predicate) and thus each such vector forms a rule.
`fsets`	A valid instance of the `fsets()` class such that all predicates in `rules` (i.e., all values of all character vectors in `rules$rules`) can be found in `colnames(fsets)`
`type`	The type of perception to use. It can be either `"local"` or `"global"` (default).
`fired`	If `type=="global"` then this argument can be NULL. If `type` is `"local"` then `fired` must be a numeric vector of values in the interval $[0, 1]$ indicating the truth values of all rules, i.e., the length of the vector must be equal to the number of rules in the `rules` argument.
`vars`	A deprecated parameter that must be `NULL`. Formerly, it was a named (typically character) vector that determined which predicates originate from the same variable, i.e., which of them semantically deal with the same property. For that purpose, each value from any vector stored in the `rules` list must be present in `names(vars)`. See also `vars()` function of the `fsets()` class.
`specs`	A deprecated parameter that must be `NULL`. Formerly, it was a square numeric matrix containing values from $\{0, 1\}$. It is a specificity matrix for which each row and column corresponds to an `rules`'es predicate `specs[i][j] = 1` if and only if the i-th predicate is more specific (i.e., the corresponding fuzzy set is a subset of) than the j-th predicate (i.e. `x[, j]`). See also `specs()` function of the `fsets()` class.

Details

In other words, for each rule `x` in the `rules` list, it searches for another rule `y` such that `is.specific(y, x)` returns TRUE. If yes then `x` is removed from the list.

Value

A modified list of rules for which no other more specific rule exists. (Each rule is a vector.)

Author(s)

Michal Burda

See Also

`is.specific()`, `fsets()`, `fcut()`, `lcut()`

Examples

```
# prepare fsets
f <- lcut(data.frame(a=0:1, b=0:1, c=0:1, d=0:1))

# run perceive function: (sm.a, bi.c) has
# more specific rule (ve.sm.a, bi.c)
perceive(list(c('sm.a', 'bi.c'),
              c('ve.sm.a', 'bi.c'),
              c('sm.b', 'sm.d')),
         f)
```

---

plot.fsets *Plot membership degrees stored in the instance of the S3 class `fsets()` as a line diagram.*

Description

This function plots the membership degrees stored in the instance of the `fsets()` class. Internally, the membership degrees are transformed into a time-series object and viewed in a plot using the `ts.plot()` function. This function is useful mainly to see the shape of fuzzy sets on regularly sampled inputs.

Usage

```
## S3 method for class 'fsets'
plot(x, ...)
```

Arguments

`x`	An instance of class `fsets()`
`...`	Other arguments that are passed to the underlying `ts.plot()` function.

Value

Result of the ts.plot() method.

Author(s)

Michal Burda

See Also

fsets(), fcut(), lcut(), ts.plot()

Examples

```
d <- lcut(0:1000/1000, name='x')
plot(d)

# Additional arguments are passed to the ts.plot method
# Here thick lines represent atomic linguistic expressions,
# i.e. ``small'', ``medium'', and ``big''.
plot(d,
     ylab='membership degree',
     xlab='values',
     gpars=list(lwd=c(rep(1, 3), 5, rep(1, 5), 5, rep(1, 7), 5, rep(1,4))))
```

---

print.algebra *Print an instance of the algebra() S3 class in a human readable form.*

Description

Print an instance of the algebra() S3 class in a human readable form.

Usage

```
## S3 method for class 'algebra'
print(x, ...)
```

Arguments

x	An instance of the algebra() S3 class
...	Unused.

Value

None.

Author(s)

Michal Burda

See Also

algebra()

---

print.ctx3 *Print the linguistic context*

Description

Format an object of the ctx3(), ctx5(), ctx3bilat() and the ctx5bilat() class into human readable form and print it to the output.

Usage

```
## S3 method for class 'ctx3'
print(x, ...)

## S3 method for class 'ctx5'
print(x, ...)

## S3 method for class 'ctx3bilat'
print(x, ...)

## S3 method for class 'ctx5bilat'
print(x, ...)
```

Arguments

x	A linguistic context to be printed
...	Unused.

Value

Nothing.

Author(s)

Michal Burda

See Also

ctx3(), ctx5(), ctx3bilat(), ctx5bilat(), minmax()

Examples

```
context <- ctx3()
print(context)
```

print.farules *Print an instance of the farules() S3 class in a human readable form.*

Description

Print an instance of the farules() S3 class in a human readable form.

Usage

```
## S3 method for class 'farules'
print(x, ...)
```

Arguments

x	An instance of the farules() S3 class
...	Unused.

Value

None.

Author(s)

Michal Burda

See Also

farules(), searchrules()

print.frbe *Print an instance of the frbe() class*

Description

Format an object of the frbe() class into human readable form and print it to the output.

Usage

```
## S3 method for class 'frbe'
print(x, ...)
```

Arguments

x	An instance of frbe() class
...	Unused.

Details

Format an object of the `frbe()` class into human readable form and print it to the output.

Value

None.

Author(s)

Michal Burda

References

Štěpnička, M., Burda, M., Štěpničková, L. Fuzzy Rule Base Ensemble Generated from Data by Linguistic Associations Mining. FUZZY SET SYST. 2015.

See Also

`frbe()`

Examples

```
# prepare data (from the forecast package)
library(forecast)
horizon <- 10
train <- wineind[-1 * (length(wineind)-horizon+1):length(wineind)]
test <- wineind[(length(wineind)-horizon+1):length(wineind)]
f <- frbe(ts(train, frequency=frequency(wineind)), h=horizon)
print(f)
print(test)
```

---

print.fsets *Print an instance of the `fsets()` class*

Description

Format an object of the `fsets()` class into human readable form and print it to the output.

Usage

```
## S3 method for class 'fsets'
print(x, ...)
```

Arguments

x	An instance of the `fsets()` class
...	Unused.

Value

Nothing

Author(s)

Michal Burda

See Also

fsets(), fcut(), lcut()

Examples

```
d <- fcut(CO2[, 1:2])
print(d)
```

---

quantifier — *A quantifier is a function that computes a fuzzy truth value of a claim about the quantity. This function creates the <1>-type quantifier. (See the examples below on how to use it as a quantifier of the <1,1> type.)*

Description

A quantifier is a function that computes a fuzzy truth value of a claim about the quantity. This function creates the <1>-type quantifier. (See the examples below on how to use it as a quantifier of the <1,1> type.)

Usage

```
quantifier(
 quantity = c("all", "almost.all", "most", "many", "few", "several", "some", "at.least"),
  n = NULL,
  alg = c("lukasiewicz", "goedel", "goguen")
)
```

Arguments

quantity	the quantity to be evaluated. 'all' computes the degree of truth to which all elements of the universe have the given property, 'almost.all', #' 'most', and 'many' evaluate whether the property is present in extremely big, very big, or not small number of elements from the universe, where these linguistic expressions are internally modeled using the lingexpr() function. 'at.least' quantity requires the 'n' argument to be specified, as it computes the truth value that at least n elements from the universe have the given property.
n	the number of elements in the 'at.least n' quantifier

`alg` the underlying algebra in which to compute the quantifier. Note that the algebra must have properly defined the `order` function, as in the case of 'goedel', 'goguen', or 'lukasiewicz' algebra, (see the `algebra()` function) or as in the `dragonfly()` or `lowerEst()` algebra.

Value

A two-argument function, which expects two numeric vectors of equal length (the vector elements are recycled to ensure equal lengths). The first argument, `x`, is a vector of membership degrees to be measured, the second argument, `w`, is the vector of weights to which the element belongs to the universe.

Let U be the set of input vector indices (1 to `length(x)`). Then the quantifier computes the truth values according to the following formula: $\vee_{z \subseteq U} \wedge_{u \in z} (x[u]) \wedge measure(m_z)$, where $m_z = sum(w)$ for `"some"` and `"at.least` and $m_z = sum(w[z])/sum(w)$ otherwise. See `sugeno()` for more details on how the quantifier is evaluated.

Setting `w` to 1 yields to operation of the <1> quantifier as developed by Dvořák et al. To compute the <1,1> quantifier as developed by Dvořák et al., e.g. "almost all A are B", `w` must be set again to 1 and `x` to the result of the implication $A \Rightarrow B$. To compute the <1,1> quantifier as proposed by Murinová et al., e.g. "almost all A are B", `x` must be set to the result of the implication $A \Rightarrow B$ and `w` to the membership degrees of A. See the examples below.

Author(s)

Michal Burda

References

Dvořák, A., Holčapek, M. L-fuzzy quantifiers of type <1> determined by fuzzy measures. Fuzzy Sets and Systems vol.160, issue 23, 3425-3452, 2009.

Dvořák, A., Holčapek, M. Type <1,1> fuzzy quantifiers determined by fuzzy measures. IEEE International Conference on Fuzzy Systems (FuzzIEEE), 2010.

Murinová, P., Novák, V. The theory of intermediate quantifiers in fuzzy natural logic revisited and the model of "Many". Fuzzy Sets and Systems, vol 388, 2020.

See Also

`sugeno()`, `lingexpr()`

Examples

```
# Dvorak <1> "almost all" quantifier
f <- quantifier('almost.all')
a <- c(0.9, 1, 1, 0.2, 1)
f(x=a, w=1)

# Dvorak <1,1> "almost all" quantifier (w set to 1)
a <- c(0.9, 1, 1, 0.2, 1)
b <- c(0.2, 1, 0, 0.5, 0.8)
f <- quantifier('almost.all')
```

```
f(x=lukas.residuum(a, b), w=1)

# Murinová <1,1> "almost all" quantifier (note w set to a)
a <- c(0.9, 1, 1, 0.2, 1)
b <- c(0.2, 1, 0, 0.5, 0.8)
f <- quantifier('almost.all')
f(x=lukas.residuum(a, b), w=a)

# Murinová <1,1> "some" quantifier
a <- c(0.9, 1, 1, 0.2, 1)
b <- c(0.2, 1, 0, 0.5, 0.8)
f <- quantifier('some')
f(x=plukas.tnorm(a, b), w=a)
```

`rbcoverage` *Compute rule base coverage of data*

Description

This function computes rule base coverage, i.e., a an average of maximum membership degree at which each row of data fires the rules in rule base.

Usage

```
rbcoverage(
  x,
  rules,
  tnorm = c("goedel", "goguen", "lukasiewicz"),
  onlyAnte = TRUE
)
```

Arguments

`x`	Data for the rules to be evaluated on. Could be either a numeric matrix or numeric vector. If matrix is given then the rules are evaluated on rows. Each value of the vector or column of the matrix represents a predicate - it's numeric value represents the truth values (values in the interval $[0, 1]$).
`rules`	Either an object of class "farules" or list of character vectors where each vector is a rule with consequent being the first element of the vector. Elements of the vectors (predicate names) must correspond to the `x`'s names (of columns if `x` is a matrix).
`tnorm`	A character string representing a triangular norm to be used (either `"goedel"`, `"goguen"`, or `"lukasiewicz"`) or an arbitrary function that takes a vector of truth values and returns a t-norm computed of them.
`onlyAnte`	TRUE if only antecedent-part of a rule should be evaluated. Antecedent-part of a rule are all predicates in rule vector starting from the 2nd position. (First element of a rule is the consequent - see above.) If FALSE, then the whole rule will be evaluated (antecedent part together with consequent).

Details

Let f_{ij} be a truth value of i-th rule on j-th row of data `x`. Then $m_j = max(f_{.j})$ is a maximum truth value that is reached for the j-th data row with the rule base. Then the rule base coverage is a mean of that truth values, i.e. $rbcoverage = mean(m_.)$.

Value

A numeric value of the rule base coverage of given data.

Author(s)

Michal Burda

References

M. Burda, M. Štěpnička, Reduction of Fuzzy Rule Bases Driven by the Coverage of Training Data, in: Proc. 16th World Congress of the International Fuzzy Systems Association and 9th Conference of the European Society for Fuzzy Logic and Technology (IFSA-EUSFLAT 2015), Advances in Intelligent Systems Research, Atlantic Press, Gijon, 2015.

See Also

`fire()`, `reduce()`

Examples

```
x <- matrix(1:20 / 20, nrow=2)
colnames(x) <- letters[1:10]

rules <- list(c('a', 'c', 'e'),
              c('b'),
              c('d', 'a'),
              c('c', 'a', 'b'))
rbcoverage(x, rules, "goguen", TRUE)  # returns 1


rules <- list(c('d', 'a'),
              c('c', 'a', 'b'))
rbcoverage(x, rules, "goguen", TRUE)  # returns 0.075)
```

`reduce` *Reduce the size of rule base*

Description

From given rule base, select such set of rules that influence mostly the rule base coverage of the input data.

Usage

```
reduce(
  x,
  rules,
  ratio,
  tnorm = c("goedel", "goguen", "lukasiewicz"),
  tconorm = c("goedel", "goguen", "lukasiewicz"),
  numThreads = 1
)
```

Arguments

x	Data for the rules to be evaluated on. Could be either a numeric matrix or numeric vector. If matrix is given then the rules are evaluated on rows. Each value of the vector or column of the matrix represents a predicate - it's numeric value represents the truth values (values in the interval $[0, 1]$).
rules	Either an object of class "farules" or list of character vectors where each vector is a rule with consequent being the first element of the vector. Elements of the vectors (predicate names) must correspond to the x's names (of columns if x is a matrix).
ratio	A percentage of rule base coverage that must be preserved. It must be a value within the $[0, 1]$ interval. Value of 1 means that the rule base coverage of the result must be the same as coverage of input rules. A sensible value is e.g. 0.9.
tnorm	Which t-norm to use as a conjunction of antecedents. The default is "goedel".
tconorm	Which t-norm to use as a disjunction, i.e., to combine multiple antecedents to get coverage of the rule base. The default is "goedel".
numThreads	How many threads to use for computation. Value higher than 1 causes that the algorithm runs in several parallel threads (using the OpenMP library).

Details

From a given rulebase, a rule with greatest coverage is selected. After that, additional rules are selected that increase the rule base coverage the most. Addition stops after the coverage exceeds $originalcoverage * ratio$.

Note that the size of the resulting rule base is not necessarily minimal because the algorithm does not search all possible combination of rules. It only finds a local minimum of rule base size.

Value

Function returns an instance of class farules() or a list depending on the type of the rules argument.

Author(s)

Michal Burda

References

M. Burda, M. Štěpnička, Reduction of Fuzzy Rule Bases Driven by the Coverage of Training Data, in: Proc. 16th World Congress of the International Fuzzy Systems Association and 9th Conference of the European Society for Fuzzy Logic and Technology (IFSA-EUSFLAT 2015), Advances in Intelligent Systems Research, Atlantic Press, Gijon, 2015.

See Also

`rbcoverage()`, `farules()`

`rmse` *Compute Root Mean Squared Error (RMSE)*

Description

RMSE is computed as $sqrt(mean((forecast - validation)^2))$.

Usage

```
rmse(forecast, validation)
```

Arguments

`forecast`	A numeric vector of forecasted values
`validation`	A numeric vector of actual (real) values

Value

A Root Mean Squared Error (RMSE)

Author(s)

Michal Burda

See Also

`smape()`, `mase()`, `frbe()`

searchrules *Searching for fuzzy association rules*

Description

This function searches the given `fsets()` object `d` for all fuzzy association rules that satisfy defined constraints. It returns a list of fuzzy association rules together with some statistics characterizing them (such as support, confidence etc.).

Usage

```
searchrules(
  d,
  lhs = 2:ncol(d),
  rhs = 1,
  tnorm = c("goedel", "goguen", "lukasiewicz"),
  n = 100,
  best = c("confidence"),
  minSupport = 0.02,
  minConfidence = 0.75,
  maxConfidence = 1,
  maxLength = 4,
  numThreads = 1,
  trie = (maxConfidence < 1)
)
```

Arguments

`d`	An object of class `fsets()` - it is basically a matrix where columns represent the fuzzy sets and values are the membership degrees. For creation of such object, use `fcut()` or `lcut()` function.
`lhs`	Indices of fuzzy attributes that may appear on the left-hand-side (LHS) of association rules, i.e., in the antecedent.
`rhs`	Indices of fuzzy attributes that may appear on the right-hand-side (RHS) of association rules, i.e., in the consequent.
`tnorm`	A t-norm to be used for computation of conjunction of fuzzy attributes. (Only the starting letters of "lukasiewicz", "goedel" and "goguen" are allowed).
`n`	The non-negative number of rules to be found. If zero, the function returns all rules satisfying the given conditions. If positive, only the `n` best rules are returned. The criterion of what is "best" is specified with the `best` argument.
`best`	Specifies the measure according to which the rules are ordered from best to worst. This argument is used mainly in combination with the `n` argument. Currently, only a single value ("confidence") can be used.
`minSupport`	The minimum support degree of a rule. Rules with support below that number are filtered out. It must be a numeric value from interval $[0, 1]$. See below for details on how the support degree is computed.

`minConfidence` The minimum confidence degree of a rule. Rules with confidence below that number are filtered out. It must be a numeric value from interval $[0, 1]$. See below for details on how the confidence degree is computed.

`maxConfidence` Maximum confidence threshold. After finding a rule that has confidence degree above the `maxConfidence` threshold, no other rule is produced based on adding some additional attribute to its antecedent part. I.e., if "Sm.age & Me.age => Sm.height" has confidence above the `maxConfidence` threshold, no other rule containing "Sm.age & Me.age" will be produced regardless of its interest measures.

If you want to disable this feature, set `maxConfidence` to 1.

`maxLength` Maximum allowed length of the rule, i.e., maximum number of predicates that are allowed on the left-hand + right-hand side of the rule. If negative, the maximum length of rules is unlimited.

`numThreads` Number of threads used to perform the algorithm in parallel. If greater than 1, the OpenMP library (not to be confused with Open MPI) is used for parallelization. Please note that there are known problems of using OpenMP together with other means of parallelization that may be used within R. Therefore, if you plan to use the `searchrules` function with some external parallelization mechanisms such as the `doMC` library, make sure that `numThreads` equals 1. This feature is available only on systems that have the OpenMP library installed.

`trie` Whether or not to use the internal mechanism of Tries. If FALSE, then a rule may appear in the output that is a descendant of a rule that has confidence above the `maxConfidence` threshold.

Tries consume a large amount of memory, so if you encounter problems with insufficient memory, set this argument to FALSE. On the other hand, the size of the result (if `n` is set to 0) can be very high if trie is set to FALSE.

Details

The function searches data frame `d` for fuzzy association rules that satisfy conditions specified by the parameters.

Value

A list of the following elements: `rules` and `statistics`.

`rules` is a list of mined fuzzy association rules. Each element of that list is a character vector with consequent attribute being on the first position.

`statistics` is a data frame of statistical characteristics about mined rules. Each row corresponds to a rule in the `rules` list. Let us consider a rule "a & b => c", let $\otimes$ be a t-norm specified with the `tnorm` parameter and i goes over all rows of a data table `d`. Then columns of the `statistics` data frame are as follows:

- support: a rule's support degree: $1/nrow(d) * \sum_{\forall i} a(i) \otimes b(i) \otimes c(i)$
- lhsSupport: a support of rule's antecedent (LHS): $1/nrow(d) * \sum_{\forall i} a(i) \otimes b(i)$
- rhsSupport: a support of rule's consequent (RHS): $1/nrow(d) * \sum_{\forall i} c(i)$
- confidence: a rule's confidence degree: $support/lhsSupport$

Author(s)

Michal Burda

See Also

fcut(), lcut(), farules(), fsets(), pbld()

Examples

```
d <- lcut(CO2)
searchrules(d, lhs=1:ncol(d), rhs=1:ncol(d))
```

---

slices *Return vector of values from given interval*

Description

Returns an ordered vector of values from given interval, of given size, generated by equal steps.

Usage

```
slices(from, to, n)
```

Arguments

from	The lower bound of the interval.
to	The upper bound of the interval.
n	The length of the vector to be produced.

Details

Returns a vector of values from from to to (inclusive), with equal difference between two consecutive values, with total length n. Function is useful e.g., together with the pbld or defuzz functions (for the values argument; see also lcut or fcut) or defuzz).

Value

A vector of numbers in the given interval and size.

Author(s)

Michal Burda

See Also

pbld, defuzz, fcut, lcut

Examples

```
## Not run:
    slices(1, 5, 10) # 1, 1.5, 2, 2.5, 3, 3.5 4, 4.5, 5

## End(Not run)
# is the same as
seq(1, 5, length.out=10)
```

---

smape *Compute Symmetric Mean Absolute Percentage Error (SMAPE)*

Description

SMAPE is computed as $mean(abs(forecast - validation)/((abs(forecast) + abs(validation))/2))$.

Usage

```
smape(forecast, validation)
```

Arguments

`forecast`	A numeric vector of forecasted values
`validation`	A numeric vector of actual (real) values

Value

A Symmetric Mean Absolute Percentage Error (SMAPE)

Author(s)

Michal Burda

See Also

`rmse()`, `mase()`, `frbe()`

sobocinski *Modify algebra's way of computing with* NA *values.*

Description

By default, the objects created with the algebra() function represent a mathematical algebra capable to work on the $[0, 1]$ interval. If NA appears as a value instead, it is propagated to the result. That is, any operation with NA results in NA, by default. This scheme of handling missing values is also known as Bochvar's. To change this default behavior, the following functions may be applied.

Usage

```
sobocinski(algebra)

kleene(algebra)

dragonfly(algebra)

nelson(algebra)

lowerEst(algebra)

lowerBound(algebra)

upperBound(algebra)
```

Arguments

algebra	the underlying algebra object to be modified – see the algebra() function

Details

The sobocinski(), kleene(), nelson(), dragonfly(), lowerEst() and lowerBound() functions modify the algebra to handle the NA in a different way than is the default. Sobocinski's algebra simply ignores NA values whereas Kleene's algebra treats NA as "unknown value". Dragonfly approach is a combination of Sobocinski's and Bochvar's approach, which preserves the ordering 0 <= NA <= 1 to obtain from compositions (see compose()) the lower-estimate in the presence of missing values.

In detail, the behavior of the algebra modifiers is defined as follows:

Sobocinski's negation for n being the underlying algebra:

a	n(a)
NA	0

Sobocinski's operation for op being one of t, pt, c, pc, i, pi, s, ps from the underlying algebra:

	b	NA
a	op(a, b)	a
NA	b	NA

Sobocinski's operation for `r` from the underlying algebra:

	b	NA
a	r(a, b)	n(a)
NA	b	NA

Kleene's negation is identical to `n` from the underlying algebra.

Kleene's operation for `op` being one of `t`, `pt`, `i`, `pi` from the underlying algebra:

	b	NA	0
a	op(a, b)	NA	0
NA	NA	NA	0
0	0	0	0

Kleene's operation for `op` being one of `c`, `pc`, `s`, `ps` from the underlying algebra:

	b	NA	1
a	op(a, b)	NA	1
NA	NA	NA	1
1	1	1	1

Kleene's operation for `r` from the underlying algebra:

	b	NA	1
a	r(a, b)	NA	1
NA	NA	NA	1
0	1	1	1

Dragonfly negation is identical to `n` from the underlying algebra.

Dragonfly operation for `op` being one of `t`, `pt`, `i`, `pi` from the underlying algebra:

	b	NA	0	1
a	op(a, b)	NA	0	a
NA	NA	NA	0	NA
0	0	0	0	0
1	b	NA	0	1

Dragonfly operation for `op` being one of `c`, `pc`, `s`, `ps` from the underlying algebra:

	b	NA	0	1
a	op(a, b)	a	a	1
NA	b	NA	NA	1

0	b	NA	0	1
1	1	1	1	1

Dragonfly operation for `r` from the underlying algebra:

	b	NA	0	1
a	r(a, b)	NA	n(a)	1
NA	b	1	NA	1
0	1	1	1	1
1	b	NA	0	1

LowerEst and LowerBound algebras are equivalent to Dragonfly's algebra in `t`, `pt`, `i`, `pi`, `c`, `pc`, `s`, `ps` operations, but differ in the `r` and `n` operations. `n` is equal to the Sobocinski's negation for both LowerEst and LowerBound algebras.

LowerEst's operation for `r` from the underlying algebra:

	b	NA	0	1
a	r(a, b)	NA	n(a)	1
NA	b	NA	0	1
0	1	1	1	1
1	b	NA	0	1

LowerBound's operation for `r` from the underlying algebra:

	b	NA	0	1
a	r(a, b)	NA	n(a)	1
NA	1	1	0	1
0	1	1	1	1
1	b	NA	0	1

Value

A list of function of the same structure as is the list returned from the `algebra()` function

Author(s)

Michal Burda

Examples

```
a <- algebra('lukas')
b <- sobocinski(a)

a$t(0.3, NA)  # NA
b$t(0.3, NA)  # 0.3
```

str.fsets — *Compactly display the internal structure of an instance of the fsets() class*

Description

This function is a method for the generic function str() for objects of the fsets() class. It is a compact way to display the internal structure of an instance of the fsets() class.

Usage

```
## S3 method for class 'fsets'
str(object, ...)
```

Arguments

object	An instance of the fsets() class
...	Unused.

Value

Nothing

Author(s)

Michal Burda

See Also

fsets(), fcut(), lcut()

sugeno — *A factory function for creation of sugeno-integrals.*

Description

A factory function for creation of sugeno-integrals.

Usage

```
sugeno(
  measure,
  relative = TRUE,
  strong = FALSE,
  alg = c("lukasiewicz", "goedel", "goguen")
)
```

Arguments

measure	A non-decreasing function that assigns a truth value from the $[0, 1]$ interval to the either relative or absolute quantity
relative	Whether the measure assumes relative or absolute quantity. Relative quantity is always a number from the $[0, 1]$ interval
strong	Whether to use the strong conjunction (TRUE) or the weak conjunction (FALSE)
alg	The underlying algebra must be either a string (one from 'lukasiewicz', 'goedel' or 'goguen') or an instance of the S3 class algebra().

Value

A two-argument function, which expects two numeric vectors of equal length (the vector elements are recycled to ensure equal lengths). The first argument, x, is a vector of membership degrees to be measured, the second argument, w, is the vector of weights.

Let U be the set of input vector indices (1 to length(x)). Then the Sugeno integral computes the truth values according to the following formula: $\vee_{z \subseteq U} \wedge_{u \in z} (x[u]) CONJ measure(m_z)$, where $m_z = sum(w[z])/sum(w)$ if relative==TRUE or $m_z = sum(w)$ if relative==FALSE and where CONJ is a strong conjunction (i.e., alg$pt) or a weak conjunction (i.e., alg$pi) according to the strong parameter.

Author(s)

Michal Burda

See Also

quantifier(), lingexpr()

Examples

```
# Dvorak <1> "almost all" quantifier
q <- sugeno(lingexpr(ctx3(), atomic='bi', hedge='ex'))
a <- c(0.9, 1, 1, 0.2, 1)
q(x=a, w=1)

# Dvorak <1,1> "almost all" quantifier
a <- c(0.9, 1, 1, 0.2, 1)
b <- c(0.2, 1, 0, 0.5, 0.8)
q <- sugeno(lingexpr(ctx3(), atomic='bi', hedge='ex'))
q(x=lukas.residuum(a, b), w=1)

# Murinová <1,1> "almost all" quantifier
a <- c(0.9, 1, 1, 0.2, 1)
b <- c(0.2, 1, 0, 0.5, 0.8)
q <- sugeno(lingexpr(ctx3(), atomic='bi', hedge='ex'))
q(x=lukas.residuum(a, b), w=a)
```

`triangle` *Deprecated functions to compute membership degrees of numeric fuzzy sets*

Description

These functions compute membership degrees of numeric fuzzy sets with triangular or raised-cosine shape. These functions are *deprecated*. Please use `triangular()` or `raisedcosine()` functions instead.

Usage

```
triangle(x, lo, center, hi)

raisedcos(x, lo, center, hi)
```

Arguments

`x`	A numeric vector to be transformed.
`lo`	A lower bound (can be -Inf).
`center`	A peak value.
`hi`	An upper bound (can be Inf).

Value

A numeric vector of membership degrees of `x` to a fuzzy set with the shape determined with `lo`, `center`, `hi`.

Author(s)

Michal Burda

See Also

`triangular()`, `raisedcosine()`

`triangular` *Factories for functions that convert numeric data into membership degrees of fuzzy sets*

Description

These functions create functions with a single argument `x` that compute membership degrees of `x` to a fuzzy set of either triangular or raised-cosine shape that is defined by `lo`, `center`, and `hi`.

Usage

```
triangular(lo, center, hi)

raisedcosine(lo, center, hi)
```

Arguments

`lo`	A lower bound (can be -Inf).
`center`	A peak value.
`hi`	An upper bound (can be Inf).

Details

The arguments must satisfy `lo <= center <= hi`. Functions compute membership degrees of triangular or raised-cosine fuzzy sets. `x` values equal to `center` obtain membership degree equal to `1`, `x`values lower than

If `lo == -Inf` then any value that is lower or equal to center gets membership degree 1. Similarly, if `hi == Inf` then any value that is greater or equal to center gets membership degree 1. `NA` and `NaN` values remain unchanged.

`triangular()` produces fuzzy sets of a triangular shape (with peak at `center`), `raisedcosine()` produces fuzzy sets defined as a raised cosine hill.

Value

A function with single argument `x` that should be a numeric vector to be converted.

Author(s)

Michal Burda

See Also

`fcut()`

Examples

```
tr <- triangular(1, 2, 3)
tr(1:30 / 3)

rc <- raisedcosine(1, 2, 3)
rc(1:30 / 3)

plot(triangular(-1, 0, 1), from=-2, to=3)
plot(triangular(-1, 0, 2), from=-2, to=3)
plot(triangular(-Inf, 0, 1), from=-2, to=3)
plot(triangular(-1, 0, Inf), from=-2, to=3)

plot(raisedcosine(-1, 0, 1), from=-2, to=3)
plot(raisedcosine(-1, 0, 2), from=-2, to=3)
plot(raisedcosine(-Inf, 0, 1), from=-2, to=3)
plot(raisedcosine(-1, 0, Inf), from=-2, to=3)
```

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