Package: gaussratiovegind (via r-universe)

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Title Distribution of Gaussian Ratios

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Description It is well known that the distribution of a Gaussian ratio does not follow a Gaussian distribution. The lack of awareness among users of vegetation indices about this non-Gaussian nature could lead to incorrect statistical modeling and interpretation. This package provides tools to accurately handle and analyse such ratios: density function, parameter estimation, simulation. An example on the study of chlorophyll fluorescence can be found in A. El Ghaziri et al. (2023) <doi:10.3390/rs15020528>.

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URL https://forgemia.inra.fr/imhorphen/gaussratiovegind

BugReports https://forgemia.inra.fr/imhorphen/gaussratiovegind/-/issues

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`dnormratio` *Ratio of two Gaussian distributions*

Description

Density of the ratio of two Gaussian distributions.

Usage

```
dnormratio(z, bet, rho, delta)
```

Arguments

`z`	length p numeric vector.
`bet, rho, delta`	numeric values. The parameters (β, ρ, δ) of the distribution, see Details.

Details

Let two independant random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.
If we denote $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$ and $\delta_y = \frac{\sigma_y}{\mu_y}$, the probability distribution function of the ratio $Z = \frac{X}{Y}$ is given by:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1+\rho^2 z^2)} \left[\exp\left(-\frac{\rho^2\beta^2 + 1}{2\delta_y^2} \right) + \sqrt{\frac{\pi}{2}}\, q \operatorname{erf}\left(\frac{q}{\sqrt{2}} \right) \exp\left(-\frac{\rho^2 (z-\beta)^2}{2\delta_y^2 (1+\rho^2 z^2)} \right) \right]$$

with $q = \dfrac{1 + \beta\rho^2 z}{\delta_y \sqrt{1+\rho^2 z^2}}$ and $\operatorname{erf}\left(\dfrac{q}{\sqrt{2}} \right) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^{\frac{q}{\sqrt{2}}} \exp\left(-t^2\right) dt$

Value

Numeric: the value of density.

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

Díaz-Francés, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. Stat Papers 54, 309–323 (2013). doi:10.1007/s0036201204292

See Also

`rnormratio()`: sample simulation.

`estparnormratio()`: parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
dnormratio(0, bet = beta1, rho = rho1, delta = delta1)
dnormratio(0.5, bet = beta1, rho = rho1, delta = delta1)
curve(dnormratio(x, bet = beta1, rho = rho1, delta = delta1), from = -0.1, to = 0.7)

# Second example
beta2 <- 2
rho2 <- 2
delta2 <- 2
dnormratio(0, bet = beta2, rho = rho2, delta = delta2)
dnormratio(0.5, bet = beta2, rho = rho2, delta = delta2)
curve(dnormratio(x, bet = beta2, rho = rho2, delta = delta2), from = -0.1, to = 0.7)
```

---

estparnormratio — *Estimation of the Parameters of a Normal Ratio Distribution*

Description

Estimation of the parameters of a ratio $Z = \frac{X}{Y}$, X and Y being two independant random variables distributed according to Gaussian distributions, using the EM (estimation-maximization) algorithm.

Usage

```
estparnormratio(z, eps = 1e-6)
```

Arguments

`z`	numeric matrix or data frame.
`eps`	numeric. Precision for the estimation of the parameters.

Details

Let a random variable: $Z = \frac{X}{Y}$,

X and Y being normally distributed: $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$.

The density probability of Z is:

$$f_Z(z;\beta,\rho,\delta_y) = \frac{\rho}{\pi(1+\rho^2 z^2)} \exp\left(-\frac{\rho^2\beta^2+1}{2\delta_y^2}\right) {}_1F_1\left(1, \frac{1}{2}; \frac{1}{2\delta_y^2}\frac{(1+\beta\rho^2 z)^2}{1+\rho^2 z^2}\right)$$

with: $\beta = \frac{\mu_x}{\mu_y}$, $\rho = \frac{\sigma_y}{\sigma_x}$, $\delta_y = \frac{\sigma_y}{\mu_y}$.

and ${}_1F_1(a,b;x)$ is the confluent hypergeometric function (Kummer's function):

$${}_1F_1(a,b;x) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n}\frac{x^n}{n!}$$

The parameters β, ρ, δ_y of the Z distribution are estimated with the EM algorithm, as presented in El Ghaziri et al. The computation uses the `kummerM` function.

This uses an iterative algorithm.

The precision for the estimation of the parameters is given by the `eps` parameter.

Value

A list of 3 elements `beta`, `rho`, `delta`: the estimated parameters of the Z distribution $\hat{\beta}$, $\hat{\rho}$, $\hat{\delta}_y$, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

See Also

`dnormratio()`: probability density of a normal ratio.

`rnormratio()`: sample simulation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22

set.seed(1234)
z1 <- rnormratio(800, bet = beta1, rho = rho1, delta = delta1)

estparnormratio(z1)

# Second example
```

```
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25

set.seed(1234)
z2 <- rnormratio(800, bet = beta2, rho = rho2, delta = delta2)

estparnormratio(z2)
```

---

kummerM *Confluent D-Hypergeometric Function*

Description

Computes the Kummer's function, or confluent hypergeometric function.

Usage

```
kummerM(a, b, z, eps = 1e-06)
```

Arguments

a	numeric.
b	numeric
z	numeric vector.
eps	numeric. Precision for the sum (default 1e-06).

Details

The Kummer's confluent hypergeometric function is given by:

$$ {}_1F_1\left(a, b; z\right) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{z^n}{n!} $$

where $(z)_p$ is the Pochhammer symbol (see `pochhammer`).

The eps argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

Value

A numeric value: the value of the Kummer's function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

`lnpochhammer` *Logarithm of the Pochhammer Symbol*

Description

Computes the logarithm of the Pochhammer symbol.

Usage

```
lnpochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

So, if $n > 0$:

$$log\left((x)_n\right) = log(x) + log(x+1) + ... + log(x+n-1)$$

If $n = 0$, $log\left((x)_n\right) = log(1) = 0$

Value

Numeric value. The logarithm of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

pochhammer, kummerM

Examples

```
lnpochhammer(2, 0)
lnpochhammer(2, 1)
lnpochhammer(2, 3)
```

`pochhammer` *Pochhammer Symbol*

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

x	numeric.
n	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

lnpochhammer, kummerM

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

rnormratio *Ratio of two Gaussian distributions*

Description

Simulate data from a ratio of two Gaussian distributions.

Usage

```
rnormratio(n, bet, rho, delta)
```

Arguments

n	integer. Number of observations. If `length(n)` > 1, the length is taken to be the nmber required.
bet, rho, delta	numeric values. The parameters (β, ρ, δ) of the distribution, see Details.

Details

Let two random variables $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$

with probability densities f_X and f_Y.

The parameters of the distribution of the ratio $Z = \frac{X}{Y}$ are: $\beta = \dfrac{\mu_x}{\mu_y}$, $\rho = \dfrac{\sigma_y}{\sigma_x}$, $\delta_y = \dfrac{\sigma_y}{\mu_y}$.

μ_x, σ_x, μ_y and σ_y are computed from β, ρ and δ_y (by fixing arbitrarily $\mu_x = 1$) and two random samples $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are simulated.

Then $\left(\dfrac{x_1}{y_1}, \ldots, \dfrac{x_n}{y_n}\right)$ is returned.

Value

A numeric vector: the produced sample.

Author(s)

Pierre Santagostini, Angélina El Ghaziri, Nizar Bouhlel

References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. Remote Sensing 15(2), 528 (2023). doi:10.3390/rs15020528

See Also

`dnormratio()`: probability density of a normal ratio.

`estparnormratio()`: parameter estimation.

Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
rnormratio(20, bet = beta1, rho = rho1, delta = delta1)

# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25
rnormratio(20, bet = beta2, rho = rho2, delta = delta2)
```

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