Package: exdqlm (via r-universe)

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Title Extended Dynamic Quantile Linear Models

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Description Bayesian quantile-regression routines for dynamic state-space models and static regression under the extended asymmetric Laplace (exAL) error distribution. The dynamic state-space models are extended dynamic quantile linear models (exDQLMs). The package combines dynamic exDQLM inference via LDVB, MCMC, and legacy ISVB with static exAL regression via LDVB and MCMC, reduced AL/DQLM paths through fixed skewness, component builders for trend/seasonality/regression blocks, static shrinkage priors including ridge, regularized horseshoe, and 'rhs_ns', evidence lower bound diagnostics, optional C++ accelerators, and posterior predictive synthesis across separately fitted quantiles through 'quantileSynthesis()'. Dynamic exDQLM methods are described in Barata et al. (2020) <doi:10.1214/21-AOAS1497>.

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Contents

exdqlm-package *exdqlm: Extended Dynamic Quantile Linear Models*

Description

Bayesian quantile-regression tools for dynamic state-space models and static regression under the extended asymmetric Laplace error distribution (exAL).

Details

The package centers on native dynamic quantile state-space modeling for univariate time series and also provides a static exAL regression workflow. Across these settings, `exdqlm` combines model construction helpers, multiple Bayesian inference engines, shrinkage priors for static coefficients, and post hoc synthesis of several fitted quantiles.

Main workflows

- Dynamic/state-space quantile modeling via `exdqlmLDVB()` and `exdqlmMCMC()`, with legacy `exdqlmISVB()` retained for backward compatibility and transfer-function extensions through `exdqlmTransferLDVB()`, `exdqlmTransferMCMC()`, and legacy `exdqlmTransferISVB()`.
- Static Bayesian exAL regression via `exalStaticLDVB()` and `exalStaticMCMC()`, with static fitted-quantile and coefficient summaries through `exalStaticDiagnostics()`.
- Modular state-space construction via `polytrendMod()`, `seasMod()`, and `regMod()`.
- Multi-quantile post-processing via `quantileSynthesis()` for post hoc posterior-predictive synthesis from separately fitted quantiles into a unified predictive distribution.

Distinctive features

- Dynamic Bayesian quantile state-space inference with LDVB as the main VB engine, MCMC for posterior simulation, and legacy ISVB retained for compatibility and historical comparisons.
- A unified package covering both dynamic exDQLM models and static exAL regression.
- Static shrinkage priors including ridge, regularized horseshoe (`"rhs"`), and `rhs_ns`.
- Reduced AL/DQLM paths through `dqlm.ind = TRUE` in both dynamic and static APIs.
- Standardized VB diagnostics traces via `fit$diagnostics$vb_trace` for ELBO, `sigma`, `gamma`, and convergence deltas across VB engines.
- Conservative automatic warmup defaults for the most failure-prone shared blocks: RHS-family `tau` scheduling plus exAL `(sigma, gamma)` warmup in VB and MCMC entry points, with explicit controls available only when users need to override the defaults.
- Optional C++ acceleration for selected state-space computations.

Release changes in 1.0.0

- Dynamic diagnostics report CRPS through a finite integrated quantile-score approximation over posterior predictive empirical quantiles, with user-configurable quantile levels and weights in `exdqlmDiagnostics()`.
- Held-out forecast diagnostics are available for forecast objects through `exdqlmForecastDiagnostics()`.
- Static diagnostics store fitted-quantile summaries and coefficient intervals, with `plot(..., type = "coefficients")` available for comparing static LDVB/MCMC coefficient summaries.
- Dynamic KL normality diagnostics are deterministic for fixed fitted objects and no longer depend on stochastic reference samples. The top-level diagnostic object exposes `KL` as the primary calibration diagnostic and keeps advanced KL sensitivity details under `kl.details`.

Runtime options

- `options(exdqlm.use_cpp_kf = TRUE|FALSE)` – C++ Kalman bridge (optional; default TRUE).
- `options(exdqlm.compute_elbo = TRUE|FALSE)` – Compute ELBO (optional; default TRUE).
- `options(exdqlm.tol_elbo = numeric)` – Positive ELBO convergence tolerance used when `exdqlm.compute_elbo = TRUE`; smaller values enforce stricter ELBO stabilization checks (default `1e-6`).
- `options(exdqlm.use_cpp_builders = TRUE|FALSE)` – C++ model builders (optional; default FALSE).
- `options(exdqlm.use_cpp_samplers = TRUE|FALSE)` – C++ samplers (optional; default FALSE).
- `options(exdqlm.use_cpp_postpred = TRUE|FALSE)` – C++ posterior predictive sampler (optional; default FALSE).
- `options(exdqlm.use_cpp_mcmc = TRUE|FALSE)` – MCMC backend routing (optional; default TRUE).
- `options(exdqlm.cpp_mcmc_mode = "strict"|"fast")` – strict keeps legacy R-kernel parity; fast enables C++ FFBS in MCMC (default "fast").
- `options(exdqlm.cpp_threads = numeric)` – Positive integer thread cap for eligible OpenMP-enabled C++ paths (`1L` forces single-thread; default `1L`).

Author(s)

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See Also

Useful links:

- https://github.com/AntonioAPDL/exdqlm
- Report bugs at https://github.com/AntonioAPDL/exdqlm/issues

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`+.exdqlm` *Addition for* `exdqlm` *objects*

Description

Combines two state space blocks into a single state space model for an exDQLM.

Usage

```
## S3 method for class 'exdqlm'
m1 + m2
```

Arguments

`m1`	object of class "`exdqlm`" containing the first model to be combined.
`m2`	object of class "`exdqlm`" containing the second model to be combined.

Value

An object of class "`exdqlm`" containing the new combined state space model components:

- FF - Observational vector.
- GG - Evolution matrix.
- m0 - Prior mean of the state vector.
- C0 - Prior covariance of the state vector.

Examples

```
trend.comp = polytrendMod(2, rep(0, 2), 10*diag(2))
seas.comp = seasMod(365, c(1,2,4), C0 = 10*diag(6))
model = trend.comp + seas.comp
```

as.exdqlm — exdqlm *objects*

Description

`as.exdqlm` attempts to turn a list into an `exdqlm` object. Works for time-invariant `dlm` objects created using the **dlm** package.

Usage

```
as.exdqlm(m)
```

Arguments

`m`	a list containing named elements m0, C0, FF and GG.

Value

An object of class "`exdqlm`" containing the state space model components:

- FF - Observational vector.
- GG - Evolution matrix.
- m0 - Prior mean of the state vector.
- C0 - Prior covariance of the state vector.

BTflow — *Monthly streamflow at the Big Trees gauge*

Description

Observed monthly mean streamflow, in cubic feet per second, at the Big Trees gauge on the San Lorenzo River in Santa Cruz County, California. The monthly values were obtained by averaging the daily USGS streamflow observations within each calendar month.

Usage

```
BTflow
```

Format

A monthly time series (`ts`) of length 471, spanning January 1987 through March 2026.

Source

U.S. Geological Survey, National Water Information System (USGS Water Data for the Nation), https://waterdata.usgs.gov/.

References

U.S. Geological Survey (2016). National Water Information System data available on the World Wide Web (USGS Water Data for the Nation). https://waterdata.usgs.gov/.

`climateIndices` *Monthly climate indices for streamflow examples*

Description

Monthly atmospheric and oceanic climate indices used as external predictors in the Big Trees streamflow examples. Values are stored on their original scales; examples that combine indices standardize the relevant columns within the analysis code.

Usage

```
climateIndices
```

Format

A data frame with 516 rows and 3 variables:

date First day of the calendar month.

noi Northern Oscillation Index.

amo Atlantic Multidecadal Oscillation index.

The data frame spans January 1980 through December 2022.

Source

Compiled from the NOAA Physical Sciences Laboratory monthly climate indices collection, https://psl.noaa.gov/data/climateindices/.

`compPlot` *Plot a component of an exDQLM*

Description

The function plots the dynamic MAP estimates and 95% credible intervals (CrIs) of a specified component of an exDQLM. Alternatively, if `just.theta=TRUE` the MAP estimates and 95% credible intervals (CrIs) of a single element of the dynamic state vector are plotted.

Usage

```
compPlot(
  m1,
  index,
  add = FALSE,
  col = "purple",
  just.theta = FALSE,
  cr.percent = 0.95,
  plot = TRUE,
  xlim = NULL,
  ylim = NULL,
  xlab = "time",
  ylab = NULL,
  lwd = 1.5,
  lwd.interval = 0.75,
  lty.interval = 2
)
```

Arguments

m1	An object of class "exdqlmLDVB", "exdqlmMCMC", or legacy "exdqlmISVB".
index	Vector of consecutive integers in $\{1, \ldots, q\}$ indicating the component or element of the state vector to be plotted.
add	Logical value indicating whether the dynamic component will be added to existing plot. Default is FALSE.
col	Character vector of length 1 giving color of the dynamic component to be plotted. Default is purple.
just.theta	Logical; if TRUE, the function plots the dynamic distribution of the index element of the state vector. If just.theta=TRUE, index must have length 1.
cr.percent	Numeric in (0, 1) indicating the probability mass for the credible intervals (e.g., 0.95). Default 0.95.
plot	Logical value indicating whether to draw the plot. If FALSE, the function only returns the plotted summaries. Default is TRUE.
xlim, ylim	Optional limits passed to the base plotting call when plot = TRUE.
xlab, ylab	Optional axis labels passed to the base plotting call when plot = TRUE.
lwd, lwd.interval	Line widths for the dynamic component and credible interval bounds, respectively.
lty.interval	Line type for the credible interval bounds.

Value

A list of the following is returned:

- map.comp - MAP estimate of the dynamic component (or element of the state vector).

- lb.comp - Lower bound of the 95% CrIs of the dynamic component (or element of the state vector).
- ub.comp - Upper bound of the 95% CrIs of the dynamic component (or element of the state vector).
- x - Time/index values used for plotting.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:80]
trend.comp = polytrendMod(2, rep(0, 2), 10*diag(2))
seas.comp = seasMod(365, c(1, 2), C0 = 10*diag(4))
model = trend.comp + seas.comp
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98, 1), dim.df = c(2, 4),
                   gam.init = -3.5, sig.init = 15,
                   n.samp = 20, tol = 0.2, verbose = FALSE)
# plot first harmonic component
compPlot(M0, index = c(3, 4), col = "blue")
c.summary = compPlot(M0, index = c(3, 4), plot = FALSE)
options(old)
```

---

dexal *Density Function for the Extended Asymmetric Laplace (exAL) Distribution*

Description

Computes the PDF of the Extended Asymmetric Laplace (exAL) distribution. Vectorized over x.

Usage

```
dexal(x, p0 = 0.5, mu = 0, sigma = 1, gamma = 0, log = FALSE)
```

Arguments

x	Numeric vector of quantiles.
p0	Probability level used in the quantile parametrization. Scalar in (0, 1). Default 0.5.
mu	Location parameter (scalar). Default 0.
sigma	Scale parameter (scalar, strictly positive). Default 1.
gamma	Skewness parameter controlling asymmetry (scalar). Must be within valid bounds implied by p0. Default 0.
log	Logical scalar; if TRUE return log-density. Default FALSE.

Value

Numeric vector of densities (same length as `x`).

Examples

```
dexal(0)
dexal(1, p0 = 0.75, mu = 0, sigma = 2, gamma = 0.25)
dexal(seq(-3, 3, by = 0.1), p0 = 0.3, mu = 0, sigma = 1, gamma = -0.45)
```

ELIanoms *Daily time-series of ELI anomalies.*

Description

ELI anomalies on the daily time-scale from January 1, 1979 to December 31, 2019 with all February 29ths omitted.

Usage

```
ELIanoms
```

Format

A time series of length 14965.

Source

https://portal.nersc.gov/archive/home/projects/cascade/www/ELI

References

Patricola, C.M., O'Brien, J.P., Risser, M.D. et al. *Maximizing ENSO as a source of western US hydroclimate predictability.* Clim Dyn 54, 351–372 (2020). doi:10.1007/s00382019050048

`exal_make_mcmc_control` *Build advanced MCMC control*

Description

Returns a readable `mcmc_control` list for `exalStaticMCMC()` and `exdqlmMCMC()`. This keeps the warmup surface explicit instead of relying on ad hoc nested lists.

Usage

```
exal_make_mcmc_control(
  n_burn = 2000L,
  n_mcmc = 1500L,
  thin = 1L,
  verbose = FALSE,
  progress_every = NULL,
  init_from_vb = TRUE,
  vb_warm_start_control = NULL,
  sigmagam = NULL,
  theta = NULL,
  latent_state = NULL,
  dqlm_sigma = NULL,
  control = NULL
)
```

Arguments

`n_burn`, `n_mcmc`, `thin`, `verbose`	Core MCMC controls.
`progress_every`	Optional progress cadence for callers that support it.
`init_from_vb`	Logical; initialize from a VB warm start.
`vb_warm_start_control`	Optional VB warm-start control list, often from `exal_make_vb_control()`.
`sigmagam`	Optional list, usually from `exal_make_mcmc_sigmagam_control()`.
`theta`	Optional list, usually from `exal_make_mcmc_theta_control()`.
`latent_state`	Optional list, usually from `exal_make_mcmc_latent_state_control()`.
`dqlm_sigma`	Optional list, usually from `exal_make_mcmc_dqlm_sigma_control()`.
`control`	Optional existing control list to update.

Value

A normalized list suitable for `mcmc_control`.

`exal_make_mcmc_dqlm_sigma_control` *Build DQLM sigma-only MCMC warmup control*

Description

Returns a normalized `mcmc_control$dqlm_sigma` block for `exdqlmMCMC()` in the reduced AL / DQLM branch.

Usage

```
exal_make_mcmc_dqlm_sigma_control(
  freeze_burnin_iters = 0L,
  freeze_only_during_burn = TRUE,
  force_after_warmup = TRUE,
  trace = TRUE
)
```

Arguments

`freeze_burnin_iters`	Non-negative integer; number of burn-in iterations to hold the sigma-only block fixed.
`freeze_only_during_burn`	Logical; if `TRUE`, hard freeze only applies during burn-in.
`force_after_warmup`	Logical; force one immediate post-warmup update.
`trace`	Logical; record diagnostics traces.

Value

A normalized list suitable for `mcmc_control$dqlm_sigma`.

`exal_make_mcmc_latent_state_control` *Build dynamic MCMC latent-state warmup control*

Description

Returns a normalized `mcmc_control$latent_state` block for `exdqlmMCMC()`.

Usage

```
exal_make_mcmc_latent_state_control(
  mode = c("u_only", "u_st_pair"),
  freeze_burnin_iters = 0L,
  freeze_only_during_burn = TRUE,
  force_after_warmup = TRUE,
  min_postwarmup_updates = 0L,
  trace = TRUE
)
```

Arguments

mode	One of "u_only" or "u_st_pair".
freeze_burnin_iters	Non-negative integer; number of burn-in iterations to hold the latent-state block fixed.
freeze_only_during_burn	Logical; if TRUE, hard freeze only applies during burn-in.
force_after_warmup	Logical; force one immediate post-warmup update.
min_postwarmup_updates	Non-negative integer; minimum number of post-warmup updates required before chain-health style gates can fire.
trace	Logical; record diagnostics traces.

Value

A normalized list suitable for mcmc_control$latent_state.

---

exal_make_mcmc_sigmagam_control

Build MCMC sigmagam warmup control

Description

Returns a normalized mcmc_control$sigmagam block for exalStaticMCMC() and exdqlmMCMC().

Usage

```
exal_make_mcmc_sigmagam_control(
  freeze_burnin_iters = NULL,
  freeze_only_during_burn = NULL,
  force_after_warmup = NULL,
  delay_adapt_until_after_warmup = NULL,
  delay_laplace_refresh_until_after_warmup = NULL
)
```

Arguments

freeze_burnin_iters
: Non-negative integer; number of burn-in iterations to hold the (sigma, gamma) block fixed.

freeze_only_during_burn
: Logical; if TRUE, hard freeze only applies during burn-in.

force_after_warmup
: Logical; force one post-warmup update.

delay_adapt_until_after_warmup
: Logical; keep proposal adaptation off until warmup ends.

delay_laplace_refresh_until_after_warmup
: Logical; keep Laplace refresh off until warmup ends.

Value

A normalized list suitable for mcmc_control$sigmagam.

When called with no arguments, this returns the package's conservative default exAL (sigma, gamma) MCMC warmup profile.

---

exal_make_mcmc_theta_control

Build MCMC theta warmup control

Description

Returns a normalized mcmc_control$theta block for exdqlmMCMC().

Usage

```
exal_make_mcmc_theta_control(
  enabled = FALSE,
  freeze_burnin_iters = 0L,
  freeze_only_during_burn = TRUE,
  sparse_update_every = 1L,
  sparse_update_until_iter = 0L,
  force_first_postwarmup_update = TRUE,
  trace = TRUE
)
```

Arguments

enabled
: Logical; explicit on/off switch.

freeze_burnin_iters
: Non-negative integer; number of burn-in iterations to hold the theta block fixed.

freeze_only_during_burn
: Logical; if TRUE, hard freeze only applies during burn-in.

`sparse_update_every`
: Positive integer; sparse-update period during the warmup window.

`sparse_update_until_iter`
: Non-negative integer; last iteration where the sparse schedule is active.

`force_first_postwarmup_update`
: Logical; force one update immediately after the hard freeze / sparse schedule ends.

`trace`
: Logical; record diagnostics traces.

Value

A normalized list suitable for `mcmc_control$theta`.

`exal_make_vb_control` *Build advanced VB control*

Description

Returns a readable `vb_control` list for `exalStaticLDVB()` and `exdqlmLDVB()`. This keeps the warmup surface explicit instead of relying on ad hoc nested lists.

Usage

```
exal_make_vb_control(
  max_iter = 150L,
  tol = 1e-04,
  n_samp_xi = 200L,
  verbose = FALSE,
  sigmagam = NULL,
  sts = NULL,
  control = NULL
)
```

Arguments

`max_iter`, `tol`, `n_samp_xi`, `verbose`
: Core VB controls.

`sigmagam`
: Optional list, usually from `exal_make_vb_sigmagam_control()`.

`sts`
: Optional list, usually from `exal_make_vb_sts_control()`, for the dynamic latent `s_t` block in `exdqlmLDVB()`.

`control`
: Optional existing control list to update.

Value

A normalized list suitable for `vb_control`.

exal_make_vb_sigmagam_control *Build VB sigmagam warmup control*

Description

Returns a normalized `sigmagam` block for `vb_control` lists used by `exalStaticLDVB()`, `exdqlmLDVB()`, and VB warm-start paths in `exalStaticMCMC()` and `exdqlmMCMC()`.

Usage

```
exal_make_vb_sigmagam_control(
  freeze_warmup_iters = NULL,
  force_after_warmup = NULL,
  postwarmup_damping = NULL,
  postwarmup_damping_iters = NULL,
  min_postwarmup_updates = NULL
)
```

Arguments

`freeze_warmup_iters`
: Non-negative integer; number of early VB iterations during which the `(sigma, gamma)` block is held fixed.

`force_after_warmup`
: Logical; force one immediate post-warmup update.

`postwarmup_damping`
: Numeric in `(0, 1]`; damping applied after warmup.

`postwarmup_damping_iters`
: Non-negative integer; number of damped post-warmup iterations.

`min_postwarmup_updates`
: Non-negative integer; minimum number of post-warmup updates required before convergence-style gates can fire.

Value

A normalized list suitable for `vb_control$sigmagam`.

When called with no arguments, this returns the package's conservative default exAL `(sigma, gamma)` warmup profile.

exal_make_vb_sts_control *Build dynamic VB latent-state warmup control*

Description

Returns a normalized `sts` block for `vb_control` lists used by `exdqlmLDVB()`.

Usage

```
exal_make_vb_sts_control(
  freeze_warmup_iters = 0L,
  force_after_warmup = TRUE,
  min_postwarmup_updates = 0L
)
```

Arguments

`freeze_warmup_iters`	Non-negative integer; number of early VB iterations during which the latent `s_t` block is held fixed.
`force_after_warmup`	Logical; force one immediate post-warmup update.
`min_postwarmup_updates`	Non-negative integer; minimum number of post-warmup updates required before convergence-style gates can fire.

Value

A normalized list suitable for `vb_control$sts`.

exalStaticDiagnostics *exAL Diagnostics*

Description

Static diagnostics companion for `exalStaticLDVB()` and `exalStaticMCMC()`. The function summarizes fitted quantiles on a shared design matrix, reports mean check loss against observed responses when available, and can optionally compare the fitted quantile curve against a known reference quantile function. The returned diagnostic object also stores posterior summaries for the static regression coefficients, which can be plotted with `plot(..., type = "coefficients")`.

Usage

```
exalStaticDiagnostics(
  m1,
  m2 = NULL,
  X = NULL,
  y = NULL,
  ref = NULL,
  plot = TRUE,
  cols = c("red", "blue"),
  cr.percent = 0.95
)
```

Arguments

m1	An object of class "exalStaticLDVB" or "exalStaticMCMC".
m2	Optional second fitted static model to compare against m1.
X	Optional design matrix. If omitted, the function uses m1$X when available.
y	Optional response vector. If omitted, the function uses m1$y when available.
ref	Optional reference quantile vector on the same rows as X.
plot	Logical; if TRUE, produce a compact static-diagnostics plot.
cols	Character vector of length 1 or 2 giving colors for plotted diagnostics.
cr.percent	Credible-interval mass used when summarizing fitted quantiles.

Details

Unlike exdqlmDiagnostics, which is built around one-step-ahead dynamic forecast diagnostics, exalStaticDiagnostics() is designed for the static regression setting. It reports fitted quantile summaries on a common design matrix, optional mean check loss against observed responses, optional reference-curve errors, coefficient posterior summaries, and compact comparison plots. The ref argument is a reference conditional quantile evaluated on the rows of X; it is distinct from the optional beta.ref argument of plot.exalStaticDiagnostic, which is used only to overlay known coefficient values in simulation examples.

Value

An object of class "exalStaticDiagnostic" containing fitted-quantile summaries, residual summaries (when y is provided), optional reference-curve error metrics, coefficient posterior summaries, and run-time metadata for m1 and m2 (if supplied).

Examples

```
set.seed(1)
x <- seq(-2, 2, length.out = 60)
X <- cbind(1, x)
y <- 0.5 * x + (1.2 + 0.35 * x) * stats::rnorm(length(x))
q_true <- 0.5 * x + (1.2 + 0.35 * x) * stats::qnorm(0.25)
```

```
fit_ldvb <- exalStaticLDVB(
  y = y, X = X, p0 = 0.25,
  max_iter = 60, tol = 1e-3,
  verbose = FALSE
)
fit_mcmc <- exalStaticMCMC(
  y = y, X = X, p0 = 0.25,
  n.burn = 60, n.mcmc = 60,
  mh.proposal = "slice",
  verbose = FALSE
)
out <- exalStaticDiagnostics(fit_ldvb, fit_mcmc, ref = q_true, plot = FALSE)
print(out)
plot(out, type = "coefficients")
```

`exalStaticLDVB` *Static exAL Regression - LDVB Approximation*

Description

The function applies a mean-field variational approximation to static Extended Asymmetric Laplace (exAL) regression. Closed-form block updates are combined with a Laplace-Delta approximation for the joint (σ, γ) block, yielding the package's static LDVB routine.

Usage

```
exalStaticLDVB(
  y,
  X,
  p0,
  max_iter = 1000,
  tol = 1e-04,
  b0 = NULL,
  V0 = NULL,
  beta_prior = c("ridge", "rhs", "rhs_ns"),
  beta_prior_controls = NULL,
  a_sigma = 1,
  b_sigma = 1,
  gamma_bounds = c(L.fn(p0), U.fn(p0)),
  log_prior_gamma = NULL,
  init = NULL,
  dqlm.ind = FALSE,
  al.ind = NULL,
  n.samp = 200,
  n_samp_xi = 200,
  ld_controls = NULL,
  vb_control = NULL,
```

```
  verbose = TRUE
)
```

Arguments

y	Numeric vector (length n).
X	Numeric matrix (n x p).
p0	Target quantile in (0,1).
max_iter	Integer; maximum LDVB iterations (default 1000).
tol	Numeric; convergence tolerance based on relative ELBO changes (default 1e-4).
b0, V0	Prior mean and covariance for $\beta \sim \mathcal{N}(b_0, V_0)$.
beta_prior	Coefficient prior type: "ridge" (default), "rhs" (regularized horseshoe), or "rhs_ns" (Nishimura-Suchard regularized horseshoe with a closed-form inverse-gamma hierarchy for static inference).
beta_prior_controls	Optional list of prior-specific controls. For RHS-family priors (that is, when beta_prior is "rhs" or "rhs_ns"), supported keys include: tau0, nu, s or s2, shrink_intercept, intercept_prec, n_inner, eta_bounds, freeze_tau_iters, freeze_tau_warmup_iters, update_every, update_every_warmup, update_every_warmup_iters, force_tau_after_warmup, collapse_tau_ratio_tol, collapse_beta_max_abs_tol, collapse_invV_med_tol, collapse_beta_l2_tol, collapse_small_beta_frac_tol, small_beta_abs_tol, warn_on_collapse, var_floor, h_curv, verbose, init_lambda, init_log_lambda, init_tau, init_log_tau, init_c2, and init_log_c2. For beta_prior = "rhs_ns", optional slab controls a_zeta, b_zeta, and zeta2_fixed are also supported. In this mode, the local-global-slab block is represented by $(\lambda_j^2, \nu_j, \tau^2, \xi, \zeta^2)$, with closed-form inverse-gamma updates in VB and MCMC. When beta_prior is "rhs" or "rhs_ns", b0 and V0 are retained only for backward-compatible ridge behavior and are ignored for the shrunk coefficients. If both init_log_tau and init_tau are omitted (or NULL), the RHS global scale initializes at tau = 1 (init_log_tau = 0) instead of tau0. By default (shrink_intercept = FALSE), the intercept is excluded from horseshoe shrinkage and uses intercept_prec.
a_sigma, b_sigma	Prior for $\sigma \sim IG(a_\sigma, b_\sigma)$ with density $p(\sigma) \propto \sigma^{-(a_\sigma+1)} e^{-b_\sigma/\sigma}$.
gamma_bounds	Two-vector (L, U) support for gamma. Defaults to c(L.fn(p0), U.fn(p0)).
log_prior_gamma	Function g -> log pi(gamma=g). Default is a truncated Student-t prior centered at 0 on the admissible gamma support.
init	Optional list with starting values: beta, sigma, gamma; if missing, reasonable defaults are used.
dqlm.ind	Logical; if TRUE, fit the reduced AL model (gamma = 0). In that special case the nonconjugate block drops out and the remaining variational updates are available in closed form. This argument is retained for consistency with the dynamic exDQLM API; in static examples, al.ind = TRUE is the clearer spelling for this AL special case.

al.ind	Optional static-model alias for dqlm.ind. Prefer this argument when requesting the static AL special case. When supplied, this flag maps directly to dqlm.ind. If both are given, they must agree.
n.samp	Number of samples to draw from the approximated posterior distribution after convergence. Default is n.samp = 200.
n_samp_xi	Integer; retained for backward compatibility in the Laplace-Delta block. Under the current delta-only implementation this value is ignored.
ld_controls	Optional list of controls for the Laplace-Delta block. Supported keys include optimizer_method ("lbfgsb" or "bfgs"), direct_commit, damping, xi_damping, optimizer_maxit, eig_floor, eig_cap, step_cap_eta, step_cap_ell, eta_lo, eta_hi, sigma_bounds, sigma_init_mode, gamma_init_mode, sigma_floor_abs, sigma_min_mult, sigma_max_mult, sigma_bound_ratio_min, gamma_init_pad_frac, logit_eps, init_cov_diag, reuse_seed, mode_grad_tol, mode_min_eig, auto_stabilize, cycle_window, cycle_lag1_max, cycle_lag2_min, cycle_gamma_min_amp, cycle_sigma_min_amp, cycle_s_min_amp, cycle_tau2_min_amp, stabilize_damping, stabilize_xi_damping, stabilize_step_cap_eta, stabilize_step_cap_ell, and store_trace.
vb_control	Optional normalized VB control list, usually from exal_make_vb_control(). When supplied, the core VB arguments and warmup blocks are read from vb_control first and then merged with the explicit function arguments. When omitted, the package applies its conservative default warmup profile for exAL (sigma, gamma) updates while retaining the built-in RHS-family tau warmup defaults.
verbose	Logical; print progress.

Details

Mean-field factorization:

$$q(\beta)\ \prod_{i=1}^{n} q(v_i)\ q(s_i)\ q(\sigma,\gamma).$$

This factorization induces a blockwise coordinate-ascent variational inference scheme. The (σ,γ) block is the only nonconjugate component, so LDVB approximates it in transformed coordinates $\eta = \mathrm{logit}((\gamma - L)/(U - L))$ and $\ell = \log\sigma$. The xi expectations needed by the remaining block updates are then computed with a second-order Delta approximation. The xi expectations used in the updates can be computed either from a deterministic second-order Delta approximation in (η,ℓ) or from a Gaussian Monte Carlo sample. The Laplace-Delta controls also allow bounded optimization in the transformed (η,ℓ) block to better mimic the stabilized RHS-family readout implementation. For RHS-family priors, the prior block uses tau warmup/freeze scheduling to avoid the early-collapse regime where global shrinkage drives all slope coefficients toward zero before the likelihood-side variational factors stabilize.

Value

An object of class "exalStaticLDVB" containing:

- qbeta: list with m, V.
- samp.beta: posterior sample from $q(\beta)$ with n.samp rows.
- qv: list with chi (length n), psi (scalar), E_v and E_inv_v (moments).

- qs: list with mu (length n), tau2 (length n), E_s, E_s2.
- qsiggam: list with eta_hat, ell_hat, Sigma (2x2), approximate means gamma_mean, sigma_mean, and the xi expectations.
- samp.sigma, samp.gamma: posterior samples from the variational approximation for the scale and skewness parameters. In the AL special case (dqlm.ind = TRUE), samp.gamma is a vector of zeros.
- converged, iter, run.time, and misc (including p0, bounds L,U, dimensions, and ELBO trace).
- beta_prior: prior metadata and, for RHS-family priors, posterior summaries of the shrinkage hyperparameters, including warmup/freeze-aware tau summaries and collapse diagnostics (collapse_flag, tau_near_zero, beta_collapse, and warning when triggered). For "rhs_ns", the state also tracks lambda2, nu, tau2, xi, and zeta2 with the corresponding inverse moments.
- diagnostics: ELBO and joint-convergence diagnostics including a standardized VB iteration trace at diagnostics$vb_trace (iteration-wise ELBO / sigma / gamma / convergence deltas), state/sigma/gamma/ELBO deltas, stopping reason, and Laplace-Delta block trace diagnostics, including replicated-xi controls, automatic stabilization / cycle-detection fields, and final local-mode quality checks. For RHS fits this also includes diagnostics$rhs with the resolved preflight configuration and collapse diagnostics.

Examples

```
set.seed(123)
n <- 60
X <- cbind(1, seq(-1, 1, length.out = n))
y <- as.numeric(X %*% c(0.2, -0.1) + rnorm(n, sd = 0.15))
fit <- exalStaticLDVB(y = y, X = X, p0 = 0.5, max_iter = 60, tol = 1e-3, verbose = FALSE)
fit$converged
head(fit$diagnostics$vb_trace)

fit_rhs <- exalStaticLDVB(
  y = y, X = X, p0 = 0.5,
  beta_prior = "rhs",
  beta_prior_controls = list(tau0 = 0.5, nu = 3, s2 = 1, shrink_intercept = FALSE),
  max_iter = 50, tol = 5e-3, verbose = FALSE
)
fit_rhs$beta_prior$type

fit_rhs_ns <- exalStaticLDVB(
  y = y, X = X, p0 = 0.5,
  beta_prior = "rhs_ns",
  beta_prior_controls = list(tau0 = 0.5, a_zeta = 1.5, b_zeta = 1, zeta2_fixed = 1),
  max_iter = 50, tol = 5e-3, verbose = FALSE
)
fit_rhs_ns$beta_prior$type

fit_al <- exalStaticLDVB(
  y = y, X = X, p0 = 0.5,
  al.ind = TRUE,
  max_iter = 50, tol = 5e-3, verbose = FALSE
```

```
)
fit_al$dqlm.ind
```

`exalStaticMCMC` *exAL (static) - MCMC algorithm*

Description

The function applies a Gibbs sampler for static Extended Asymmetric Laplace regression (exAL). We update β, v, s from their full conditionals, then update (σ, γ) either jointly on transformed coordinates $(\ell, \eta) = (\log\sigma, \mathrm{logit}((\gamma - L)/(U - L)))$ (for `"rw"` and `"laplace_rw"`) or with univariate gamma kernels (`"slice"`, `"slice_eta"`, `"laplace_local"`). Optional multi-refresh and global-jump controls are available to improve exAL mixing in hard cases.

Usage

```
exalStaticMCMC(
  y,
  X,
  p0,
  b0 = NULL,
  V0 = NULL,
  beta_prior = c("ridge", "rhs", "rhs_ns"),
  beta_prior_controls = NULL,
  a_sigma = 1,
  b_sigma = 1,
  gamma_bounds = c(L.fn(p0), U.fn(p0)),
  log_prior_gamma = NULL,
  init = NULL,
  dqlm.ind = FALSE,
  al.ind = NULL,
  n.burn = 2000,
  n.mcmc = 1500,
  thin = 1,
  init.from.vb = FALSE,
  vb_init_controls = NULL,
  vb_init_fit = NULL,
  mcmc_control = NULL,
  sigmagam_controls = NULL,
  mh.proposal = c("slice", "laplace_rw", "rw", "slice_eta", "laplace_local"),
  mh.adapt = TRUE,
  mh.adapt.interval = 50L,
  mh.target.accept = c(0.2, 0.45),
  mh.scale.bounds = c(0.1, 10),
  mh.max_scale.step = 0.35,
  mh.min_burn_adapt = 50L,
```

```
  slice.width = 0.1,
  slice.max.steps = Inf,
  gamma.substeps = 1L,
  p.global.eta.jump = 0,
  global.eta.jump.scale = 1,
  trace.diagnostics = TRUE,
  trace.every = 1L,
  verbose = TRUE,
  progress_callback = NULL
)
```

Arguments

y	Numeric vector of length n.
X	Numeric matrix $n \times p$ (design).
p0	Quantile level in $(0, 1)$.
b0, V0	Prior mean and covariance for β (Normal). Defaults: $b_0 = \mathbf{0}_p$, $V_0 = 10^6 I_p$.
beta_prior	Coefficient prior type: "ridge" (default), "rhs" (regularized horseshoe), or "rhs_ns" (Nishimura-Suchard regularized horseshoe with a closed-form inverse-gamma hierarchy for static inference).
beta_prior_controls	Optional list of prior-specific controls. For RHS-family priors (that is, when beta_prior is "rhs" or "rhs_ns"), supported keys include: tau0, nu, s or s2, shrink_intercept, intercept_prec, n_inner, eta_bounds, var_floor, h_curv, verbose, init_lambda, init_log_lambda, init_tau, init_log_tau, init_c2, init_log_c2, collapse_tau_ratio_tol, collapse_beta_max_abs_tol, collapse_invV_med_tol, collapse_beta_l2_tol, collapse_small_beta_frac_tol, small_beta_abs_tol, slice_width, and slice_max_steps. For beta_prior = "rhs_ns", optional slab controls a_zeta, b_zeta, and zeta2_fixed are also supported. In this mode, the local-global-slab block is represented by $(\lambda_j^2, \nu_j, \tau^2, \xi, \zeta^2)$ and updated with closed-form Gibbs steps. When beta_prior is "rhs" or "rhs_ns", b0 and V0 are ignored for the shrunk coefficients and retained only for backward-compatible ridge behavior. If both init_log_tau and init_tau are omitted (or NULL), the RHS global scale initializes at tau = 1 (init_log_tau = 0) instead of tau0. By default (shrink_intercept = FALSE), the intercept is excluded from horseshoe shrinkage and uses intercept_prec.
a_sigma, b_sigma	Hyperparameters for an inverse-gamma prior on sigma, with density proportional to sigma^{-(a_sigma+1)} exp(-b_sigma/sigma).
gamma_bounds	Numeric length-2 vector (L, U) for gamma. Defaults to c(L.fn(p0), U.fn(p0)).
log_prior_gamma	Function function(g) log pi(g) for gamma on (L, U). Default is a truncated Student-t prior centered at 0 on the admissible support.
init	Optional list with starting values: beta, sigma, gamma, v (length n), s (length n), and for RHS-family priors optionally lambda, tau, and c2. For beta_prior = "rhs_ns", the squared-scale parameterization lambda2, tau2, zeta2, and optional auxiliaries nu, xi are also accepted. Missing pieces are filled sensibly.

`dqlm.ind` Logical; if `TRUE`, fit the reduced AL model (`gamma = 0`), corresponding to Bayesian linear quantile regression under the AL working likelihood. This removes the `gamma`- and `s`-blocks and leaves conjugate Gibbs updates for `beta`, `sigma`, and `v`. This argument is retained for consistency with the dynamic exDQLM API; in static examples, `al.ind = TRUE` is the clearer spelling for this AL special case.

`al.ind` Optional static-model alias for `dqlm.ind`. Prefer this argument when requesting the static AL special case. When supplied, this flag maps directly to `dqlm.ind`. If both are given, they must agree.

`n.burn` Number of burn-in iterations. Default `2000`.

`n.mcmc` Number of kept MCMC iterations (after burn). Default `1500`.

`thin` Integer; save every `thin`-th iteration after burn. We internally run `n.burn + n.mcmc * thin` iterations to return exactly `n.mcmc` saved draws.

`init.from.vb` Logical; if `TRUE`, run static VB first and use its posterior moments as MCMC initialization.

`vb_init_controls`
Optional list controlling VB warm start. Supported keys: `max_iter`, `tol`, `n_samp_xi`, `verbose`, and `ld_controls` (passed through to `exalStaticLDVB()`).

`vb_init_fit` Optional precomputed static VB fit object used as the warm-start reference when `init.from.vb = TRUE`.

`mcmc_control` Optional normalized MCMC control list, usually from `exal_make_mcmc_control()`. When supplied, the core MCMC arguments and warmup blocks are read from `mcmc_control` first and then merged with the explicit function arguments. When omitted, the package applies its conservative default exAL `(sigma, gamma)` warmup profile and keeps the RHS-family `tau` warmup defaults active through the shared prior layer.

`sigmagam_controls`
Optional list controlling warmup/freeze behavior for the nonconjugate `(sigma, gamma)` block. Prefer `exal_make_mcmc_sigmagam_control()` or the `sigmagam` block of `exal_make_mcmc_control()` for new code; this argument remains available as a direct advanced override.

`mh.proposal` Character string controlling the exAL nonconjugate update kernel. `"slice"` (default) uses an exact bounded univariate slice sampler on `gamma` (with `sigma` updated from its conditional), and `"slice_eta"` does the same on transformed `eta`. `"laplace_rw"` uses a Laplace-informed adaptive random-walk MH update on the transformed joint block $(\eta, \ell) = (\mathrm{logit}((\gamma - L)/(U - L)), \log\sigma)$. `"rw"` uses the same exact joint update with identity base covariance. `"laplace_local"` reproduces the prior approximate local-Gaussian gamma draw retained for compatibility and not recommended for routine use. Only `"laplace_local"` is approximate.

`mh.adapt` Logical; adapt the random-walk proposal scale during burn-in for `"rw"` and `"laplace_rw"`. Ignored for `"laplace_local"`, `"slice"`, and `"slice_eta"`.

`mh.adapt.interval`
Integer adaptation window for RW-based kernels.

`mh.target.accept`
Numeric length-2 target acceptance band.

`mh.scale.bounds`
: Numeric length-2 lower/upper bounds for RW proposal scale multiplier.

`mh.max_scale.step`
: Numeric multiplicative adaptation cap in `(0,1)`.

`mh.min_burn_adapt`
: Integer minimum burn-in before adaptation starts.

`slice.width`
: Positive numeric width for bounded slice updates when `mh.proposal = "slice"` or `"slice_eta"`.

`slice.max.steps`
: Positive integer or `Inf`; maximum stepping-out expansions for the slice sampler.

`gamma.substeps`
: Positive integer; number of consecutive gamma-kernel refreshes per outer MCMC iteration. Default `1`.

`p.global.eta.jump`
: Numeric in `[0,1]`; per-substep probability of proposing a global independence-MH move on eta (the logit transform of gamma) using a Laplace proposal with MH correction. Default `0`.

`global.eta.jump.scale`
: Positive numeric scale multiplier applied to the Laplace proposal SD used in global eta jumps.

`trace.diagnostics`
: Logical; if `TRUE`, retain per-iteration gamma/s diagnostics under `mh.diagnostics$trace`. Set `FALSE` for lighter-weight runs.

`trace.every`
: Positive integer; when `trace.diagnostics=TRUE`, record one diagnostics row every `trace.every` iterations.

`verbose`
: Print progress every `progress_every` iterations.

`progress_callback`
: Optional callback invoked with a named list at MCMC start, at each progress checkpoint, and on completion. Intended for workflow-level per-case progress logging.

Value

An object of class `"exalStaticMCMC"` containing:

- `run.time` - total wall time in seconds.
- `X`, `p0`, `bounds` - design, quantile, and (L, U).
- `samp.beta` - posterior sample of `beta` as `coda::mcmc` (n.mcmc x p).
- `samp.sigma` - posterior sample of `sigma` as `coda::mcmc`.
- `samp.gamma` - posterior sample of `gamma` as `coda::mcmc`.
- `samp.v` - latent `v` draws as `coda::mcmc` (`n.mcmc` x `n`).
- `samp.s` - latent `s` draws as `coda::mcmc` (`n.mcmc` x `n`).
- `samp.lambda`, `samp.tau`, `samp.c2` - RHS latent draws when an RHS-family prior is used; otherwise `NULL`.
- `beta_prior` - prior metadata and, for RHS-family priors, posterior summaries of the shrinkage block. For `"rhs_ns"`, the state tracks `lambda2`, `nu`, `tau2`, `xi`, and `zeta2`.

- `mh.diagnostics` - proposal kernel diagnostics for the exAL gamma update, including whether the saved kernel is exact/signoff-ready.
- `rhs.diagnostics` - RHS latent summaries and optional trace metadata when an RHS-family prior is used, including the resolved preflight configuration used at fit start.
- `last` - last state of the chain (useful for restarts).

Examples

```
set.seed(123)
n <- 60; p <- 3
X <- cbind(1, rnorm(n), rnorm(n))
beta0 <- c(0.5, -1, 0.8); sigma0 <- 1.2
y <- as.numeric(X %*% beta0 + rnorm(n, 0, sigma0))
fit <- exalStaticMCMC(
  y, X, p0 = 0.5, n.burn = 60, n.mcmc = 60, thin = 1, verbose = FALSE
)
summary(fit$samp.beta)

fit_rhs <- exalStaticMCMC(
  y, X, p0 = 0.5,
  beta_prior = "rhs",
  beta_prior_controls = list(tau0 = 0.5, nu = 3, s2 = 1, shrink_intercept = FALSE),
  n.burn = 50, n.mcmc = 50, thin = 1, mh.proposal = "slice", verbose = FALSE
)
fit_rhs$beta_prior$type

fit_rhs_ns <- exalStaticMCMC(
  y, X, p0 = 0.5,
  beta_prior = "rhs_ns",
  beta_prior_controls = list(tau0 = 0.5, a_zeta = 1.5, b_zeta = 1, zeta2_fixed = 1),
  n.burn = 40, n.mcmc = 40, thin = 1, mh.proposal = "slice", verbose = FALSE
)
fit_rhs_ns$beta_prior$type

fit_al <- exalStaticMCMC(
  y, X, p0 = 0.5,
  al.ind = TRUE,
  n.burn = 50, n.mcmc = 50, thin = 1, verbose = FALSE
)
fit_al$dqlm.ind
```

`exdqlmDiagnostics` *exDQLM Diagnostics*

Description

The function computes the following for the model(s) provided: the posterior predictive loss criterion based off the check loss, the CRPS approximated as a finite integrated quantile score over

posterior predictive empirical quantiles, the one-step-ahead distribution sequence, and deterministic semiclosed KL normality diagnostics for the MAP standardized forecast errors. The function also plots the following: the qq-plot and ACF plot corresponding to the one-step-ahead distribution sequence, and a time series plot of the MAP standard forecast errors.

Usage

```
exdqlmDiagnostics(
  m1,
  m2 = NULL,
  plot = TRUE,
  cols = c("red", "blue"),
  ref = NULL,
  crps_probs = seq(0.01, 0.99, by = 0.01),
  crps_weights = NULL,
  kl_k = NULL
)
```

Arguments

m1	An object of class "exdqlmLDVB", "exdqlmMCMC", or legacy "exdqlmISVB".
m2	An optional additional object of class "exdqlmLDVB", "exdqlmMCMC", or legacy "exdqlmISVB" to compare with m1.
plot	Logical value indicating whether the following will be plotted for m1 and m2 (if provided): a qq-plot and ACF plot of the MAP one-step-ahead distribution sequence, and a time series plot of the standardized forecast errors. Default is TRUE.
cols	Character vector of length 1 or 2 giving color(s) used to plot diagnostics. Default c("red","blue").
ref	Optional finite reference sample of size length(m1$y) from a standard normal distribution. Used for the reversed KL diagnostic. When NULL, a deterministic standard-normal quantile grid is used.
crps_probs	Numeric vector of quantile levels used to approximate CRPS through the integrated quantile-score identity. Values must be strictly between 0 and 1. Default is seq(0.01, 0.99, by = 0.01).
crps_weights	Optional non-negative numeric weights for crps_probs. When NULL, equal weights are used. When provided, weights are normalized to sum to 1.
kl_k	Optional positive integer vector of nearest-neighbor values used for the KL entropy and cross-entropy estimates. When NULL, the default grid c(3, 5, 10, 20, 30) is filtered to values supported by the finite standardized-error sample size, falling back to 1 for very small samples.

Details

The primary KL summary is computed from the MAP standardized one-step-ahead forecast errors map.standard.forecast.errors. The reported KL value is the user-facing calibration diagnostic and estimates $KL(P_e||N(0,1))$, where P_e is the continuous diagnostic-error law represented by the

standardized errors. It uses the semiclosed identity $KL(P_e||N(0,1)) = CE(P_e, N(0,1)) - H(P_e)$, with the normal cross-entropy term evaluated analytically and the entropy estimated by a one-dimensional k-nearest-neighbor estimator. The reported `KL.flip` estimates the reversed diagnostic $KL(N(0,1)||P_e)$ using kNN cross-entropy. The reversed direction is more sensitive and should be read as a secondary sensitivity diagnostic, not as a replacement for `KL`. Advanced by-k sensitivity tables and Gaussian plug-in checks are stored under `kl.details` so the top-level diagnostic object exposes a single primary KL value. Negative finite-sample estimates are not clamped; they indicate estimator bias or instability for the current sample.

Value

An object of class "`exdqlmDiagnostic`" containing the following:

- `m1.uts` - The one-step-ahead distribution sequence of `m1`.
- `m1.KL` - The forward KL normality diagnostic `KL(P_error || N(0,1))` for the MAP standardized forecast errors.
- `m1.KL.flip` - The reversed ("flipped") KL diagnostic `KL(N(0,1) || P_error)` for the MAP standardized forecast errors; this is a secondary sensitivity diagnostic.
- `m1.CRPS` - The mean CRPS approximated by a finite integrated quantile score over posterior predictive empirical quantiles.
- `m1.pplc` - The posterior predictive loss criterion of `m1` based off the check loss function.
- `m1.qq` - The ordered pairs of the qq-plot comparing `m1.uts` with a standard normal distribution.
- `m1.acf` - The autocorrelations of `m1.uts` by lag.
- `m1.rt` - Run-time of the original model `m1` in seconds.
- `m1.msfe` - MAP standardized one-step-ahead forecast errors from the original model `m1`.
- `y` - The original time-series used to fit `m1`.
- `crps.method` - The CRPS approximation method.
- `crps.probs` - The quantile levels used for the CRPS approximation.
- `crps.weights` - The normalized weights used for the CRPS approximation.
- `kl.method`, `kl.k`, `kl.aggregate`, and `kl.reference` - KL estimator metadata.
- `kl.n_finite`, `kl.n_ref`, and `kl.zero_distance_count` - KL diagnostic sample-size and distance-floor metadata.
- `kl.details` - Advanced KL estimator details by model. For each model this includes primary/flipped definitions, by-k sensitivity tables, a Gaussian plug-in check, and estimator metadata.

If `m2` is provided, analogous results for `m2` are also included in the list.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
```

```
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.95), dim.df = c(1),
                  gam.init = -3.5, sig.init = 15,
                  n.samp = 20, tol = 0.2, verbose = FALSE)
M0.diags = exdqlmDiagnostics(M0, plot = FALSE)
options(old)
```

`exdqlmForecast` *k-step-ahead quantile forecasts*

Description

Computes filtered and k-step-ahead forecast quantiles from a fitted dynamic quantile model and optionally adds them to an existing plot.

Usage

```
exdqlmForecast(
  start.t,
  k,
  m1,
  fFF = NULL,
  fGG = NULL,
  plot = TRUE,
  add = FALSE,
  cols = c("purple", "magenta"),
  cr.percent = 0.95,
  return.draws = FALSE,
  n.samp = NULL,
  seed = NULL
)
```

Arguments

`start.t`	Integer index at which forecasts start (must be within the span of the fitted model in `m1`).
`k`	Integer number of steps ahead to forecast.
`m1`	A fitted exDQLM model object, returned by `exdqlmLDVB()`, `exdqlmMCMC()`, or legacy `exdqlmISVB()`.
`fFF`	Optional state vector(s) for the forecast steps. A numeric matrix with q rows and either 1 column (non–time-varying) or k columns (time-varying). Its dimension must match the fitted model in `m1`.
`fGG`	Optional evolution matrix/matrices for the forecast steps. Either a numeric $q \times q$ matrix (non–time-varying) or a $q \times q \times k$ array (time-varying). Its dimensions must match the fitted model in `m1`.

plot	Logical value indicating whether to plot filtered and forecast quantiles with equal–tailed credible intervals. Default is TRUE.
add	Logical value indicating whether to add the forecasted quantiles to the current plot. Default is FALSE.
cols	Character vector of length 2 giving the colors for filtered and forecasted quantiles respectively. Default c("purple","magenta").
cr.percent	Numeric in (0, 1) indicating the probability mass for the credible intervals (e.g., 0.95). Default 0.95.
return.draws	Logical; if TRUE, the function also returns a matrix of posterior predictive forecast draws in samp.fore. Default is FALSE.
n.samp	Optional positive integer specifying how many forecast draws to return when return.draws = TRUE. If omitted, all available posterior (σ, γ) draws from m1 are used.
seed	Optional integer random seed used only for forecast-draw generation when return.draws = TRUE. If provided, the previous R RNG state is restored on exit.

Value

An object of class "exdqlmForecast" containing the following:

- start.t Integer index at which forecasts start (within the span of the fitted model in m1).
- k Integer number of steps ahead forecasted.
- m1 The fitted exDQLM model object used to initialize the forecast.
- cr.percent The probability mass for the credible intervals (e.g., 0.95).
- fa Forecast state mean vectors ($q \times k$ matrix).
- fR Forecast state covariance matrices ($q \times q \times k$ array).
- ff Forecast quantile means (length-k numeric).
- fQ Forecast quantile variances (length-k numeric).
- samp.fore Optional posterior predictive forecast draws (k x n.samp) returned when return.draws = TRUE.

Examples

```
# Toy example
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 20L)
y = scIVTmag[1:100]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15, n.samp = 30,
                verbose = FALSE)
exdqlmForecast(start.t = 90, k = 10, m1 = M0)
M0.forecast = exdqlmForecast(start.t = 90, k = 10, m1 = M0,
                             return.draws = TRUE, n.samp = 50, seed = 123)
dim(M0.forecast$samp.fore)
options(old)
```

exdqlmForecastDiagnostics

Held-out forecast diagnostics for exDQLM forecasts

Description

Computes held-out forecast scores from one or two `exdqlmForecast` objects returned by `exdqlmForecast()`. Unlike `exdqlmDiagnostics()`, which summarizes fitted one-step-ahead forecast errors and their KL normality diagnostics, this function evaluates posterior predictive forecast draws against observations reserved outside the fitted sample.

Usage

```
exdqlmForecastDiagnostics(
  m1,
  m2 = NULL,
  y,
  p0 = NULL,
  crps_probs = seq(0.01, 0.99, by = 0.01),
  crps_weights = NULL
)
```

Arguments

`m1`	An object of class "`exdqlmForecast`", returned by `exdqlmForecast()` with `return.draws = TRUE`.
`m2`	An optional second object of class "`exdqlmForecast`" to compare with `m1`.
`y`	Numeric vector or time series of held-out observations. Its length must equal the forecast horizon.
`p0`	Optional quantile level used for the check-loss calculation. When `NULL`, the value is taken from `m1$m1$p0`. If `m2` is supplied, its fitted quantile level must agree with the resolved value.
`crps_probs`	Numeric vector of quantile levels used to approximate CRPS through the integrated quantile-score identity. Values must be strictly between 0 and 1. Default is `seq(0.01, 0.99, by = 0.01)`.
`crps_weights`	Optional non-negative numeric weights for `crps_probs`. When `NULL`, equal weights are used. When provided, weights are normalized to sum to 1.

Details

The check loss is computed at the target quantile level `p0` using the forecast quantile means `ff` stored in each forecast object. CRPS is computed from `samp.fore` using the same finite integrated quantile-score approximation used by `exdqlmDiagnostics()`. This function does not compute KL diagnostics because KL in **exdqlm** is defined for fitted one-step-ahead MAP standardized forecast errors, not for arbitrary held-out forecast draws.

Value

An object of class "exdqlmForecastDiagnostic" containing:

- y - Held-out observations used for scoring.
- p0 - Quantile level used for check loss.
- horizon - Forecast horizon.
- m1.check_loss - Mean target-quantile check loss for m1.
- m1.CRPS - Mean CRPS approximation for m1.
- m1.pointwise - Pointwise held-out scores for m1.
- crps.method, crps.probs, and crps.weights - CRPS approximation metadata.

If m2 is supplied, analogous m2.* fields are included.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:65]
y_train = y[1:60]
y_holdout = y[61:65]
model = polytrendMod(1, stats::quantile(y_train, 0.85), 10)
M0 = exdqlmLDVB(y_train, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.samp = 20, tol = 0.2, verbose = FALSE)
fFF = model$FF[, 1, drop = FALSE]
fGG = model$GG
M0.forecast = exdqlmForecast(start.t = 60, k = 5, m1 = M0,
                             fFF = fFF, fGG = fGG,
                             return.draws = TRUE, n.samp = 20, seed = 123,
                             plot = FALSE)
exdqlmForecastDiagnostics(M0.forecast, y = y_holdout)
options(old)
```

---

exdqlmISVB *exDQLM - legacy ISVB algorithm*

Description

The function applies an Importance Sampling Variational Bayes (ISVB) algorithm to estimate the posterior of an exDQLM. This legacy VB engine is retained for backward compatibility and historical comparisons; for standard exDQLM VB fits, exdqlmLDVB() is the main default technique.

Usage

```
exdqlmISVB(
  y,
  p0,
  model,
  df,
  dim.df,
  fix.gamma = FALSE,
  gam.init = NA,
  fix.sigma = FALSE,
  sig.init = NA,
  dqlm.ind = FALSE,
  exps0,
  tol = 0.1,
  n.IS = 500,
  n.samp = 200,
  PriorSigma = NULL,
  PriorGamma = NULL,
  verbose = TRUE,
  debug_shapes = FALSE,
  debug_every = 5
)
```

Arguments

y	A univariate time-series.
p0	The quantile of interest, a value between 0 and 1.
model	List of the state-space model including GG, FF, prior parameters m0 and C0.
df	Discount factors for each block.
dim.df	Dimension of each block of discount factors.
fix.gamma	Logical value indicating whether to fix gamma at gam.init. Default is FALSE.
gam.init	Initial value for gamma (skewness parameter), or value at which gamma will be fixed if fix.gamma=TRUE.
fix.sigma	Logical value indicating whether to fix sigma at sig.init. Default is FALSE.
sig.init	Initial value for sigma (scale parameter), or value at which sigma will be fixed if fix.sigma=TRUE.
dqlm.ind	Logical value indicating whether to fix gamma at 0, reducing the exDQLM to the special case of the DQLM. Default is FALSE.
exps0	Initial value for dynamic quantile. If exps0 is not specified, it is set to the DLM estimate of the p0 quantile.
tol	Tolerance for convergence of dynamic quantile estimates. Default is tol=0.1.
n.IS	Number of particles for the importance sampling of joint variational distribution of sigma and gamma. Default is n.IS=500.
n.samp	Number of samples to draw from the approximated posterior distribution. Default is n.samp=200.

PriorSigma	List of parameters for inverse gamma prior on sigma; shape a_sig and scale b_sig. Default is an inverse gamma with mean 1 (or sig.init if provided) and variance 10.
PriorGamma	List of parameters for truncated student-t prior on gamma; center m_gam, scale s_gam and degrees of freedom df_gam. Default is a standard student-t with 1 degree of freedom, truncated to the support of gamma.
verbose	Logical value indicating whether progress should be displayed.
debug_shapes	Logical; if TRUE, print KF input/output shapes every debug_every iterations.
debug_every	Integer; frequency (in iterations) for shape prints when debug_shapes=TRUE.

Details

Advanced options (set via options()):

- exdqlm.use_cpp_kf: use the C++ Kalman filter bridge (default TRUE).
- exdqlm.compute_elbo: compute ELBO every iteration (default TRUE).
- exdqlm.tol_elbo: ELBO convergence tolerance (default 1e-6).
- exdqlm.tol_sigma: sigma-delta convergence tolerance (default: tol).
- exdqlm.tol_gamma: gamma-delta convergence tolerance (default: tol).
- exdqlm.vb.min_iter: minimum iterations before convergence can trigger (default 10).
- exdqlm.vb.patience: number of consecutive joint-converged iterations required (default 3).
- exdqlm.use_cpp_samplers: use C++ samplers for s_t, u_t, theta (default FALSE). The GIG-based u_t sampler always uses the package C++ Devroye implementation; when FALSE, the remaining samplers fall back to R implementations.
- exdqlm.use_cpp_postpred: use C++ posterior predictive sampler (default FALSE).

Value

An object of class "exdqlmISVB" containing the following:

- y - Time-series data used to fit the model.
- run.time - Algorithm run time in seconds.
- iter - Number of iterations until convergence was reached.
- dqlm.ind - Logical value indicating whether gamma was fixed at 0, reducing the exDQLM to the special case of the DQLM.
- model - List of the state-space model including GG, FF, prior parameters m0 and C0.
- p0 - The quantile which was estimated.
- df - Discount factors used for each block.
- dim.df - Dimension used for each block of discount factors.
- sig.init - Initial value for sigma, or value at which sigma was fixed if fix.sigma=TRUE.
- seq.sigma - Sequence of sigma estimated by the algorithm until convergence.
- samp.theta - Posterior sample of the state vector variational distribution.
- samp.post.pred - Sample of the posterior predictive distributions.

- map.standard.forecast.errors - MAP standardized one-step-ahead forecast errors.
- samp.sigma - Posterior sample of scale parameter sigma variational distribution.
- samp.vts - Posterior sample of latent parameters, v_t, variational distributions.
- theta.out - List containing the variational distribution of the state vector including filtered distribution parameters (fm and fC) and smoothed distribution parameters (sm and sC).
- vts.out - List containing the variational distributions of latent parameters v_t.
- fix.sigma Logical value indicating whether sigma was fixed at sig.init.
- diagnostics - List containing ELBO trace, standardized VB iteration trace diagnostics$vb_trace (iteration-wise ELBO / sigma / gamma / convergence deltas), and convergence diagnostics (joint stopping status, deltas for state/sigma/gamma/ELBO, and criteria used).

If dqlm.ind=FALSE, the object also contains:

- gam.init - Initial value for gamma, or value at which gamma was fixed if fix.gamma=TRUE.
- seq.gamma - Sequence of gamma estimated by the algorithm until convergence.
- samp.gamma - Posterior sample of skewness parameter gamma variational distribution.
- samp.sts - Posterior sample of latent parameters, s_t, variational distributions.
- gammasig.out - List containing the IS estimate of the variational distribution of sigma and gamma.
- sts.out - List containing the variational distributions of latent parameters s_t.
- fix.gamma Logical value indicating whether gamma was fixed at gam.init.

Or if dqlm.ind=TRUE, the object also contains:

- sig.out - As above but for the DQLM case (gamma = 0); list containing the IS estimate of the variational distribution of sigma.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 20L)
y = scIVTmag[1:120]
trend.comp = polytrendMod(1, stats::quantile(y, 0.85), 10)
seas.comp = seasMod(365, c(1,2), C0 = 10*diag(4))
model = trend.comp + seas.comp
# Legacy ISVB fit retained for backward-compatible comparisons
M0 = exdqlmISVB(y, p0 = 0.85, model, df = c(1,1), dim.df = c(1,4),
                gam.init = -3.5, sig.init = 15, tol = 0.2,
                n.IS = 20, n.samp = 20, verbose = FALSE)
head(M0$diagnostics$vb_trace)

M0_al = exdqlmISVB(y, p0 = 0.85, model, df = c(1,1), dim.df = c(1,4),
                   dqlm.ind = TRUE, sig.init = 15, tol = 0.2,
                   n.IS = 20, n.samp = 20, verbose = FALSE)
tail(M0_al$diagnostics$vb_trace$elbo, 2)
options(old)
```

exdqlmLDVB *exDQLM - LDVB algorithm (Laplace-Delta)*

Description

The function applies a Laplace-Delta Variational Bayes (LDVB) algorithm to estimate the posterior of an exDQLM.

Arguments

y	A univariate time-series.
p0	The quantile of interest, a value between 0 and 1.
model	List of the state-space model including GG, FF, prior parameters m0 and C0.
df	Discount factors for each block.
dim.df	Dimension of each block of discount factors.
fix.gamma	Logical value indicating whether to fix gamma at gam.init. Default is FALSE.
gam.init	Initial value for gamma (skewness parameter), or value at which gamma will be fixed if fix.gamma=TRUE.
fix.sigma	Logical value indicating whether to fix sigma at sig.init. Default is FALSE.
sig.init	Initial value for sigma (scale parameter), or value at which sigma will be fixed if fix.sigma=TRUE.
dqlm.ind	Logical value indicating whether to fix gamma at 0, reducing the exDQLM to the special case of the DQLM. Default is FALSE.
exps0	Initial value for dynamic quantile. If exps0 is not specified, it is set to the DLM estimate of the p0 quantile.
tol	Tolerance for convergence of dynamic quantile estimates. Default is tol=0.1.
n.samp	Number of samples to draw from the approximated posterior distribution. Default is n.samp=200.
PriorSigma	List of parameters for inverse gamma prior on sigma; shape a_sig and scale b_sig. Default is an inverse gamma with mean 1 (or sig.init if provided) and variance 10.
PriorGamma	List of parameters for truncated student-t prior on gamma; center m_gam, scale s_gam and degrees of freedom df_gam. Default is a standard student-t with 1 degree of freedom, truncated to the support of gamma.
vb_control	Optional normalized VB control list, usually from exal_make_vb_control(). When supplied, the core VB arguments and warmup blocks are read from vb_control first and then merged with the explicit function arguments. When omitted, exAL-style VB fits use the package's conservative default (sigma, gamma) warmup profile automatically; explicit controls remain the advanced override path.
verbose	Logical value indicating whether progress should be displayed.
debug_shapes	Logical; if TRUE, print KF input/output shapes every debug_every iterations.
debug_every	Integer; frequency (in iterations) for shape prints when debug_shapes=TRUE.

Details

Advanced options (set via `options()`):

- `exdqlm.use_cpp_kf`: use the C++ Kalman filter bridge (default TRUE).
- `exdqlm.compute_elbo`: compute ELBO every iteration (default TRUE).
- `exdqlm.tol_elbo`: ELBO convergence tolerance (default 1e-6).
- `exdqlm.tol_sigma`: sigma-delta convergence tolerance (default: `tol`).
- `exdqlm.tol_gamma`: gamma-delta convergence tolerance (default: `tol`).
- `exdqlm.vb.min_iter`: minimum iterations before convergence can trigger (default 10).
- `exdqlm.vb.patience`: number of consecutive joint-converged iterations required (default 3).
- `exdqlm.use_cpp_samplers`: use C++ samplers for s_t, u_t, theta (default FALSE). The GIG-based u_t sampler always uses the package C++ Devroye implementation; when FALSE, the remaining samplers fall back to R implementations.
- `exdqlm.use_cpp_postpred`: use C++ posterior predictive sampler (default FALSE).
- `exdqlm.dynamic.ldvb.sts`: optional warmup/freeze controls for the exDQLM latent s_t VB block. Supported fields are `freeze_warmup_iters`, `force_after_warmup`, and `min_postwarmup_updates`.

Value

An object of class "`exdqlmLDVB`" containing the following:

- `y` - Time-series data used to fit the model.
- `run.time` - Algorithm run time in seconds.
- `iter` - Number of iterations until convergence was reached.
- `dqlm.ind` - Logical value indicating whether gamma was fixed at `0`, reducing the exDQLM to the special case of the DQLM.
- `model` - List of the state-space model including `GG`, `FF`, prior parameters `m0` and `C0`.
- `p0` - The quantile which was estimated.
- `df` - Discount factors used for each block.
- `dim.df` - Dimension used for each block of discount factors.
- `sig.init` - Initial value for sigma, or value at which sigma was fixed if `fix.sigma=TRUE`.
- `seq.sigma` - Sequence of sigma estimated by the algorithm until convergence.
- `samp.theta` - Posterior sample of the state vector variational distribution.
- `samp.post.pred` - Sample of the posterior predictive distributions.
- `map.standard.forecast.errors` - MAP standardized one-step-ahead forecast errors.
- `samp.sigma` - Posterior sample of scale parameter sigma variational distribution.
- `samp.vts` - Posterior sample of latent parameters, v_t, variational distributions.
- `theta.out` - List containing the variational distribution of the state vector including filtered distribution parameters (`fm` and `fC`) and smoothed distribution parameters (`sm` and `sC`).
- `vts.out` - List containing the variational distributions of latent parameters v_t.
- `fix.sigma` Logical value indicating whether sigma was fixed at `sig.init`.

- diagnostics - List containing ELBO trace, standardized VB iteration trace diagnostics$vb_trace (iteration-wise ELBO / sigma / gamma / convergence deltas), and convergence diagnostics (joint stopping status, deltas for state/sigma/gamma/ELBO, and criteria used).

If dqlm.ind=FALSE, the list also contains:

- gam.init - Initial value for gamma, or value at which gamma was fixed if fix.gamma=TRUE.
- seq.gamma - Sequence of gamma estimated by the algorithm until convergence.
- samp.gamma - Posterior sample of skewness parameter gamma variational distribution.
- samp.sts - Posterior sample of latent parameters, s_t, variational distributions.
- gammasig.out - List containing the LD (Laplace-Delta) approximation for the variational distribution of sigma and gamma (means, transformed Hessian, and ELBO components).
- sts.out - List containing the variational distributions of latent parameters s_t.
- fix.gamma Logical value indicating whether gamma was fixed at gam.init.

Or if dqlm.ind=TRUE, the list also contains:

- sig.out - As above but for the DQLM case (gamma = 0), the LD approximation for sigma.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 20L)
y = scIVTmag[1:80]
trend.comp = polytrendMod(1, stats::quantile(y, 0.85), 10)
seas.comp = seasMod(365, c(1,2), C0 = 10*diag(4))
model = trend.comp + seas.comp
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(1,1), dim.df = c(1,4),
                gam.init = -3.5, sig.init = 15, tol = 0.2,
                n.samp = 20, verbose = FALSE)

M0_al = exdqlmLDVB(y, p0 = 0.85, model, df = c(1,1), dim.df = c(1,4),
                   dqlm.ind = TRUE, sig.init = 15, tol = 0.2,
                   n.samp = 20, verbose = FALSE)
options(old)
```

---

exdqlmMCMC *exDQLM - MCMC algorithm*

Description

The function applies a Markov chain Monte Carlo (MCMC) algorithm to sample the posterior of an exDQLM.

Arguments

y	A univariate time-series.
p0	The quantile of interest, a value between 0 and 1.
model	List of the state-space model including GG, FF, prior parameters m0 and C0.
df	Discount factors for each block.
dim.df	Dimension of each block of discount factors.
fix.gamma	Logical value indicating whether to fix gamma at gam.init. Default is FALSE.
gam.init	Initial value for gamma (skewness parameter), or value at which gamma will be fixed if fix.gamma = TRUE.
fix.sigma	Logical value indicating whether to fix sigma at sig.init. Default is FALSE.
sig.init	Initial value for sigma (scale parameter), or value at which sigma will be fixed if fix.sigma = TRUE.
dqlm.ind	Logical value indicating whether to fix gamma at 0, reducing the exDQLM to the AL/DQLM special case. Default is FALSE.
Sig.mh	Covariance matrix used in the random walk MH step to jointly sample sigma and gamma.
joint.sample	Logical value indicating whether or not to recompute Sig.mh based off the initial burn-in samples of gamma and sigma. Default is FALSE.
n.burn	Number of MCMC iterations to burn. Default is n.burn = 2000.
n.mcmc	Number of MCMC iterations to sample. Default is n.mcmc = 1500.
init.from.isvb	Logical value indicating whether to use the legacy ISVB warm start when init.from.vb = TRUE. Default is FALSE, which favors LDVB as the default VB warm start. This flag only chooses the warm-start source; it does not change the subsequent MCMC proposal kernel.
init.from.vb	Optional logical. If TRUE, run a VB pre-initialization step (LDVB by default, or ISVB when init.from.isvb = TRUE) and initialize MCMC from converged VB moments. Default is TRUE. If explicitly set to NULL, it falls back to init.from.isvb behavior for backward compatibility.
vb_init_controls	Optional list controlling VB warm start. Supported keys: method ("isvb" or "ldvb"), tol, n.IS, n.samp, max_iter, verbose.
vb_init_fit	Optional precomputed VB fit object. If supplied, warm start uses this object directly and does not rerun VB internally.
mcmc_control	Optional normalized MCMC control list, usually from exal_make_mcmc_control(). When supplied, the core MCMC arguments and warmup blocks are read from mcmc_control first and then merged with the explicit function arguments. When omitted, exAL-style dynamic MCMC uses the package's conservative default (sigma, gamma) warmup profile automatically; explicit controls remain the advanced override path.
sigmagam_controls	Optional list controlling warmup/freeze for the exDQLM sigma/gamma block during MCMC.

`latent_state_controls`
: Optional list controlling early latent-state warmup/freeze in dynamic MCMC. Supported keys include `freeze_burnin_iters`, `freeze_only_during_burn`, `force_after_warmup`, and `mode` (`"u_only"` or `"u_st_pair"`).

`theta_state_controls`
: Optional list controlling early theta-state warmup/freeze in dynamic MCMC. Supported keys include `freeze_burnin_iters`, `freeze_only_during_burn`, and `force_after_warmup`.

`dqlm_sigma_controls`
: Optional list controlling sigma-only warmup/freeze in the DQLM branch. Supported keys mirror `sigmagam_controls`.

`mh.proposal`
: Character; proposal kernel for the exDQLM scale/skew block. `"slice"` (default) uses an exact sigma GIG update plus a bounded univariate slice sampler directly on `gamma`; `"laplace_rw"` uses a Laplace-informed covariance then RW; and `"rw"` uses joint random-walk MH on `(log sigma, logit gamma)`. This choice is separate from the VB warm-start method.

`mh.adapt`
: Logical; adapt MH proposal scale during burn-in.

`mh.adapt.interval`
: Integer; adaptation interval (iterations).

`mh.target.accept`
: Numeric length-2 vector with lower/upper target acceptance rates.

`mh.scale.bounds`
: Numeric length-2 vector with min/max global scaling for MH covariance.

`mh.max_scale.step`
: Numeric in (0,1); maximum fractional scale change per adaptation step.

`mh.min_burn_adapt`
: Minimum burn-in iterations required to enable adaptation.

`slice.width`
: Positive numeric width for the bounded slice sampler when `mh.proposal = "slice"`. Default `0.1` for parity with `bqrgal`.

`slice.max.steps`
: Positive integer or `Inf`; maximum stepping-out expansions for the slice sampler.

`trace.diagnostics`
: Logical; if `TRUE`, retain per-iteration sigma/gamma/s/u diagnostics under `mh.diagnostics$trace`. Set `FALSE` for lighter-weight runs.

`trace.every`
: Positive integer; when `trace.diagnostics = TRUE`, record one diagnostics row every `trace.every` iterations.

`verbose.every`
: Positive integer controlling how often console progress is printed when `verbose = TRUE`. Default `500`, independent of `trace.every`.

`progress_callback`
: Optional callback invoked with a named list at MCMC start, at each progress checkpoint, and on completion. Intended for workflow-level progress logging.

`PriorSigma`
: List of parameters for inverse gamma prior on sigma; shape `a_sig` and scale `b_sig`. Default is an inverse gamma with mean 1, or `sig.init` when supplied, and variance 10.

PriorGamma	List of parameters for truncated Student-t prior on gamma; center m_gam, scale s_gam, and degrees of freedom df_gam. Default is a standard Student-t with 1 degree of freedom, truncated to the support of gamma.
verbose	Logical value indicating whether progress should be displayed.

Value

An object of class "exdqlmMCMC" containing the following:

- y - Time-series data used to fit the model.
- run.time - Algorithm run time in seconds.
- dqlm.ind - Logical value indicating whether gamma was fixed at 0, reducing the exDQLM to the special case of the DQLM.
- model - List of the state-space model including GG, FF, prior parameters m0 and C0.
- p0 - The quantile which was estimated.
- df - Discount factors used for each block.
- dim.df - Dimension used for each block of discount factors.
- samp.theta - Posterior sample of the state vector.
- samp.post.pred - Sample of the posterior predictive distributions.
- map.standard.forecast.errors - MAP standardized one-step-ahead forecast errors.
- samp.sigma - Posterior sample of scale parameter sigma.
- samp.vts - Posterior sample of latent parameters, v_t.
- theta.out - List containing the distributions of the state vector including filtered distribution parameters (fm and fC) and smoothed distribution parameters (sm and sC).
- n.burn Number of MCMC iterations that were burned.
- n.mcmc Number of MCMC iterations that were sampled.

If dqlm.ind=FALSE, the object also contains the following:

- samp.gamma - Posterior sample of skewness parameter gamma.
- samp.sts - Posterior sample of latent parameters, s_t.
- init.log.sigma - Burned samples of log sigma from the random walk MH joint sampling of sigma and gamma.
- init.logit.gamma - Burned samples of logit gamma from the random walk MH joint sampling of sigma and gamma.
- accept.rate - Acceptance rate of the MH step.
- accept.rate.burn - MH acceptance rate during burn-in.
- accept.rate.keep - MH acceptance rate in kept MCMC samples.
- Sig.mh - Covariance matrix used in MH step to jointly sample sigma and gamma.
- mh.diagnostics - MH tuning diagnostics (proposal mode, scaling path, adaptation summary).
- diagnostics - ESS and chain-ready summaries for sigma/gamma.

Examples

```
data("scIVTmag", package = "exdqlm")
y = scIVTmag[1:80]
trend.comp = polytrendMod(order = 1, m0 = stats::quantile(y, 0.85), C0 = 10)
seas.comp = seasMod(p = 365, h = c(1,2), C0 = 10*diag(4))
model = trend.comp + seas.comp
M2 = exdqlmMCMC(y, p0=0.85, model, df = c(1,1), dim.df = c(1,4),
                gam.init = -3.5, sig.init = 15,
                n.burn = 40, n.mcmc = 40,
                init.from.vb = FALSE, verbose = FALSE)

M2_al = exdqlmMCMC(y, p0=0.85, model, df = c(1,1), dim.df = c(1,4),
                   dqlm.ind = TRUE, sig.init = 15,
                   n.burn = 30, n.mcmc = 30,
                   init.from.vb = FALSE, verbose = FALSE)
```

`exdqlmPlot` *Plot exDQLM*

Description

The function plots the MAP estimates and 95% credible intervals (CrIs) of the dynamic quantile of an exDQLM.

Usage

```
exdqlmPlot(
  m1,
  add = FALSE,
  col = "purple",
  cr.percent = 0.95,
  plot = TRUE,
  xlim = NULL,
  ylim = NULL,
  xlab = "time",
  ylab = NULL,
  lwd = 1.5,
  lwd.interval = 0.75,
  lty.interval = 2
)
```

Arguments

`m1`	An object of class "`exdqlmLDVB`", "`exdqlmMCMC`", or legacy "`exdqlmISVB`".
`add`	Logical value indicating whether the dynamic quantile will be added to existing plot. Default is `FALSE`.

col	Character vector of length 1 giving color of the dynamic quantile to be plotted. Default is purple.
cr.percent	Numeric in (0, 1) indicating the probability mass for the credible intervals (e.g., 0.95). Default 0.95.
plot	Logical value indicating whether to draw the plot. If FALSE, the function only returns the plotted summaries. Default is TRUE.
xlim, ylim	Optional limits passed to the base plotting call when plot = TRUE.
xlab, ylab	Optional axis labels passed to the base plotting call when plot = TRUE.
lwd, lwd.interval	Line widths for the dynamic quantile and credible interval bounds, respectively.
lty.interval	Line type for the credible interval bounds.

Value

A list of the following is returned:

- map.quant - MAP estimate of the dynamic quantile.
- lb.quant - Lower bound of the 95% CrIs of the dynamic quantile.
- ub.quant - Upper bound of the 95% CrIs of the dynamic quantile.
- x - Time/index values used for plotting.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.samp = 20, tol = 0.2, verbose = FALSE)
exdqlmPlot(M0, col = "blue")
q.summary = exdqlmPlot(M0, plot = FALSE)
options(old)
```

---

exdqlmTransferISVB *Transfer Function exDQLM - legacy ISVB algorithm*

Description

The function applies an Importance Sampling Variational Bayes (ISVB) algorithm to estimate the posterior of an exDQLM with exponential-decay transfer function component. This transfer wrapper is retained as a legacy path; exdqlmTransferLDVB() is the main VB transfer entry point.

Usage

```
exdqlmTransferISVB(
  y,
  p0,
  model,
  X,
  df,
  dim.df,
  lam,
  tf.df,
  fix.gamma = FALSE,
  gam.init = NA,
  fix.sigma = FALSE,
  sig.init = NA,
  dqlm.ind = FALSE,
  exps0,
  tol = 0.1,
  n.IS = 500,
  n.samp = 200,
  PriorSigma = NULL,
  PriorGamma = NULL,
  tf.m0 = NULL,
  tf.C0 = NULL,
  verbose = TRUE
)
```

Arguments

y	A univariate time-series.
p0	The quantile of interest, a value between 0 and 1.
model	List of the state-space model including GG, FF, prior parameters m0 and C0.
X	A numeric vector or matrix of transfer-function inputs. Vectors are treated as a univariate input series. Matrices should have one row per time point and one column per covariate.
df	Discount factors for each block.
dim.df	Dimension of each block of discount factors.
lam	Transfer function rate parameter lambda, a value between 0 and 1.
tf.df	Discount factor specification for the transfer function component. If length(tf.df) = 1, the value is shared by the ζ_t state and the whole ψ_t block. If length(tf.df) = 2, it is interpreted as c(df_zeta, df_psi_shared). If length(tf.df) = k + 1, where $k = ncol(X)$, the values are applied componentwise to $(\zeta_t, \psi_{1,t}, \ldots, \psi_{k,t})$.
fix.gamma	Logical value indicating whether to fix gamma at gam.init. Default is FALSE.
gam.init	Initial value for gamma (skewness parameter), or value at which gamma will be fixed if fix.gamma=TRUE.
fix.sigma	Logical value indicating whether to fix sigma at sig.init. Default is FALSE.

sig.init	Initial value for sigma (scale parameter), or value at which sigma will be fixed if fix.sigma=TRUE.
dqlm.ind	Logical value indicating whether to fix gamma at 0, reducing the exDQLM to the special case of the DQLM. Default is FALSE.
exps0	Initial value for dynamic quantile. If exps0 is not specified, it is set to the DLM estimate of the p0 quantile.
tol	Tolerance for convergence of dynamic quantile estimates. Default is tol=0.1.
n.IS	Number of particles for the importance sampling of joint variational distribution of sigma and gamma. Default is n.IS=500.
n.samp	Number of samples to draw from the approximated posterior distribution. Default is n.samp=200.
PriorSigma	List of parameters for inverse gamma prior on sigma; shape a_sig and scale b_sig. Default is an inverse gamma with mean 1 (or sig.init if provided) and variance 10.
PriorGamma	List of parameters for truncated student-t prior on gamma; center m_gam, scale s_gam and degrees of freedom df_gam. Default is a standard student-t with 1 degree of freedom, truncated to the support of gamma.
tf.m0	Prior mean of the transfer function component. Defaults to a zero vector of length $k+1$, where $k = ncol(X)$.
tf.C0	Prior covariance of the transfer function component. Defaults to the $(k+1) \times (k+1)$ identity matrix.
verbose	Logical value indicating whether progress should be displayed.

Details

Advanced options (set via options()):

- exdqlm.use_cpp_kf: use the C++ Kalman filter bridge (default TRUE).
- exdqlm.compute_elbo: compute ELBO every iteration (default TRUE).
- exdqlm.tol_elbo: ELBO convergence tolerance (default 1e-6).
- exdqlm.use_cpp_samplers: use C++ samplers for s_t, u_t, theta (default FALSE). The GIG-based u_t sampler always uses the package C++ Devroye implementation; when FALSE, the remaining samplers fall back to R implementations.
- exdqlm.use_cpp_postpred: use C++ posterior predictive sampler (default FALSE).

Value

An object of class "exdqlmISVB" containing the following:

- run.time - Algorithm run time in seconds.
- iter - Number of iterations until convergence was reached.
- dqlm.ind - Logical value indicating whether gamma was fixed at 0, reducing the exDQLM to the special case of the DQLM.
- model - List of the augmented state-space model including GG, FF, prior parameters m0 and C0.

- p0 - The quantile which was estimated.
- df - Discount factors used for each block, including transfer function component.
- dim.df - Dimension used for each block of discount factors, including transfer function component.
- lam - Transfer function rate parameter lambda.
- sig.init - Initial value for sigma, or value at which sigma was fixed if fix.sigma=TRUE.
- seq.sigma - Sequence of sigma estimated by the algorithm until convergence.
- samp.theta - Posterior sample of the state vector variational distribution.
- samp.post.pred - Sample of the posterior predictive distributions.
- map.standard.forecast.errors - MAP standardized one-step-ahead forecast errors.
- samp.sigma - Posterior sample of scale parameter sigma variational distribution.
- samp.vts - Posterior sample of latent parameters, v_t, variational distributions.
- theta.out - List containing the variational distribution of the state vector including filtered distribution parameters (fm and fC) and smoothed distribution parameters (sm and sC).
- vts.out - List containing the variational distributions of latent parameters v_t.
- median.kt - Median number of time steps until the aggregated transfer effect $|x_t^\top \psi_{t-1}|$ is less than or equal to 1e-3.

If dqlm.ind=FALSE, the object also contains:

- gam.init - Initial value for gamma, or value at which gamma was fixed if fix.gamma=TRUE.
- seq.gamma - Sequence of gamma estimated by the algorithm until convergence.
- samp.gamma - Posterior sample of skewness parameter gamma variational distribution.
- samp.sts - Posterior sample of latent parameters, s_t, variational distributions.
- gammasig.out - List containing the IS estimate of the variational distribution of sigma and gamma.
- sts.out - List containing the variational distributions of latent parameters s_t.

Or if dqlm.ind=TRUE, the object also contains:

- sig.out - As above but for the DQLM case (gamma = 0); list containing the IS estimate of the variational distribution of sigma.

Examples

```
data("scIVTmag", package = "exdqlm")
data("ELIanoms", package = "exdqlm")
old = options(exdqlm.max_iter = 20L)
y = scIVTmag[1:120]
X = ELIanoms[1:120]
trend.comp = polytrendMod(1, stats::quantile(y, 0.85), 10)
seas.comp = seasMod(365, c(1,2), C0 = 10*diag(4))
model = trend.comp + seas.comp
# Legacy ISVB transfer fit retained for backward-compatible comparisons
M1 = exdqlmTransferISVB(y, p0 = 0.85, model = model,
```

```
                          X, df = c(1,1), dim.df = c(1,4),
                          gam.init = -3.5, sig.init = 15,
                          lam = 0.38, tf.df = c(0.97,0.97),
                          n.IS = 20, n.samp = 20, tol = 0.2,
                          verbose = FALSE)
X_multi = cbind(ELIanoms[1:120], scale(scIVTmag[1:120])[, 1])
M2 = exdqlmTransferISVB(y, p0 = 0.85, model = model,
                          X_multi, df = c(1,1), dim.df = c(1,4),
                          gam.init = -3.5, sig.init = 15,
                          lam = 0.38, tf.df = c(0.97, 0.99),
                          n.IS = 20, n.samp = 20, tol = 0.2,
                          verbose = FALSE)
options(old)
```

`exdqlmTransferLDVB` *Transfer Function exDQLM - LDVB algorithm*

Description

The function applies a Laplace-Delta Variational Bayes (LDVB) algorithm to estimate the posterior of an exDQLM with an exponential-decay transfer function component. For multivariate transfer inputs, each column of `X` has its own instantaneous coefficient state in ψ_t, while a single scalar decay rate `lam` controls persistence of the accumulated transfer effect ζ_t.

Usage

```
exdqlmTransferLDVB(
  y,
  p0,
  model,
  X,
  df,
  dim.df,
  lam,
  tf.df,
  fix.gamma = FALSE,
  gam.init = NA,
  fix.sigma = FALSE,
  sig.init = NA,
  dqlm.ind = FALSE,
  exps0,
  tol = 0.1,
  n.samp = 200,
  PriorSigma = NULL,
  PriorGamma = NULL,
  tf.m0 = NULL,
```

```
  tf.C0 = NULL,
  verbose = TRUE,
  debug_shapes = FALSE,
  debug_every = 5
)
```

Arguments

y	A univariate time-series.
p0	The quantile of interest, a value between 0 and 1.
model	List of the state-space model including GG, FF, prior parameters m0 and C0.
X	A numeric vector or matrix of transfer-function inputs. Vectors are treated as a univariate input series. Matrices should have one row per time point and one column per covariate.
df	Discount factors for each block.
dim.df	Dimension of each block of discount factors.
lam	Single transfer-function decay-rate parameter λ, a value between 0 and 1. This scalar is shared across all transfer inputs and controls propagation of the accumulated transfer effect ζ_t.
tf.df	Discount factor specification for the transfer function component. These discount factors control the evolution variances of the transfer states, separately from the deterministic decay rate lam. If length(tf.df) = 1, the value is shared by the ζ_t state and the whole ψ_t block. If length(tf.df) = 2, it is interpreted as c(df_zeta, df_psi_shared). If length(tf.df) = k + 1, where $k = ncol(X)$, the values are applied componentwise to $(\zeta_t, \psi_{1,t}, \ldots, \psi_{k,t})$.
fix.gamma	Logical value indicating whether to fix gamma at gam.init. Default is FALSE.
gam.init	Initial value for gamma (skewness parameter), or value at which gamma will be fixed if fix.gamma=TRUE.
fix.sigma	Logical value indicating whether to fix sigma at sig.init. Default is FALSE.
sig.init	Initial value for sigma (scale parameter), or value at which sigma will be fixed if fix.sigma=TRUE.
dqlm.ind	Logical value indicating whether to fix gamma at 0, reducing the exDQLM to the special case of the DQLM. Default is FALSE.
exps0	Initial value for dynamic quantile. If exps0 is not specified, it is set to the DLM estimate of the p0 quantile.
tol	Tolerance for convergence of dynamic quantile estimates. Default is tol=0.1.
n.samp	Number of samples to draw from the approximated posterior distribution. Default is n.samp=200.
PriorSigma	List of parameters for inverse gamma prior on sigma; shape a_sig and scale b_sig. Default is an inverse gamma with mean 1 (or sig.init if provided) and variance 10.
PriorGamma	List of parameters for truncated student-t prior on gamma; center m_gam, scale s_gam and degrees of freedom df_gam. Default is a standard student-t with 1 degree of freedom, truncated to the support of gamma.

tf.m0	Prior mean of the transfer function component. Defaults to a zero vector of length $k+1$, where $k = ncol(X)$.
tf.C0	Prior covariance of the transfer function component. Defaults to the $(k+1) \times (k+1)$ identity matrix.
verbose	Logical value indicating whether progress should be displayed.
debug_shapes	Logical; if TRUE, print KF input/output shapes every debug_every iterations.
debug_every	Integer; frequency (in iterations) for shape prints when debug_shapes=TRUE.

Value

An object of class "exdqlmLDVB" containing the following:

- y - Time-series data used to fit the model.
- run.time - Algorithm run time in seconds.
- iter - Number of iterations until convergence was reached.
- dqlm.ind - Logical value indicating whether gamma was fixed at 0, reducing the exDQLM to the special case of the DQLM.
- model - List of the state-space model including GG, FF, prior parameters m0 and C0.
- p0 - The quantile which was estimated.
- df - Discount factors used for each block.
- dim.df - Dimension used for each block of discount factors.
- sig.init - Initial value for sigma, or value at which sigma was fixed if fix.sigma=TRUE.
- seq.sigma - Sequence of sigma estimated by the algorithm until convergence.
- samp.theta - Posterior sample of the state vector variational distribution.
- samp.post.pred - Sample of the posterior predictive distributions.
- map.standard.forecast.errors - MAP standardized one-step-ahead forecast errors.
- samp.sigma - Posterior sample of scale parameter sigma variational distribution.
- samp.vts - Posterior sample of latent parameters, v_t, variational distributions.
- theta.out - List containing the variational distribution of the state vector including filtered distribution parameters (fm and fC) and smoothed distribution parameters (sm and sC).
- vts.out - List containing the variational distributions of latent parameters v_t.
- fix.sigma Logical value indicating whether sigma was fixed at sig.init.
- diagnostics - List containing ELBO trace, standardized VB iteration trace diagnostics$vb_trace (iteration-wise ELBO / sigma / gamma / convergence deltas), and convergence diagnostics (joint stopping status, deltas for state/sigma/gamma/ELBO, and criteria used).

If dqlm.ind=FALSE, the list also contains:

- gam.init - Initial value for gamma, or value at which gamma was fixed if fix.gamma=TRUE.
- seq.gamma - Sequence of gamma estimated by the algorithm until convergence.
- samp.gamma - Posterior sample of skewness parameter gamma variational distribution.
- samp.sts - Posterior sample of latent parameters, s_t, variational distributions.

- gammasig.out - List containing the LD (Laplace-Delta) approximation for the variational distribution of sigma and gamma (means, transformed Hessian, and ELBO components).
- sts.out - List containing the variational distributions of latent parameters s_t.
- fix.gamma Logical value indicating whether gamma was fixed at gam.init.

Or if dqlm.ind=TRUE, the list also contains:

- sig.out - As above but for the DQLM case (gamma = 0), the LD approximation for sigma.

Transfer-function return fields

In addition to the standard exdqlmLDVB() return values, the returned model, df, and dim.df entries correspond to the transfer-function-augmented state-space model, with appended ζ_t and ψ_t states. The object also contains:

- lam - Single transfer-function decay-rate parameter λ.
- median.kt - Median number of time steps until the aggregated transfer effect $|x_t^\top \psi_{t-1}|$ is less than or equal to 1e-3.
- transfer_input_names - Column names of the transfer inputs after normalization of X.

Examples

```
data("scIVTmag", package = "exdqlm")
data("ELIanoms", package = "exdqlm")
old = options(exdqlm.max_iter = 20L)
y = scIVTmag[1:120]
X = ELIanoms[1:120]
trend.comp = polytrendMod(1, stats::quantile(y, 0.85), 10)
seas.comp = seasMod(365, c(1,2), C0 = 10*diag(4))
model = trend.comp + seas.comp
M1 = exdqlmTransferLDVB(
  y, p0 = 0.85, model = model, X = X,
  df = c(1,1), dim.df = c(1,4),
  gam.init = -3.5, sig.init = 15,
  lam = 0.38, tf.df = c(0.97,0.97),
  n.samp = 20, tol = 0.2, verbose = FALSE
)
X_multi = cbind(ELIanoms[1:120], scale(scIVTmag[1:120])[, 1])
M2 = exdqlmTransferLDVB(
  y, p0 = 0.85, model = model, X = X_multi,
  df = c(1,1), dim.df = c(1,4),
  gam.init = -3.5, sig.init = 15,
  lam = 0.38, tf.df = c(0.97, 0.99),
  n.samp = 20, tol = 0.2, verbose = FALSE
)
options(old)
```

exdqlmTransferMCMC *Transfer Function exDQLM - MCMC algorithm*

Description

The function applies a Markov chain Monte Carlo (MCMC) algorithm to sample the posterior of an exDQLM with an exponential-decay transfer function component for a fixed transfer rate parameter `lam`. For multivariate transfer inputs, each column of `X` has its own instantaneous coefficient state in ψ_t, while a single scalar decay rate `lam` controls persistence of the accumulated transfer effect ζ_t.

Usage

```
exdqlmTransferMCMC(
  y,
  p0,
  model,
  X,
  df,
  dim.df,
  lam,
  tf.df,
  fix.gamma = FALSE,
  gam.init = NA,
  fix.sigma = FALSE,
  sig.init = NA,
  dqlm.ind = FALSE,
  Sig.mh,
  joint.sample = FALSE,
  n.burn = 2000,
  n.mcmc = 1500,
  init.from.isvb = FALSE,
  PriorSigma = NULL,
  PriorGamma = NULL,
  verbose = TRUE,
  init.from.vb = TRUE,
  vb_init_controls = NULL,
  vb_init_fit = NULL,
  mh.proposal = c("slice", "laplace_rw", "rw"),
  mh.adapt = TRUE,
  mh.adapt.interval = 50L,
  mh.target.accept = c(0.2, 0.45),
  mh.scale.bounds = c(0.1, 10),
  mh.max_scale.step = 0.35,
  mh.min_burn_adapt = 50L,
  slice.width = 0.1,
  slice.max.steps = Inf,
  trace.diagnostics = TRUE,
```

```
  trace.every = 1L,
  verbose.every = 500L,
  progress_callback = NULL,
  tf.m0 = NULL,
  tf.C0 = NULL
)
```

Arguments

y	A univariate time-series.
p0	The quantile of interest, a value between 0 and 1.
model	List of the state-space model including GG, FF, prior parameters m0 and C0.
X	A numeric vector or matrix of transfer-function inputs. Vectors are treated as a univariate input series. Matrices should have one row per time point and one column per covariate.
df	Discount factors for each block.
dim.df	Dimension of each block of discount factors.
lam	Single transfer-function decay-rate parameter λ, a value between 0 and 1. This scalar is shared across all transfer inputs and controls propagation of the accumulated transfer effect ζ_t.
tf.df	Discount factor specification for the transfer function component. These discount factors control the evolution variances of the transfer states, separately from the deterministic decay rate lam. If length(tf.df) = 1, the value is shared by the ζ_t state and the whole ψ_t block. If length(tf.df) = 2, it is interpreted as c(df_zeta, df_psi_shared). If length(tf.df) = k + 1, where $k = ncol(X)$, the values are applied componentwise to $(\zeta_t, \psi_{1,t}, \ldots, \psi_{k,t})$.
fix.gamma	Logical value indicating whether to fix gamma at gam.init. Default is FALSE.
gam.init	Initial value for gamma (skewness parameter), or value at which gamma will be fixed if fix.gamma = TRUE.
fix.sigma	Logical value indicating whether to fix sigma at sig.init. Default is FALSE.
sig.init	Initial value for sigma (scale parameter), or value at which sigma will be fixed if fix.sigma = TRUE.
dqlm.ind	Logical value indicating whether to fix gamma at 0, reducing the exDQLM to the AL/DQLM special case. Default is FALSE.
Sig.mh	Covariance matrix used in the random walk MH step to jointly sample sigma and gamma.
joint.sample	Logical value indicating whether or not to recompute Sig.mh based off the initial burn-in samples of gamma and sigma. Default is FALSE.
n.burn	Number of MCMC iterations to burn. Default is n.burn = 2000.
n.mcmc	Number of MCMC iterations to sample. Default is n.mcmc = 1500.
init.from.isvb	Logical value indicating whether to use the legacy ISVB warm start when init.from.vb = TRUE. Default is FALSE, which favors LDVB as the default VB warm start. This flag only chooses the warm-start source; it does not change the subsequent MCMC proposal kernel.

`PriorSigma` List of parameters for inverse gamma prior on sigma; shape `a_sig` and scale `b_sig`. Default is an inverse gamma with mean 1, or `sig.init` when supplied, and variance 10.

`PriorGamma` List of parameters for truncated Student-t prior on gamma; center `m_gam`, scale `s_gam`, and degrees of freedom `df_gam`. Default is a standard Student-t with 1 degree of freedom, truncated to the support of gamma.

`verbose` Logical value indicating whether progress should be displayed.

`init.from.vb` Optional logical. If `TRUE`, run a VB pre-initialization step (`LDVB` by default, or `ISVB` when `init.from.isvb = TRUE`) and initialize MCMC from converged VB moments. Default is `TRUE`. If explicitly set to `NULL`, it falls back to `init.from.isvb` behavior for backward compatibility.

`vb_init_controls`
Optional list controlling VB warm start. Supported keys: `method` (`"isvb"` or `"ldvb"`), `tol`, `n.IS`, `n.samp`, `max_iter`, `verbose`.

`vb_init_fit` Optional precomputed VB fit object. If supplied, warm start uses this object directly and does not rerun VB internally.

`mh.proposal` Character; proposal kernel for the exDQLM scale/skew block. `"slice"` (default) uses an exact sigma GIG update plus a bounded univariate slice sampler directly on `gamma`; `"laplace_rw"` uses a Laplace-informed covariance then RW; and `"rw"` uses joint random-walk MH on `(log sigma, logit gamma)`. This choice is separate from the VB warm-start method.

`mh.adapt` Logical; adapt MH proposal scale during burn-in.

`mh.adapt.interval`
Integer; adaptation interval (iterations).

`mh.target.accept`
Numeric length-2 vector with lower/upper target acceptance rates.

`mh.scale.bounds`
Numeric length-2 vector with min/max global scaling for MH covariance.

`mh.max_scale.step`
Numeric in (0,1); maximum fractional scale change per adaptation step.

`mh.min_burn_adapt`
Minimum burn-in iterations required to enable adaptation.

`slice.width` Positive numeric width for the bounded slice sampler when `mh.proposal = "slice"`. Default `0.1` for parity with `bqrgal`.

`slice.max.steps`
Positive integer or `Inf`; maximum stepping-out expansions for the slice sampler.

`trace.diagnostics`
Logical; if `TRUE`, retain per-iteration sigma/gamma/s/u diagnostics under `mh.diagnostics$trace`. Set `FALSE` for lighter-weight runs.

`trace.every` Positive integer; when `trace.diagnostics = TRUE`, record one diagnostics row every `trace.every` iterations.

`verbose.every` Positive integer controlling how often console progress is printed when `verbose = TRUE`. Default `500`, independent of `trace.every`.

`progress_callback`
: Optional callback invoked with a named list at MCMC start, at each progress checkpoint, and on completion. Intended for workflow-level progress logging.

`tf.m0`
: Prior mean of the transfer function component. Defaults to a zero vector of length $k + 1$, where $k = ncol(X)$.

`tf.C0`
: Prior covariance of the transfer function component. Defaults to the $(k + 1) \times (k + 1)$ identity matrix.

Value

An object of class "`exdqlmMCMC`" containing the following:

- `y` - Time-series data used to fit the model.
- `run.time` - Algorithm run time in seconds.
- `dqlm.ind` - Logical value indicating whether gamma was fixed at `0`, reducing the exDQLM to the special case of the DQLM.
- `model` - List of the state-space model including `GG`, `FF`, prior parameters `m0` and `C0`.
- `p0` - The quantile which was estimated.
- `df` - Discount factors used for each block.
- `dim.df` - Dimension used for each block of discount factors.
- `samp.theta` - Posterior sample of the state vector.
- `samp.post.pred` - Sample of the posterior predictive distributions.
- `map.standard.forecast.errors` - MAP standardized one-step-ahead forecast errors.
- `samp.sigma` - Posterior sample of scale parameter sigma.
- `samp.vts` - Posterior sample of latent parameters, v_t.
- `theta.out` - List containing the distributions of the state vector including filtered distribution parameters (`fm` and `fC`) and smoothed distribution parameters (`sm` and `sC`).
- `n.burn` Number of MCMC iterations that were burned.
- `n.mcmc` Number of MCMC iterations that were sampled.

If `dqlm.ind=FALSE`, the object also contains the following:

- `samp.gamma` - Posterior sample of skewness parameter gamma.
- `samp.sts` - Posterior sample of latent parameters, s_t.
- `init.log.sigma` - Burned samples of log sigma from the random walk MH joint sampling of sigma and gamma.
- `init.logit.gamma` - Burned samples of logit gamma from the random walk MH joint sampling of sigma and gamma.
- `accept.rate` - Acceptance rate of the MH step.
- `accept.rate.burn` - MH acceptance rate during burn-in.
- `accept.rate.keep` - MH acceptance rate in kept MCMC samples.
- `Sig.mh` - Covariance matrix used in MH step to jointly sample sigma and gamma.
- `mh.diagnostics` - MH tuning diagnostics (proposal mode, scaling path, adaptation summary).
- `diagnostics` - ESS and chain-ready summaries for sigma/gamma.

Transfer-function return fields

In addition to the standard `exdqlmMCMC()` return values, the returned `model`, `df`, and `dim.df` entries correspond to the transfer-function-augmented state-space model, with appended ζ_t and ψ_t states. The object also contains:

- `lam` - Single transfer-function decay-rate parameter λ.
- `median.kt` - Median number of time steps until the aggregated transfer effect $|x_t^\top \psi_{t-1}|$ is less than or equal to `1e-3`.
- `transfer_input_names` - Column names of the transfer inputs after normalization of `X`.

Examples

```
data("scIVTmag", package = "exdqlm")
data("ELIanoms", package = "exdqlm")
y = scIVTmag[1:120]
X = ELIanoms[1:120]
trend.comp = polytrendMod(1, stats::quantile(y, 0.85), 10)
seas.comp = seasMod(365, c(1,2), C0 = 10*diag(4))
model = trend.comp + seas.comp
M1 = exdqlmTransferMCMC(
  y, p0 = 0.85, model = model, X = X,
  df = c(1,1), dim.df = c(1,4),
  gam.init = -3.5, sig.init = 15,
  lam = 0.38, tf.df = c(0.97,0.97),
  n.burn = 40, n.mcmc = 40,
  init.from.vb = FALSE, verbose = FALSE
)
X_multi = cbind(ELIanoms[1:120], scale(scIVTmag[1:120])[, 1])
M2 = exdqlmTransferMCMC(
  y, p0 = 0.85, model = model, X = X_multi,
  df = c(1,1), dim.df = c(1,4),
  gam.init = -3.5, sig.init = 15,
  lam = 0.38, tf.df = c(0.97, 0.99),
  n.burn = 40, n.mcmc = 40,
  init.from.vb = FALSE, verbose = FALSE
)
```

---

get_gamma_bounds *Bounds for the exAL shape parameter gamma*

Description

Returns valid lower/upper bounds (`L`, `U`) for the shape parameter `gamma` of the standardized extended Asymmetric Laplace (exAL), given `p0` in (0,1).

Usage

```
get_gamma_bounds(p0)
```

Arguments

p0	Numeric scalar in (0, 1); typically the target quantile level.

Details

This is a user-facing convenience wrapper around the C++ routine `get_gamma_bounds_cpp()`, which performs the actual computation.

Value

A numeric vector of length 2 named `c("L","U")`.

Examples

```
get_gamma_bounds(0.5)
get_gamma_bounds(0.9)
```

---

is.exalStaticDiagnostic — exalStaticDiagnostic *objects*

Description

`is.exalStaticDiagnostic` tests if its argument is an `exalStaticDiagnostic` object.

Usage

```
is.exalStaticDiagnostic(x)
```

Arguments

x	an **R** object

---

is.exalStaticLDVB — exalStaticLDVB *objects*

Description

`is.exalStaticLDVB` tests if its argument is an `exalStaticLDVB` object.

Usage

```
is.exalStaticLDVB(m)
```

Arguments

m	an **R** object

is.exalStaticMCMC exalStaticMCMC *objects*

Description

is.exalStaticMCMC tests if its argument is an exalStaticMCMC object.

Usage

```
is.exalStaticMCMC(m)
```

Arguments

m	an **R** object

is.exdqlm exdqlm *objects*

Description

is.exdqlm tests if its argument is a exdqlm object.

Usage

```
is.exdqlm(m)
```

Arguments

m	an **R** object

is.exdqlmDiagnostic exdqlmDiagnostic *objects*

Description

is.exdqlmDiagnostic tests if its argument is a exdqlmDiagnostic object.

Usage

```
is.exdqlmDiagnostic(x)
```

Arguments

x	an **R** object

is.exdqlmForecast exdqlmForecast *objects*

Description

is.exdqlmForecast tests if its argument is a exdqlmForecast object.

Usage

```
is.exdqlmForecast(x)
```

Arguments

x	an **R** object

is.exdqlmForecastDiagnostic exdqlmForecastDiagnostic *objects*

Description

is.exdqlmForecastDiagnostic tests if its argument is an exdqlmForecastDiagnostic object.

Usage

```
is.exdqlmForecastDiagnostic(x)
```

Arguments

x	an **R** object.

is.exdqlmISVB exdqlmISVB *objects*

Description

is.exdqlmISVB tests if its argument is a exdqlmISVB object.

Usage

```
is.exdqlmISVB(m)
```

Arguments

m	an **R** object

is.exdqlmLDVB exdqlmLDVB *objects*

Description

is.exdqlmLDVB tests if its argument is a exdqlmLDVB object.

Usage

```
is.exdqlmLDVB(m)
```

Arguments

m	an **R** object

is.exdqlmMCMC exdqlmMCMC *objects*

Description

is.exdqlmMCMC tests if its argument is a exdqlmMCMC object.

Usage

```
is.exdqlmMCMC(m)
```

Arguments

m	an **R** object

is.exdqlmSynthesis exdqlmSynthesis *objects*

Description

is.exdqlmSynthesis tests if its argument is an exdqlmSynthesis object returned by quantileSynthesis.

Usage

```
is.exdqlmSynthesis(x)
```

Arguments

x	an **R** object

`pexal` *Cumulative Distribution Function (CDF) for the exAL Distribution*

Description

Vectorized over `q`.

Usage

```
pexal(
  q,
  p0 = 0.5,
  mu = 0,
  sigma = 1,
  gamma = 0,
  lower.tail = TRUE,
  log.p = FALSE
)
```

Arguments

`q`	Numeric vector of quantiles.
`p0`	Probability level used in the quantile parametrization. Scalar in (0, 1). Default `0.5`.
`mu`	Location parameter (scalar). Default `0`.
`sigma`	Scale parameter (scalar, strictly positive). Default `1`.
`gamma`	Skewness parameter controlling asymmetry (scalar). Must be within valid bounds implied by `p0`. Default `0`.
`lower.tail`	Logical scalar; if `TRUE` (default) return $P(X \leq q)$, otherwise $P(X > q)$.
`log.p`	Logical scalar; if `TRUE`, return log-probabilities.

Value

Numeric vector of CDF values (same length as `q`).

Examples

```
pexal(0)
pexal(c(-1, 0, 1), p0 = 0.5, mu = 0, sigma = 1, gamma = 0.1)
```

plot.exalStaticDiagnostic *Plot Method for* exalStaticDiagnostic *Objects*

Description

Plot Method for exalStaticDiagnostic Objects

Usage

```
## S3 method for class 'exalStaticDiagnostic'
plot(
  x,
  cols = c("red", "blue"),
  type = c("quantile", "coefficients"),
  beta.ref = NULL,
  include.intercept = TRUE,
  coef.names = NULL,
  xlab = NULL,
  ylab = NULL,
  ylim = NULL,
  legend.labels = NULL,
  beta.ref.label = "reference",
  legend = TRUE,
  ...
)
```

Arguments

x	An exalStaticDiagnostic object.
cols	Character vector of length 1 or 2 giving color(s) used to plot diagnostics.
type	Character string; "quantile" plots fitted conditional quantile summaries, and "coefficients" plots posterior coefficient intervals.
beta.ref	Optional coefficient reference vector for type = "coefficients". This is typically available only in simulation benchmarks. It is used as a plotting overlay, not as a package diagnostic metric.
include.intercept	Logical; if FALSE, omit the first coefficient from type = "coefficients" plots.
coef.names	Optional names for coefficients in type = "coefficients" plots.
xlab, ylab	Optional axis labels.
ylim	Optional y-axis limits.
legend.labels	Optional labels for the first and second model intervals in type = "coefficients" plots.
beta.ref.label	Label for the optional beta.ref overlay.

legend	Logical; if TRUE, add a legend to coefficient plots.
...	Additional arguments passed to plotting functions.

---

plot.exalStaticLDVB *Plot Method for* exalStaticLDVB *Objects*

Description

Plot Method for exalStaticLDVB Objects

Usage

```
## S3 method for class 'exalStaticLDVB'
plot(x, X = NULL, add = FALSE, col = "purple", cr.percent = 0.95, ...)
```

Arguments

x	An exalStaticLDVB object.
X	Optional design matrix used to compute fitted quantiles. If omitted, the method uses x$X when available.
add	Logical; add to an existing plot.
col	Character vector of length 1 giving color for fitted quantiles.
cr.percent	Numeric in (0, 1) for credible-interval mass.
...	Additional arguments passed to plot when add = FALSE.

Value

A list with map.quant, lb.quant, and ub.quant.

---

plot.exalStaticMCMC *Plot Method for* exalStaticMCMC *Objects*

Description

Plot Method for exalStaticMCMC Objects

Usage

```
## S3 method for class 'exalStaticMCMC'
plot(x, add = FALSE, col = "purple", cr.percent = 0.95, ...)
```

Arguments

x	An exalStaticMCMC object.
add	Logical; add to an existing plot.
col	Character vector of length 1 giving color for fitted quantiles.
cr.percent	Numeric in (0, 1) for credible-interval mass.
...	Additional arguments passed to plot when add = FALSE.

Value

A list with map.quant, lb.quant, and ub.quant.

---

plot.exdqlmDiagnostic *Plot Method for* exdqlmDiagnostic *Objects*

Description

Plot Method for exdqlmDiagnostic Objects

Usage

```
## S3 method for class 'exdqlmDiagnostic'
plot(x, ...)
```

Arguments

x	An exdqlmDiagnostic object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.95), dim.df = c(1),
                  gam.init = -3.5, sig.init = 15,
                  n.samp = 20, tol = 0.2, verbose = FALSE)
M0.diags = exdqlmDiagnostics(M0, plot = FALSE)
plot(M0.diags)
options(old)
```

plot.exdqlmForecast *Plot Method for* exdqlmForecast *Objects*

Description

Plot Method for exdqlmForecast Objects

Usage

```
## S3 method for class 'exdqlmForecast'
plot(x, ...)
```

Arguments

x	An exdqlmForecast object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.samp = 20, tol = 0.2, verbose = FALSE)
M0.forecast = exdqlmForecast(start.t = 50, k = 5, m1 = M0)
plot(M0.forecast)
options(old)
```

plot.exdqlmISVB *Plot Method for* exdqlmISVB *Objects*

Description

Plot Method for exdqlmISVB Objects

Usage

```
## S3 method for class 'exdqlmISVB'
plot(x, ...)
```

Arguments

x	An exdqlmISVB object.
...	Additional arguments.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
# Legacy ISVB object retained for backward-compatible plotting methods
M0 = exdqlmISVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.IS = 20, n.samp = 20, tol = 0.2,
                   verbose = FALSE)
plot(M0)
options(old)
```

---

plot.exdqlmLDVB *Plot Method for* exdqlmLDVB *Objects*

Description

Plot Method for exdqlmLDVB Objects

Usage

```
## S3 method for class 'exdqlmLDVB'
plot(x, ...)
```

Arguments

x	An exdqlmLDVB object.
...	Additional arguments.

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.samp = 20, tol = 0.2, verbose = FALSE)
plot(M0)
options(old)
```

plot.exdqlmMCMC *Plot Method for* exdqlmMCMC *Objects*

Description

Plot Method for exdqlmMCMC Objects

Usage

```
## S3 method for class 'exdqlmMCMC'
plot(x, ...)
```

Arguments

x	An exdqlmMCMC object.
...	Additional arguments.

Examples

```
data("scIVTmag", package = "exdqlm")
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M2 = exdqlmMCMC(y, p0=0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.burn = 20, n.mcmc = 20,
                init.from.vb = FALSE, verbose = FALSE)
plot(M2)
```

plot.exdqlmSynthesis *Plot Method for* exdqlmSynthesis *Objects*

Description

Plot the pointwise posterior predictive interval produced by quantileSynthesis. The method is intentionally separate from quantileSynthesis() so the synthesis step remains a computation, while the returned object still has a standard plotting interface.

Usage

```
## S3 method for class 'exdqlmSynthesis'
plot(
  x,
  y = NULL,
  time = NULL,
```

```
  add = FALSE,
  interval = 0.95,
  show.median = TRUE,
  show.mean = FALSE,
  band.col = grDevices::adjustcolor("lightblue", alpha.f = 0.35),
  median.col = "blue",
  mean.col = "darkblue",
  y.col = "dark grey",
  border = NA,
  xlab = "time",
  ylab = "posterior predictive synthesis",
  main = NULL,
  xlim = NULL,
  ylim = NULL,
  ...
)
```

Arguments

x	An exdqlmSynthesis object.
y	Optional observed series to overlay.
time	Optional time vector for the synthesized summaries. If omitted, seq_len(T) is used, where T is the number of synthesized time points.
add	Logical; add the synthesis interval to an existing plot.
interval	Numeric in (0, 1) giving the plotted central interval. Currently 0.50 and 0.95 are supported from stored summaries.
show.median	Logical; draw the synthesized posterior median.
show.mean	Logical; draw the synthesized posterior mean.
band.col	Fill color for the predictive interval.
median.col	Color for the posterior median line.
mean.col	Color for the posterior mean line.
y.col	Color for the optional observed series.
border	Border color for the predictive interval polygon.
xlab, ylab, main	Graphical labels.
xlim, ylim	Optional axis limits.
...	Additional graphical arguments passed to the initial plot() call when add = FALSE.

polytrendMod *Create an n-th order polynomial exDQLM component*

Description

The function creates an n-th order polynomial exDQLM component.

Usage

```
polytrendMod(order, m0, C0, backend = c("auto", "R", "cpp"))
```

Arguments

order	Numeric order n of the polynomial model.
m0	Optional numeric prior mean. Defaults to $n \times 1$ vector of zeros.
C0	Optional numeric prior covariance. Defaults to matrix $10^3 I_n$.
backend	Backend selection for matrix construction: "auto" (default), "R", or "cpp".

Value

An object of class "exdqlm" containing the following:

- FF - $n \times 1$ observational vector.
- GG - $n \times n$ evolution matrix.
- m0 - $n \times 1$ prior mean of the state vector.
- C0 - $n \times n$ prior covariance matrix of the state vector.

Examples

```
# create a second order polynomial component
trend.comp = polytrendMod(2, rep(0, 2), 10*diag(2))
```

print.exalStaticDiagnostic *Print Method for* exalStaticDiagnostic *Objects*

Description

Print Method for exalStaticDiagnostic Objects

Usage

```
## S3 method for class 'exalStaticDiagnostic'
print(x, ...)
```

Arguments

x	An exalStaticDiagnostic object.
...	Additional arguments (unused).

---

print.exalStaticLDVB *Print Method for* exalStaticLDVB *Objects*

Description

Print Method for exalStaticLDVB Objects

Usage

```
## S3 method for class 'exalStaticLDVB'
print(x, ...)
```

Arguments

x	An exalStaticLDVB object.
...	Additional arguments (unused).

---

print.exalStaticMCMC *Print Method for* exalStaticMCMC *Objects*

Description

Print Method for exalStaticMCMC Objects

Usage

```
## S3 method for class 'exalStaticMCMC'
print(x, ...)
```

Arguments

x	An exalStaticMCMC object.
...	Additional arguments (unused).

print.exdqlm *Print exDQLM model details*

Description

Print the details of the exDQLM model.

Usage

```
## S3 method for class 'exdqlm'
print(x, ...)
```

Arguments

x	a exdqlm object.
...	further arguments (unused).

print.exdqlmDiagnostic *Print Method for* exdqlmDiagnostic *Objects*

Description

Print Method for exdqlmDiagnostic Objects

Usage

```
## S3 method for class 'exdqlmDiagnostic'
print(x, ...)
```

Arguments

x	An exdqlmDiagnostic object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.95), dim.df = c(1),
                  gam.init = -3.5, sig.init = 15,
                  n.samp = 20, tol = 0.2, verbose = FALSE)
M0.diags = exdqlmDiagnostics(M0, plot=FALSE)
print(M0.diags)
```

```
options(old)
```

`print.exdqlmForecast` *Print Method for* `exdqlmForecast` *Objects*

Description

Print Method for `exdqlmForecast` Objects

Usage

```
## S3 method for class 'exdqlmForecast'
print(x, ...)
```

Arguments

`x`	An `exdqlmForecast` object.
`...`	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.samp = 20, tol = 0.2, verbose = FALSE)
M0.forecast = exdqlmForecast(start.t = 50, k = 5, m1 = M0)
print(M0.forecast)
options(old)
```

`print.exdqlmForecastDiagnostic`
Print Method for `exdqlmForecastDiagnostic` *Objects*

Description

Print Method for `exdqlmForecastDiagnostic` Objects

Usage

```
## S3 method for class 'exdqlmForecastDiagnostic'
print(x, ...)
```

Arguments

x	An exdqlmForecastDiagnostic object.
...	Additional arguments (unused).

---

print.exdqlmISVB *Print Method for* exdqlmISVB *Objects*

Description

Print Method for exdqlmISVB Objects

Usage

```
## S3 method for class 'exdqlmISVB'
print(x, ...)
```

Arguments

x	An exdqlmISVB object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
# Legacy ISVB object retained for backward-compatible inspection methods
M0 = exdqlmISVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.IS = 20, n.samp = 20, tol = 0.2,
                   verbose = FALSE)
print(M0)
options(old)
```

print.exdqlmLDVB *Print Method for* exdqlmLDVB *Objects*

Description

Print Method for exdqlmLDVB Objects

Usage

```
## S3 method for class 'exdqlmLDVB'
print(x, ...)
```

Arguments

x	An exdqlmLDVB object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.samp = 20, tol = 0.2, verbose = FALSE)
print(M0)
options(old)
```

print.exdqlmMCMC *Print Method for* exdqlmMCMC *Objects*

Description

Print Method for exdqlmMCMC Objects

Usage

```
## S3 method for class 'exdqlmMCMC'
print(x, ...)
```

Arguments

x	An exdqlmMCMC object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M2 = exdqlmMCMC(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.burn = 20, n.mcmc = 20,
                init.from.vb = FALSE, verbose = FALSE)
print(M2)
```

---

print.exdqlmSynthesis *Print Method for* exdqlmSynthesis *Objects*

Description

Print Method for exdqlmSynthesis Objects

Usage

```
## S3 method for class 'exdqlmSynthesis'
print(x, ...)
```

Arguments

x	An exdqlmSynthesis object.
...	Additional arguments (unused).

---

qexal *Quantile Function for the exAL Distribution*

Description

Vectorized over p.

Usage

```
qexal(
  p,
  p0 = 0.5,
  mu = 0,
  sigma = 1,
  gamma = 0,
  lower.tail = TRUE,
  log.p = FALSE
)
```

Arguments

p	Numeric vector of probabilities in (0, 1).
p0	Probability level used in the quantile parametrization. Scalar in (0, 1). Default 0.5.
mu	Location parameter (scalar). Default 0.
sigma	Scale parameter (scalar, strictly positive). Default 1.
gamma	Skewness parameter controlling asymmetry (scalar). Must be within valid bounds implied by p0. Default 0.
lower.tail	Logical scalar; if TRUE (default) return $P(X \leq q)$, otherwise $P(X > q)$.
log.p	Logical scalar; if TRUE, return log-probabilities.

Value

Numeric vector of quantiles (same length as p).

Examples

```
p <- seq(0.1, 0.9, by = 0.2)
q <- qexal(p, p0 = 0.5, mu = 0, sigma = 1, gamma = 0)
all.equal(p, pexal(q, p0 = 0.5, mu = 0, sigma = 1, gamma = 0), tol = 1e-4)
```

---

quantileSynthesis *Synthesize a unified posterior predictive distribution from multiple quantile-model draws*

Description

The function synthesizes posterior predictive draws from multiple fitted quantile models into a single posterior predictive distribution. It uses a two-step correction: (i) isotonic regression at the grid of target quantiles to align the fitted quantile levels, and (ii) distributional alignment (shift each model's draws so its tau-quantile matches the isotone anchor). It then builds a single predictive quantile function per time by piecewise-linear blending across adjacent quantile models with optional global monotone rearrangement.

Usage

```
quantileSynthesis(
  draws_list,
  p,
  enforce_isotonic = TRUE,
  rearrange = TRUE,
  grid_M = 1001L,
  n_samp = 1000L,
  seed = NULL,
  T_expected = NULL
)
```

Arguments

draws_list	List of length L; each element is either: (i) a numeric matrix of posterior predictive draws (T x ns or ns x T), (ii) a dynamic fit object (exdqlmMCMC, exdqlmLDVB, or legacy exdqlmISVB) with samp.post.pred, or (iii) an exdqlmForecast object with samp.fore. Rows are coerced to time.
p	Numeric vector of target quantile levels in (0,1) of length L (same order as draws_list, not necessarily sorted). Duplicate levels are not allowed.
enforce_isotonic	Logical; apply isotonic regression (PAVA) over the grid p at each time t to remove crossing. Default TRUE.
rearrange	Logical; apply monotone rearrangement (evaluate -> sort -> reinterpolate) on a dense grid over u in (0,1). Default TRUE.
grid_M	Integer; size of dense grid M for rearrangement (u_k = k/(M+1)). Default 1001L.
n_samp	Integer; number of synthesized draws per time. Default 1000L.
seed	NULL or integer for reproducible synthesized draws. Default NULL.
T_expected	Optional integer; if provided, forces the time dimension to T_expected when orienting each matrix to T x ns. This avoids accidental transposes.

Value

An object of class "exdqlmSynthesis", which is a list containing:

- draws - Numeric matrix T x n_samp of synthesized draws.
- levels - Sorted copy of p (length L).
- quantiles - Numeric matrix T x L of isotone anchors m^*_{i,t}.
- summary - List with row-wise summaries of draws (mean, q025, q250, q500, q750, q975).
- method - List of synthesis settings used (name, isotonic, rearrange, grid_M, T_inferred).

Examples

```
# short example
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 10L)
TT = 50
y = scIVTmag[1:TT]

# create a compact trend model
trend.comp = polytrendMod(1, stats::quantile(y, 0.85), 10)
model = trend.comp

# fit quantiles using LDVB and save posterior predictive samples
fits <- draws <- NULL
p0s = c(0.10, 0.50, 0.90)
for(i in 1:length(p0s)){
  fits[[i]] = exdqlmLDVB(
    y, p0 = p0s[i], model, df = 0.98, dim.df = 1,
    sig.init = 15, n.samp = 20, tol = 0.2, verbose = FALSE
```

```
  )
  draws[[i]] = fits[[i]]$samp.post.pred
}

# synthesize posterior predictive from all quantiles
syn = quantileSynthesis(
  draws_list = draws,
  p = p0s,
  T_expected = TT)

# alternatively, pass fitted dynamic objects directly
syn2 = quantileSynthesis(
  draws_list = fits,
  p = p0s,
  T_expected = TT)

# plot the synthesized 95% posterior predictive interval
plot(syn2, y = y)
options(old)
```

`regMod` *Create a standard regression component for an exDQLM*

Description

The function constructs a regression block where the observation vector at time t is $F_t = X_t$ (row of the design matrix), and the state evolves as $\theta_t = \theta_{t-1}$ (i.e., $G_t = I_n$).

Usage

```
regMod(X, m0, C0)
```

Arguments

`X`	A numeric matrix of dimension $T \times n$ (T time points, n regressors). Vectors are accepted and treated as $T \times 1$.
`m0`	Optional numeric prior mean (length n). Defaults to zeros.
`C0`	Optional numeric prior covariance ($n \times n$). Defaults to $10^3 I_n$.

Details

Input `X` is a $T \times n$ matrix of regressors; the returned `FF` is an $n \times T$ matrix (i.e., `t(X)`), consistent with component composition via `+.exdqlm`.

Value

An object of class "exdqlm" with elements:

- FF - $n \times T$ matrix with column t equal to $F_t = X_t$.
- GG - $n \times n$ identity matrix (static coefficients).
- m0, C0 - Prior mean/covariance for regression coefficients.

Examples

```
data("climateIndices", package = "exdqlm")

T <- 150
bt_dates <- seq(as.Date("1987-01-01"), by = "month", length.out = T)
idx <- match(bt_dates, climateIndices$date)
X <- scale(climateIndices[idx, c("noi", "amo")])

# Single regressor (T x 1)
reg1 = regMod(X[, "noi"])
# Multiple regressors (T x n)
reg2 = regMod(X)

# Combine with trend/seasonal components
trend.comp = polytrendMod(order = 3, m0 = rep(0,3), C0 = diag(3))
seas.comp  = seasMod(p = 12, h = 1, C0 = diag(1, 2))
base.mod   = trend.comp + seas.comp
model.std  = base.mod + reg2
```

---

rexal *Random Sample Generation for the exAL Distribution*

Description

Random Sample Generation for the exAL Distribution

Usage

```
rexal(n, p0 = 0.5, mu = 0, sigma = 1, gamma = 0)
```

Arguments

n	Positive integer number of samples to draw (scalar).
p0	Probability level used in the quantile parametrization. Scalar in (0, 1). Default 0.5.
mu	Location parameter (scalar). Default 0.
sigma	Scale parameter (scalar, strictly positive). Default 1.
gamma	Skewness parameter controlling asymmetry (scalar). Must be within valid bounds implied by p0. Default 0.

Value

Numeric vector of length n.

Examples

```
set.seed(1)
rexal(3, p0 = 0.5, mu = c(-1, 0, 1))
```

---

scIVTmag *Time series of daily average magnitude IVT in Santa Cruz, CA.*

Description

ECMWF Re-Analysis 5 (ERA5) daily average magnitude IVT in Santa Cruz, CA (approximately 22 N, 122 W) from January 1, 1979 to December 31, 2019 with all February 29ths omitted.

Usage

```
scIVTmag
```

Format

A time series of length 14965.

Source

https://cds.climate.copernicus.eu

References

Hersbach, H, Bell, B, Berrisford, P, et al. *The ERA5 global reanalysis.* Q J R Meteorol Soc. 2020; 146: 1999– 2049. doi:10.1002/qj.3803

---

seasMod *Create Fourier representation of a periodic exDQLM component*

Description

The function creates a Fourier form periodic component for given period and harmonics.

Usage

```
seasMod(p, h, m0, C0, backend = c("auto", "R", "cpp"))
```

Arguments

p	Numeric period.
h	Numeric vector of harmonics to be included.
m0	Optional numeric prior mean. Defaults to $q \times 1$ vector of zeros where q is the dimension of the period component.
C0	Optional numeric prior covariance. Defaults to matrix $10^3 I_q$.
backend	Backend selection for matrix construction: "auto" (default), "R", or "cpp".

Value

An object of class "exdqlm" containing the following:

- FF - $q \times 1$ observational vector.
- GG - $q \times q$ evolution matrix.
- m0 - $q \times 1$ prior mean of the state vector.
- C0 - $q \times q$ prior covariance matrix of the state vector.

Examples

```
# create a seasonal component with first, second and fourth harmonics of a period of 365
seas.comp = seasMod(365, c(1, 2, 4), C0 = 10*diag(6))
```

---

summary.exalStaticDiagnostic

Summary Method for exalStaticDiagnostic *Objects*

Description

Summary Method for exalStaticDiagnostic Objects

Usage

```
## S3 method for class 'exalStaticDiagnostic'
summary(object, ...)
```

Arguments

object	An exalStaticDiagnostic object.
...	Additional arguments (unused).

summary.exalStaticLDVB *Summary Method for* exalStaticLDVB *Objects*

Description

Summary Method for exalStaticLDVB Objects

Usage

```
## S3 method for class 'exalStaticLDVB'
summary(object, ...)
```

Arguments

object	An exalStaticLDVB object.
...	Additional arguments (unused).

summary.exalStaticMCMC *Summary Method for* exalStaticMCMC *Objects*

Description

Summary Method for exalStaticMCMC Objects

Usage

```
## S3 method for class 'exalStaticMCMC'
summary(object, ...)
```

Arguments

object	An exalStaticMCMC object.
...	Additional arguments (unused).

summary.exdqlm *Summary exDQLM model details*

Description

Print the details of the exDQLM model.

Usage

```
## S3 method for class 'exdqlm'
summary(object, ...)
```

Arguments

object	a exdqlm object.
...	further arguments (unused).

summary.exdqlmDiagnostic *Summary Method for* exdqlmDiagnostic *Objects*

Description

Summary Method for exdqlmDiagnostic Objects

Usage

```
## S3 method for class 'exdqlmDiagnostic'
summary(object, ...)
```

Arguments

object	An exdqlmDiagnostic object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.95), dim.df = c(1),
                  gam.init = -3.5, sig.init = 15,
                  n.samp = 20, tol = 0.2, verbose = FALSE)
M0.diags = exdqlmDiagnostics(M0, plot = FALSE)
summary(M0.diags)
```

```
options(old)
```

`summary.exdqlmForecast` *Summary Method for* `exdqlmForecast` *Objects*

Description

Summary Method for `exdqlmForecast` Objects

Usage

```
## S3 method for class 'exdqlmForecast'
summary(object, ...)
```

Arguments

`object`	An `exdqlmForecast` object.
`...`	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.samp = 20, tol = 0.2, verbose = FALSE)
M0.forecast = exdqlmForecast(start.t = 50, k = 5, m1 = M0)
summary(M0.forecast)
options(old)
```

`summary.exdqlmForecastDiagnostic` *Summary Method for* `exdqlmForecastDiagnostic` *Objects*

Description

Summary Method for `exdqlmForecastDiagnostic` Objects

Usage

```
## S3 method for class 'exdqlmForecastDiagnostic'
summary(object, ...)
```

Arguments

object	An exdqlmForecastDiagnostic object.
...	Additional arguments (unused).

---

summary.exdqlmISVB *Summary Method for* exdqlmISVB *Objects*

Description

Summary Method for exdqlmISVB Objects

Usage

```
## S3 method for class 'exdqlmISVB'
summary(object, ...)
```

Arguments

object	An exdqlmISVB object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
# Legacy ISVB object retained for backward-compatible inspection methods
M0 = exdqlmISVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.IS = 20, n.samp = 20, tol = 0.2,
                   verbose = FALSE)
summary(M0)
options(old)
```

summary.exdqlmLDVB *Summary Method for* exdqlmLDVB *Objects*

Description

Summary Method for exdqlmLDVB Objects

Usage

```
## S3 method for class 'exdqlmLDVB'
summary(object, ...)
```

Arguments

object	An exdqlmLDVB object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
old = options(exdqlm.max_iter = 15L)
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M0 = exdqlmLDVB(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                   gam.init = -3.5, sig.init = 15,
                   n.samp = 20, tol = 0.2, verbose = FALSE)
summary(M0)
options(old)
```

summary.exdqlmMCMC *Summary Method for* exdqlmMCMC *Objects*

Description

Summary Method for exdqlmMCMC Objects

Usage

```
## S3 method for class 'exdqlmMCMC'
summary(object, ...)
```

Arguments

object	An exdqlmMCMC object.
...	Additional arguments (unused).

Examples

```
data("scIVTmag", package = "exdqlm")
y = scIVTmag[1:60]
model = polytrendMod(1, stats::quantile(y, 0.85), 10)
M2 = exdqlmMCMC(y, p0 = 0.85, model, df = c(0.98), dim.df = c(1),
                gam.init = -3.5, sig.init = 15,
                n.burn = 20, n.mcmc = 20,
                init.from.vb = FALSE, verbose = FALSE)
summary(M2)
```

---

summary.exdqlmSynthesis

Summary Method for `exdqlmSynthesis` *Objects*

Description

Summary Method for `exdqlmSynthesis` Objects

Usage

```
## S3 method for class 'exdqlmSynthesis'
summary(object, time = NULL, ...)
```

Arguments

`object`	An `exdqlmSynthesis` object.
`time`	Optional vector of time values. If supplied, it must have length equal to the number of rows in `object$draws`.
`...`	Additional arguments (unused).

Value

A data frame containing pointwise summaries of the synthesized posterior predictive draws.

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