

Package: `fourierin` (via `r-universe`)

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Type Package

Title Computes Numeric Fourier Integrals

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Description Computes Fourier integrals of functions of one and two variables using the Fast Fourier transform. The Fourier transforms must be evaluated on a regular grid for fast evaluation.

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LinkingTo RcppArmadillo, Rcpp

Imports Rcpp (>= 1.0.1),

Suggests MASS, knitr, rmarkdown, dplyr, tidyr, purrr, ggplot2, lattice, rbenchmark, testthat (>= 3.1.0), covr, spelling

RoxygenNote 7.2.3

URL <https://github.com/gbasulto/fourierin>

BugReports <https://github.com/gbasulto/fourierin/issues>

VignetteBuilder knitr

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fourierin	<i>Compute Fourier integrals</i>
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Description

It computes Fourier integrals for functions of one and two variables.

Usage

```
fourierin(
  f,
  lower_int,
  upper_int,
  lower_eval = NULL,
  upper_eval = NULL,
  const_adj,
  freq_adj,
  resolution = NULL,
  eval_grid = NULL,
  use_fft = TRUE
)
```

Arguments

f	function or a vector of size m. If a function is provided, it must be able to be evaluated at vectors. If a vector of values is provided, such evaluations must have been obtained on a regular grid and the Fourier integral is faster if m is a power of 2.
lower_int	Lower integration limit(s).
upper_int	Upper integration limit(s).
lower_eval	Lower evaluation limit(s). It can be NULL if an evaluation grid is provided.
upper_eval	Upper evaluation limit(s). It can be NULL if an evaluation grid is provided.
const_adj	Factor related to adjust definition of Fourier transform. It is usually equal to 0, -1 or 1.
freq_adj	Constant to adjust the exponent on the definition of the Fourier transform. It is usually equal to 1, -1, 2pi or -2pi.
resolution	A vector of integers (faster if powers of two) determining the resolution of the evaluation grid. Not required if f is a vector.
eval_grid	Optional matrix with d columns with the points where the Fourier integral will be evaluated. If it is provided, the FFT will not be used.
use_fft	Logical value specifying whether the FFT will be used.

Details

See plenty of detailed examples in the vignette.

Value

A list with the elements n-dimensional array and n vectors with their corresponding resolution. Specifically,

values	A n-dimensional (resol_1 x resol_2 x ... x resol_n) complex array with the values.
w1	A vector of size resol_1
...	
wn	A vector of size resol_n

Examples

```
##--- Example 1 -----
##--- Recovering std. normal from its characteristic function -----
library(fourierin)

## Function to be used in the integrand
myfnc <- function(t) exp(-t^2/2)

## Compute integral
out <- fourierin(f = myfnc, lower_int = -5, upper_int = 5,
                lower_eval = -3, upper_eval = 3, const_adj = -1,
                freq_adj = -1, resolution = 64)

## Extract grid and values
grid <- out$w
values <- Re(out$values)

## Compare with true values of Fourier transform
plot(grid, values, type = "l", col = 3)
lines(grid, dnorm(grid), col = 4)

##--- Example 2 -----
##--- Computing characteristic function of a gamma r. v. -----

library(fourierin)

## Function to be used in integrand
myfnc <- function(t) dgamma(t, shape, rate)

## Compute integral
shape <- 5
rate <- 3
out <- fourierin(f = myfnc, lower_int = 0, upper_int = 6,
                lower_eval = -4, upper_eval = 4,
                const_adj = 1, freq_adj = 1, resolution = 64)
```

```

## Extract values
grid <- out$w                                # Extract grid
re_values <- Re(out$values)                  # Real values
im_values <- Im(out$values)                  # Imag values

## Now compute the real and imaginary true values of the
## characteristic function.
true_cf <- function(t, shape, rate) (1 - 1i*t/rate)^-shape
true_re <- Re(true_cf(grid, shape, rate))
true_im <- Im(true_cf(grid, shape, rate))

## Compare them. We can see a slight discrepancy on the tails,
## but that is fixed when resolution is increased.
plot(grid, re_values, type = "l", col = 3)
lines(grid, true_re, col = 4)

                                                    # Same here
plot(grid, im_values, type = "l", col = 3)
lines(grid, true_im, col = 4)

##--- Example 3 -----
##--- Recovering std. normal from its characteristic function ---
library(fourierin)

##-Parameters of bivariate normal distribution
mu <- c(-1, 1)
sig <- matrix(c(3, -1, -1, 2), 2, 2)

##-Multivariate normal density
##-x is n x d
f <- function(x) {
  ##-Auxiliar values
  d <- ncol(x)
  z <- sweep(x, 2, mu, "-")
  ##-Get numerator and denominator of normal density
  num <- exp(-0.5*rowSums(z * (z %%% solve(sig))))
  denom <- sqrt((2*pi)^d*det(sig))
  return(num/denom)
}

## Characteristic function
## s is n x d
phi <- function(s) {
  complex(modulus = exp(- 0.5*rowSums(s*(s %%% sig))),
          argument = s %%% mu)
}

##-Approximate cf using Fourier integrals
eval <- fourierin(f, lower_int = c(-8, -6), upper_int = c(6, 8),
                 lower_eval = c(-4, -4), upper_eval = c(4, 4),
                 const_adj = 1, freq_adj = 1,
                 resolution = c(128, 128))

```

```

## Extract values
t1 <- eval$w1
t2 <- eval$w2
t <- as.matrix(expand.grid(t1 = t1, t2 = t2))
approx <- eval$values
true <- matrix(phi(t), 128, 128)      # Compute true values

## This is a section of the characteristic function
i <- 65
plot(t2, Re(approx[i, ]), type = "l", col = 2,
     ylab = "",
     xlab = expression(t[2]),
     main = expression(paste("Real part section at ",
                              t[1], "= 0")))
lines(t2, Re(true[i, ]), col = 3)
legend("topleft", legend = c("true", "approximation"),
      col = 3:2, lwd = 1)

##-Another section, now of the imaginary part
plot(t1, Im(approx[, i]), type = "l", col = 2,
     ylab = "",
     xlab = expression(t[1]),
     main = expression(paste("Imaginary part section at ",
                              t[2], "= 0")))
lines(t1, Im(true[, i]), col = 3)
legend("topleft", legend = c("true", "approximation"),
      col = 3:2, lwd = 1)

```

fourierin_1d

Univariate Fourier integrals

Description

It computes Fourier integrals of functions of one and two variables on a regular grid.

Usage

```

fourierin_1d(
  f,
  lower_int,
  upper_int,
  lower_eval = NULL,
  upper_eval = NULL,
  const_adj,
  freq_adj,
  resolution = NULL,
  eval_grid = NULL,
  use_fft = TRUE
)

```

Arguments

f	function or a vector of size m. If a function is provided, it must be able to be evaluated at vectors. If a vector of values is provided, such evaluations must have been obtained on a regular grid and the Fourier integral is faster if m is a power of 2.
lower_int	Lower integration limit(s).
upper_int	Upper integration limit(s).
lower_eval	Lower evaluation limit(s). It can be NULL if an evaluation grid is provided.
upper_eval	Upper evaluation limit(s). It can be NULL if an evaluation grid is provided.
const_adj	Factor related to adjust definition of Fourier transform. It is usually equal to 0, -1 or 1.
freq_adj	Constant to adjust the exponent on the definition of the Fourier transform. It is usually equal to 1, -1, 2pi or -2pi.
resolution	A vector of integers (faster if powers of two) determining the resolution of the evaluation grid. Not required if f is a vector.
eval_grid	Optional matrix with d columns with the points where the Fourier integral will be evaluated. If it is provided, the FFT will not be used.
use_fft	Logical value specifying whether the FFT will be used.

Details

See vignette for more detailed examples.

Value

If w is given, only the values of the Fourier integral are returned, otherwise, a list with the elements

w	A vector of size m where the integral was computed.
values	A complex vector of size m with the values of the integral

Examples

```
##--- Example 1 -----
##--- Recovering std. normal from its characteristic function ----
library(fourierin)

#' Function to to be used in integrand
myfun <- function(t) exp(-t^2/2)

                                # Compute Foeien integral
out <- fourierin_1d(f = myfun,
                  lower_int = -5, upper_int = 5,
                  lower_eval = -3, upper_eval = 3,
                  const_adj = -1, freq_adj = -1,
                  resolution = 64)

## Extract grid and values
```

```

grid <- out$w
values <- Re(out$values)

plot(grid, values, type = "l", col = 3)
lines(grid, dnorm(grid), col = 4)

##--- Example 2 -----
##--- Computing characteristic function of a gamma r. v. -----

library(fourierin)

## Function to to be used in integrand
myfun <- function(t) dgamma(t, shape, rate)

## Compute integral
shape <- 5
rate <- 3
out <- fourierin_1d(f = myfun, lower_int = 0, upper_int = 6,
                  lower_eval = -4, upper_eval = 4,
                  const_adj = 1, freq_adj = 1, resolution = 64)

grid <- out$w                                # Extract grid
re_values <- Re(out$values)                  # Real values
im_values <- Im(out$values)                  # Imag values

                                        # Now compute the real and
                                        # imaginary true values of the
                                        # characteric function.
true_cf <- function(t, shape, rate) (1 - 1i*t/rate)^-shape
true_re <- Re(true_cf(grid, shape, rate))
true_im <- Im(true_cf(grid, shape, rate))

                                        # Compare them. We can see a
                                        # slight discrepancy on the
                                        # tails, but that is fixed
                                        # when resolution is
                                        # increased.

plot(grid, re_values, type = "l", col = 3)
lines(grid, true_re, col = 4)

                                        # Same here
plot(grid, im_values, type = "l", col = 3)
lines(grid, true_im, col = 4)

```

Description

It computes Fourier integrals for functions of one and two variables.

Usage

```

fourierin_2d(
  f,
  lower_int,
  upper_int,
  lower_eval = NULL,
  upper_eval = NULL,
  const_adj,
  freq_adj,
  resolution = NULL,
  eval_grid = NULL,
  use_fft = TRUE
)

```

Arguments

f	function or a vector of size m. If a function is provided, it must be able to be evaluated at vectors. If a vector of values is provided, such evaluations must have been obtained on a regular grid and the Fourier integral is faster if m is a power of 2.
lower_int	Lower integration limit(s).
upper_int	Upper integration limit(s).
lower_eval	Lower evaluation limit(s). It can be NULL if an evaluation grid is provided.
upper_eval	Upper evaluation limit(s). It can be NULL if an evaluation grid is provided.
const_adj	Factor related to adjust definition of Fourier transform. It is usually equal to 0, -1 or 1.
freq_adj	Constant to adjust the exponent on the definition of the Fourier transform. It is usually equal to 1, -1, 2pi or -2pi.
resolution	A vector of integers (faster if powers of two) determining the resolution of the evaluation grid. Not required if f is a vector.
eval_grid	Optional matrix with d columns with the points where the Fourier integral will be evaluated. If it is provided, the FFT will not be used.
use_fft	Logical value specifying whether the FFT will be used.

Value

If w is given, only the values of the Fourier integral are returned, otherwise, a list with three elements

w1	Evaluation grid for first entry
w2	Evaluation grid for second entry
values	m1 x m2 matrix of complex numbers, corresponding to the evaluations of the integral

Examples

```

##--- Recovering std. normal from its characteristic function -----
library(fourierin)

##-Parameters of bivariate normal distribution
mu <- c(-1, 1)
sig <- matrix(c(3, -1, -1, 2), 2, 2)

##-Multivariate normal density
##-x is n x d
f <- function(x) {
  ##-Auxiliar values
  d <- ncol(x)
  z <- sweep(x, 2, mu, "-")

  ##-Get numerator and denominator of normal density
  num <- exp(-0.5*rowSums(z * (z %*% solve(sig))))
  denom <- sqrt((2*pi)^d*det(sig))

  return(num/denom)
}

##-Characteristic function
##-s is n x d
phi <- function(s) {
  complex(modulus = exp(- 0.5*rowSums(s*(s %*% sig))),
          argument = s %*% mu)
}

##-Approximate cf using Fourier integrals
eval <- fourierin_2d(f, lower_int = c(-8, -6), upper_int = c(6, 8),
                    lower_eval = c(-4, -4), upper_eval = c(4, 4),
                    const_adj = 1, freq_adj = 1,
                    resolution = c(128, 128))

## Extract values
t1 <- eval$w1
t2 <- eval$w2
t <- as.matrix(expand.grid(t1 = t1, t2 = t2))
approx <- eval$values
true <- matrix(phi(t), 128, 128)      # Compute true values

##-This is a section of the characteristic functions
i <- 65
plot(t2, Re(approx[i, ]), type = "l", col = 2,
     ylab = "",
     xlab = expression(t[2]),
     main = expression(paste("Real part section at ",
                             t[1], "= 0")))
lines(t2, Re(true[i, ]), col = 3)
legend("topleft", legend = c("true", "approximation"),
      col = 3:2, lwd = 1)

```

```
##-Another section, now of the imaginary part
plot(t1, Im(approx[, i]), type = "l", col = 2,
      ylab = "",
      xlab = expression(t[1]),
      main = expression(paste("Imaginary part section at ",
                               t[2], "= 0")))
lines(t1, Im(true[, i]), col = 3)
legend("topleft", legend = c("true", "approximation"),
      col = 3:2, lwd = 1)
```

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