

# Package: flex (via r-universe)

June 1, 2026

**Title** Fuzzy Linear Squares Estimation with Explicit Formula (FLEX)

**Version** 0.1.0

**Date** 2025-08-27

**Description** The FLEX method, developed by Yoon and Choi (2013) <[doi:10.1007/978-3-642-33042-1\\_21](https://doi.org/10.1007/978-3-642-33042-1_21)>, performs least squares estimation for fuzzy predictors and outcomes, generating crisp regression coefficients by minimizing the distance between observed and predicted outcomes. It also provides functions for fuzzifying data and inference tasks, including significance testing, fit indices, and confidence interval estimation.

**License** MIT + file LICENSE

**BugReports** <https://github.com/cwlee-quantpsych/flex/issues>

**Depends** R (>= 3.5.0)

**Encoding** UTF-8

**RoxygenNote** 7.3.2

**Imports** ggplot2, plotly, rlang, magrittr

**NeedsCompilation** no

**Author** Chaewon Lee [aut, cre], Jin Hee Yoon [ctb]

**Maintainer** Chaewon Lee <[chaewon.lee@unc.edu](mailto:chaewon.lee@unc.edu)>

**Repository** <https://cran.r-universe.dev>

**Date/Publication** 2025-09-02 06:30:07 UTC

**RemoteUrl** <https://github.com/cran/flex>

**RemoteRef** HEAD

**RemoteSha** c7d474604393fbf09130582b45eca37557f99e4a

## Contents

coefficients	2
coefficients.fuzzy_lm	3
compute_ci	4

compute_p_values . . . . .	4
compute_pred . . . . .	5
compute_res . . . . .	6
compute_t_values . . . . .	6
fuzzify_crisp_matrix . . . . .	7
fuzzify_crisp_value . . . . .	8
fuzzify_crisp_vector . . . . .	8
fuzzy_add . . . . .	9
fuzzy_crisp_mult . . . . .	10
fuzzy_d_squared . . . . .	10
fuzzy_lm . . . . .	11
fuzzy_mults . . . . .	12
plot . . . . .	13
plot.fuzzy_lm . . . . .	14
predictions . . . . .	15
predictions.fuzzy_lm . . . . .	16
residuals . . . . .	16
residuals.fuzzy_lm . . . . .	17
summary.fuzzy_lm . . . . .	18

<b>Index</b>	<b>19</b>
--------------	-----------

---

coefficients	<i>Define generic for coefficients</i>
--------------	----------------------------------------

---

## Description

A generic function to retrieve coefficients from model objects.

## Usage

```
coefficients(object, ...)
```

## Arguments

object	The model object from which to extract coefficients.
...	Additional arguments (ignored).

## Value

A data frame of coefficients and related statistics.

**Examples**

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Extract coefficients
coefficients(object)
```

---

coefficients.fuzzy\_lm *Accessor for Coefficients*

---

**Description**

Accessor for Coefficients

**Usage**

```
## S3 method for class 'fuzzy_lm'
coefficients(object, ...)
```

**Arguments**

object	An object of class fuzzy_lm.
...	Additional arguments (ignored).

**Value**

A data frame of coefficients and statistics.

**Examples**

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Extract coefficients
coefficients(object)
```

---

compute_ci	<i>Compute confidence intervals for regression coefficients</i>
------------	-----------------------------------------------------------------

---

**Description**

Compute confidence intervals for regression coefficients

**Usage**

```
compute_ci(beta_hat, se_beta, df, alpha = 0.05)
```

**Arguments**

beta_hat	Numeric vector. Estimated regression coefficients.
se_beta	Numeric vector. Standard errors of coefficients.
df	Integer. Degrees of freedom.
alpha	Numeric. Significance level (default: 0.05).

**Value**

A list containing lower and upper bounds of confidence intervals.

**Examples**

```
beta_hat <- c(0.5, 1.2) # Example regression coefficients
se_beta <- c(0.1, 0.2) # Example standard errors
df <- 30 # Example degrees of freedom
ci <- compute_ci(beta_hat, se_beta, df)
print(ci)
```

---

compute_p_values	<i>Compute p-values for regression coefficients</i>
------------------	-----------------------------------------------------

---

**Description**

Compute p-values for regression coefficients

**Usage**

```
compute_p_values(t_values, df)
```

**Arguments**

t_values	Numeric vector. T-values for regression coefficients.
df	Integer. Degrees of freedom.

**Value**

Numeric vector of p-values for each coefficient.

**Examples**

```
t_values <- c(2.5, 3.0) # Example t-values
df <- 30 # Example degrees of freedom
p_values <- compute_p_values(t_values, df)
print(p_values)
```

---

`compute_pred`*Compute Predictions from Fuzzy Linear Model*

---

**Description**

Compute Predictions from Fuzzy Linear Model

**Usage**

```
compute_pred(object, X_fuzzy)
```

**Arguments**

<code>object</code>	List. Result of fuzzy least squares regression containing <code>beta_hat</code> .
<code>X_fuzzy</code>	List. Fuzzified predictor variables.

**Value**

A list of fuzzy predictions.

**Examples**

```
# Example setup
X_fuzzy <- list(
  list(list(l = 1, x = 2, r = 3), list(l = 4, x = 5, r = 6)),
  list(list(l = 2, x = 3, r = 4), list(l = 5, x = 6, r = 7))
)
beta_hat <- c(0.5, 1.2, -0.8) # Example regression coefficients
object <- list(beta_hat = beta_hat)

# Compute predictions
predictions <- compute_pred(object, X_fuzzy)
print(head(predictions, 6))
```

compute\_res

*Compute Residuals for Fuzzy Linear Model*

---

**Description**

Compute Residuals for Fuzzy Linear Model

**Usage**

```
compute_res(Y_fuzzy, Y_pred)
```

**Arguments**

Y\_fuzzy            List. Fuzzified observed response variables.  
Y\_pred            List. Fuzzified predicted response variables.

**Value**

A list of fuzzy residuals.

**Examples**

```
# Example setup
Y_fuzzy <- list(
  list(l = 2, x = 3, r = 4),
  list(l = 5, x = 6, r = 7)
)
Y_pred <- list(
  list(l = 1.5, x = 2.5, r = 3.5),
  list(l = 4.5, x = 5.5, r = 6.5)
)

# Compute residuals
residuals <- compute_res(Y_fuzzy, Y_pred)
print(head(residuals, 6))
```

---

compute\_t\_values*Compute t-values for regression coefficients*

---

**Description**

Compute t-values for regression coefficients

**Usage**

```
compute_t_values(beta_hat, Y_fuzzy, Y_pred, XtX_inv)
```

**Arguments**

beta_hat	Numeric vector. Estimated regression coefficients.
Y_fuzzy	List. Observed fuzzy responses.
Y_pred	List. Predicted fuzzy responses.
XtX_inv	Matrix. Inverse of the XtX matrix.

**Value**

Numeric vector of t-values for the regression coefficients.

**Examples**

```
# Example setup
beta_hat <- c(0.5, 1.2) # Example regression coefficients
Y_fuzzy <- list(
  list(l = 2.1, x = 2.3, r = 2.5),
  list(l = 3.1, x = 3.3, r = 3.5),
  list(l = 4.1, x = 4.3, r = 4.5)
) # Example fuzzy response
Y_pred <- list(
  list(l = 2.0, x = 2.2, r = 2.4),
  list(l = 3.0, x = 3.2, r = 3.4),
  list(l = 4.0, x = 4.2, r = 4.4)
) # Example predicted values
XtX_inv <- matrix(c(0.1, 0.2, 0.2, 0.4), ncol = 2) # Example XtX_inv matrix
t_values <- compute_t_values(beta_hat, Y_fuzzy, Y_pred, XtX_inv)
print(t_values)
```

---

fuzzify\_crisp\_matrix *Fuzzify a matrix of crisp values*

---

**Description**

Converts a numeric matrix into a list of triangular fuzzy numbers.

**Usage**

```
fuzzify_crisp_matrix(crisp_matrix, spread = 1)
```

**Arguments**

crisp_matrix	Numeric matrix to be fuzzified.
spread	Numeric. The spread for fuzzification (default is 1).

**Value**

A list of lists representing rows of triangular fuzzy numbers.

**Examples**

```
set.seed(123)
matrix <- matrix(runif(9, 5, 15), nrow = 3, ncol = 3)
fuzzify_crisp_matrix(matrix, spread = 1.5)
```

---

fuzzify\_crisp\_value    *Fuzzify a single crisp value*

---

**Description**

Converts a crisp value into a triangular fuzzy number with a specified spread.

**Usage**

```
fuzzify_crisp_value(crisp_value, spread = 1)
```

**Arguments**

crisp\_value    Numeric. The crisp value to be fuzzified.  
spread        Numeric. The spread for fuzzification (default is 1).

**Value**

A list representing the triangular fuzzy number with components l, x, and r.

**Examples**

```
fuzzify_crisp_value(10, spread = 2)
```

---

fuzzify\_crisp\_vector    *Fuzzify a vector of crisp values*

---

**Description**

Converts a numeric vector into a list of fuzzified values using a triangular fuzzy membership function.

**Usage**

```
fuzzify_crisp_vector(crisp_vector, spread = 1, var_name = "Outcome")
```

**Arguments**

crisp_vector	A numeric vector to be fuzzified.
spread	A non-negative numeric value specifying the spread for the fuzzy membership function.
var_name	Optional. A character string specifying a common name for all fuzzified values. Default is NULL.

**Value**

A list of fuzzified values, where each value is represented as a list with components l, x, and r.

**Examples**

```
crisp_vector <- c(10, 20, 30)
fuzzify_crisp_vector(crisp_vector, spread = 1, var_name = "Variable")
```

---

fuzzy_add	<i>Add two triangular fuzzy numbers</i>
-----------	-----------------------------------------

---

**Description**

Performs the addition of two triangular fuzzy numbers.

**Usage**

```
fuzzy_add(X, Y)
```

**Arguments**

X	List. First triangular fuzzy number with components l, x, and r.
Y	List. Second triangular fuzzy number with components l, x, and r.

**Value**

A list representing the sum of the two fuzzy numbers.

**Examples**

```
X <- list(l = 1, x = 2, r = 3)
Y <- list(l = 2, x = 3, r = 4)
fuzzy_add(X, Y)
```

---

fuzzy_crisp_mult	<i>Multiply a crisp scalar by a triangular fuzzy number</i>
------------------	-------------------------------------------------------------

---

**Description**

Scales a triangular fuzzy number by a crisp scalar.

**Usage**

```
fuzzy_crisp_mult(scalar, fuzzy_num)
```

**Arguments**

scalar	Numeric. The scalar to multiply with the fuzzy number.
fuzzy_num	List. A triangular fuzzy number with components l, x, and r.

**Value**

A list representing the scaled fuzzy number.

**Examples**

```
scalar <- 3
fuzzy_num <- list(l = 1, x = 2, r = 3)
fuzzy_crisp_mult(scalar, fuzzy_num)
```

---

fuzzy_d_squared	<i>Compute the squared distance between two fuzzy numbers</i>
-----------------	---------------------------------------------------------------

---

**Description**

Calculates the squared distance between two triangular fuzzy numbers using Diamond's metric.

**Usage**

```
fuzzy_d_squared(X, Y)
```

**Arguments**

X	List. First triangular fuzzy number.
Y	List. Second triangular fuzzy number.

**Value**

Numeric. The squared distance between X and Y.

**Examples**

```
X <- list(l = 1, x = 2, r = 3)
Y <- list(l = 2, x = 3, r = 4)
fuzzy_d_squared(X, Y)
```

fuzzy\_lm

*Fuzzy Linear Regression***Description**

Fits a fuzzy linear regression model given fuzzified predictors and response variables.

**Usage**

```
fuzzy_lm(X_fuzzy, Y_fuzzy, p, X_crisp = NULL)
```

**Arguments**

X_fuzzy	A list of fuzzified predictor values.
Y_fuzzy	A list of fuzzified response values.
p	An integer specifying the number of predictors.
X_crisp	Optional. The original crisp predictor matrix or data frame. Used to retrieve variable names. Default is NULL.

**Value**

A list object of class fuzzy\_lm containing:

Coefficients	A data frame with estimated coefficients, standard errors, t-values, p-values, and significance stars.
Residuals	The residuals from the fitted model.
Predictions	The predicted fuzzified response values.
RSS	The residual sum of squares.
R_squared	The coefficient of determination (R-squared).
Sigma_squared	The estimated variance of the residuals.
Degrees_of_Freedom	The degrees of freedom for the model.

**Examples**

```

# Simulate complex data for fuzzy linear regression
set.seed(123)

# Generate a dataset with 100 observations and 4 predictors
n <- 100
X_crisp <- data.frame(
  Age = round(runif(n, 20, 70)),          # Random ages between 20 and 70
  Income = round(runif(n, 20000, 120000)), # Random incomes between 20k and 120k
  Education = round(runif(n, 10, 20)),    # Random years of education between 10 and 20
  Experience = round(runif(n, 1, 40))     # Random years of work experience between 1 and 40
)

# Define true coefficients
beta <- c(5.0, 1.2, -0.5, 0.8, 0.05) # Intercept and coefficients for the predictors

# Generate the crisp response variable with noise
Y_crisp <- round(beta[1] + as.matrix(X_crisp) %*% beta[-1] + rnorm(n, mean = 0, sd = 50))

# Fuzzify the predictor and response variables
X_fuzzy <- fuzzify_crisp_matrix(as.matrix(X_crisp), spread = 10) # Larger spread for predictors
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 20)           # Larger spread for responses

# Fit the fuzzy linear model
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 4, X_crisp = X_crisp)

# Print the coefficients
print("Fuzzy Linear Model Coefficients:")
print(object$Coefficients)

# Example residuals and predictions
print("Example Residuals:")
print(head(object$Residuals, 6))

print("Example Predictions:")
print(head(object$Predictions, 6))

```

---

fuzzy\_mults

*Multiply two triangular fuzzy numbers*


---

**Description**

Computes the scalar product of two triangular fuzzy numbers.

**Usage**

```
fuzzy_mults(X, Y)
```

**Arguments**

- X List. First triangular fuzzy number with components l, x, and r.  
 Y List. Second triangular fuzzy number with components l, x, and r.

**Value**

A scalar representing the sum of the product of the corresponding components.

**Examples**

```
X <- list(l = 1, x = 2, r = 3)
Y <- list(l = 2, x = 3, r = 4)
fuzzy_mults(X, Y)
```

---

 plot

*Generic Plot Function*


---

**Description**

This is a generic plot function that dispatches to specific plot methods based on the class of the object provided. It is used to create plots for objects such as `fuzzy_lm`.

**Usage**

```
plot(object, ...)
```

**Arguments**

- object The object to be plotted.  
 ... Additional arguments passed to specific plot methods.

**Value**

Depends on the class of object. Typically, a plot or visualization is returned.

**Examples**

```
# Example with fuzzy_lm:
set.seed(123)
x_crisp <- seq(4, 12, length.out = 20)
beta <- 1.5
intercept <- 2
y_crisp <- intercept + beta * x_crisp + rnorm(length(x_crisp), mean = 0, sd = 0.5)

# Fuzzify data
spread_x <- 0.5
spread_y <- 1.0
X_fuzzy <- fuzzify_crisp_matrix(matrix(x_crisp, ncol = 1), spread = spread_x)
```

```

Y_fuzzy <- fuzzify_crisp_vector(y_crisp, spread = spread_y)

# Fit fuzzy regression model
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 1)

# Plot

plot(object, X_fuzzy = X_fuzzy, Y_fuzzy = Y_fuzzy)

```

---

plot.fuzzy\_lm

*Plot Fuzzy Regression Results*


---

### Description

Visualizes the results of a fuzzy regression model. For simple regression (1 predictor), it generates a 2D plot with fuzzy intervals and regression lines. For multiple regression (2 predictors), it generates a 3D plot with cubes representing fuzzy intervals and a regression plane.

### Usage

```

## S3 method for class 'fuzzy_lm'
plot(object, ...)

```

### Arguments

object	An object of class fuzzy_lm.
...	Additional arguments passed to the method, including: <ul style="list-style-type: none"> <li>• X_fuzzy: A list of fuzzified predictor variables.</li> <li>• Y_fuzzy: A list of fuzzified outcome variables.</li> </ul>

### Value

A ggplot2 object for simple regression or a plotly object for multiple regression.

### Examples

```

# Example 1: Simple Regression
# See above for setup example

# Example 2: Multiple Regression
set.seed(123)
n <- 100
x1_crisp <- runif(n, 5, 15)
x2_crisp <- runif(n, 10, 20)
beta <- c(3, 1.5, -0.8)
y_crisp <- beta[1] + beta[2] * x1_crisp + beta[3] * x2_crisp + rnorm(n, mean = 0, sd = 2)

```

```
X_fuzzy <- fuzzify_crisp_matrix(cbind(x1_crisp, x2_crisp), spread = 0.5)
Y_fuzzy <- fuzzify_crisp_vector(y_crisp, spread = 1.0)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 2)

plot(object, X_fuzzy = X_fuzzy, Y_fuzzy = Y_fuzzy)
```

---

predictions

*Define generic for predictions*

---

## Description

Define generic for predictions

## Usage

```
predictions(object, ...)
```

## Arguments

`object` An object of class `fuzzy_lm`. The model object.  
`...` Additional arguments (currently ignored).

## Value

A list of fuzzy predictions.

## Examples

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Extract predictions
head(predictions(object))
```

---

predictions.fuzzy\_lm *Accessor for Predictions*

---

### Description

Accessor for Predictions

### Usage

```
## S3 method for class 'fuzzy_lm'
predictions(object, ...)
```

### Arguments

object            An object of class fuzzy\_lm. The model object.  
...                Additional arguments (currently ignored).

### Value

A list of fuzzy predictions.

### Examples

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Extract predictions
head(predictions(object))
```

---

residuals            *Define generic for residuals*

---

### Description

A generic function to retrieve residuals from model objects.

### Usage

```
residuals(object, ...)
```

**Arguments**

object            The model object from which to extract residuals.  
 ...              Additional arguments (ignored).

**Value**

A list of fuzzy residuals.

**Examples**

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Extract residuals
head(residuals(object))
```

---

residuals.fuzzy\_lm    *Accessor for Residuals*

---

**Description**

Accessor for Residuals

**Usage**

```
## S3 method for class 'fuzzy_lm'
residuals(object, ...)
```

**Arguments**

object            An object of class fuzzy\_lm. The model object.  
 ...              Additional arguments (currently ignored).

**Value**

A list of fuzzy residuals.

**Examples**

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Extract residuals
head(residuals(object))
```

---

summary.fuzzy\_lm

*Summary for Fuzzy Linear Regression*


---

**Description**

Summary for Fuzzy Linear Regression

**Usage**

```
## S3 method for class 'fuzzy_lm'
summary(object, ...)
```

**Arguments**

object            An object of class fuzzy\_lm. The model object.  
 ...              Additional arguments (currently ignored).

**Value**

Prints a summary of the fuzzy linear regression results.

**Examples**

```
# Simulate data and fit a fuzzy linear model
set.seed(123)
X_crisp <- matrix(round(runif(300, 2, 10)), nrow = 100, ncol = 3)
beta <- c(1.5, -0.8, 2.0)
Y_crisp <- round(X_crisp %*% beta + rnorm(100, mean = 0, sd = 1))
X_fuzzy <- fuzzify_crisp_matrix(X_crisp, spread = 1)
Y_fuzzy <- fuzzify_crisp_vector(Y_crisp, spread = 1)
object <- fuzzy_lm(X_fuzzy, Y_fuzzy, p = 3)

# Summarize the model
summary(object)
```

# Index

coefficients, [2](#)  
coefficients.fuzzy\_lm, [3](#)  
compute\_ci, [4](#)  
compute\_p\_values, [4](#)  
compute\_pred, [5](#)  
compute\_res, [6](#)  
compute\_t\_values, [6](#)  
  
fuzzify\_crisp\_matrix, [7](#)  
fuzzify\_crisp\_value, [8](#)  
fuzzify\_crisp\_vector, [8](#)  
fuzzy\_add, [9](#)  
fuzzy\_crisp\_mult, [10](#)  
fuzzy\_d\_squared, [10](#)  
fuzzy\_lm, [11](#)  
fuzzy\_mults, [12](#)  
  
plot, [13](#)  
plot.fuzzy\_lm, [14](#)  
predictions, [15](#)  
predictions.fuzzy\_lm, [16](#)  
  
residuals, [16](#)  
residuals.fuzzy\_lm, [17](#)  
  
summary.fuzzy\_lm, [18](#)