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Title Bayesian Spectral Analysis Models using Gaussian Process Priors

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Imports MASS, ggplot2, gridExtra

Description Contains functions to perform Bayesian inference using a spectral analysis of Gaussian process priors. Gaussian processes are represented with a Fourier series based on cosine basis functions. Currently the package includes parametric linear models, partial linear additive models with/without shape restrictions, generalized linear additive models with/without shape restrictions, and density estimation model. To maximize computational efficiency, the actual Markov chain Monte Carlo sampling for each model is done using codes written in FORTRAN 90. This software has been developed using funding supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (no. NRF-2016R1D1A1B03932178 and no. NRF-2017R1D1A3B03035235).

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 $b1q$

Bayesian Quantile Regression

Description

This function fits a Bayesian quantile regression model.

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Usage

 $blq(formula, data = NULL, p, mcmc = list(), prior = list(), marginal.likelihood = TRUE)$

Arguments

Details

This generic function fits a Bayesian quantile regression model.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. Further, let $x_{i,k}$ be the covariate related to the response, linearly. The model is as follows.

$$
y_i = w_i^T \beta + \epsilon_i, \ i = 1, \dots, n,
$$

where the error terms $\{\epsilon_i\}$ are a random sample from an asymmetric Laplace distribution, $ALD_p(0, \sigma^2)$, which has the following probability density function:

,

$$
ALD_p(\epsilon; \mu, \sigma^2) = \frac{p(1-p)}{\sigma^2} \exp\left(-\frac{(x-\mu)[p-I(x \le \mu)]}{\sigma^2}\right)
$$

where $0 < p < 1$ is the skew parameter, $\sigma^2 > 0$ is the scale parameter, $-\infty < \mu < \infty$ is the location parameter, and $I(\cdot)$ is the indication function.

The conjugate priors are assumed for β and σ :

$$
\beta|\sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)
$$

Value

An object of class blm representing the Bayesian parametric linear model fit. Generic functions such as print and fitted have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, 56, 501-514.

Kozumi, H. and Kobayashi, G. (2011) Gibbs sampling methods for Bayesian quantile regression. *Journal of Statistical Computation and Simulation*, 81(11), 1565-1578.

See Also

[blr](#page-3-1), [gblr](#page-33-1)

Examples

```
#####################
# Simulated example #
#####################
# Simulate data
set.seed(1)
n < - 100w \leftarrow runif(n)y \le -3 + 2 \times w + \text{rad}(n, \text{ scale} = 0.8, p = 0.5)# Fit median regression
fout \leftarrow blq(y \sim w, p = 0.5)
# Summary
print(fout); summary(fout)
# fitted values
fit <- fitted(fout)
# Plots
plot(fout)
```
blr *Bayesian Linear Regression*

Description

This function fits a Bayesian linear regression model using scale invariant prior.

Usage

blr(formula, data = NULL, mcmc = list(), prior = list(), marginal.likelihood = TRUE)

Arguments

a logical variable indicating whether the log marginal likelihood is calculated.

Details

This generic function fits a Bayesian linear regression model using scale invariant prior.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. The model for regression function is as follows.

$$
y_i = w_i^T \beta + \epsilon_i, \ i = 1, \dots, n,
$$

where the error terms $\{\epsilon_i\}$ are a random sample from a normal distribution, $N(0, \sigma^2)$. The conjugate priors are assumed for β and σ :

$$
\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)
$$

Value

An object of class blm representing the Bayesian spectral analysis model fit. Generic functions such as print and fitted have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws and the posterior samples of the fitted values are stored in the list fit.draws. The output list also includes the following objects:

See Also

[blq](#page-1-1), [gblr](#page-33-1)

Examples

```
#####################
# Simulated example #
#####################
# Simulate data
set.seed(1)
n < -100w \leftarrow runif(n)y \le -3 + 2 \times w + \text{rnorm}(n, \text{sd} = 0.8)# Fit the model with default priors and mcmc parameters
fout \leftarrow blr(y \sim w)
# Summary
print(fout); summary(fout)
# Fitted values
fit <- fitted(fout)
# Plots
plot(fout)
```
bsad *Bayesian Semiparametric Density Estimation*

Description

This function fits a semiparametric model, which consists of parametric and nonparametric components, for estimating density using a logistic Gaussian process.

Usage

```
bsad(x, xmin, xmax, nint, MaxNCos, mcmc = list(), prior = list(),smoother = c('geometric', 'algebraic'),
parametric = c('none', 'normal', 'gamma', 'laplace'), marginal.likelihood = TRUE,
verbose = FALSE)
```
Arguments

Details

This generic function fits a semiparametric model, which consists of parametric and nonparametric, for density estimation (Lenk, 2003):

$$
f(x|\beta, Z) = \frac{\exp[h(x)^\top \beta + Z(x)]}{\int_{\mathcal{X}} \exp[h(y)^\top \beta + Z(y)] dG(y)}
$$

where Z is a zero mean, second-order Gaussian process with bounded, continuous covariance function. i.e.,

$$
E[Z(x), Z(y)] = \sigma(x, y), \quad \int_{\mathcal{X}} ZdG = 0 \quad (a.s.)
$$

Using the Karhunen-Loeve Expansion, Z is represented as infinite series with random coefficients

$$
Z(x) = \sum_{j=1}^{\infty} \theta_j \varphi_j(x),
$$

where $\{\varphi_j\}$ is the cosine basis, $\varphi_j(x) = \sqrt{2} \cos[j\pi G(x)]$.

For the random Fourier coefficients of the expansion, two smoother priors are assumed (optional),

$$
\theta_j|\tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), j \ge 1 \text{ (geometric smoother)}
$$

$$
\theta_j|\tau, \gamma \sim N(0, \tau^2 \exp[-\ln(j+1)\gamma]), j \ge 1 \text{ (algebraic smoother)}
$$

The coefficient β have the popular normal prior,

$$
\beta|m_{0,\beta},V_{0,\beta}\sim N(m_{0,\beta},V_{0,\beta})
$$

To complete the model specification, independent hyper priors are assumed,

$$
\tau^2 | r_0, s_0 \sim IGa(r_0/2, s_0/2)
$$

$$
\gamma | w_0 \sim Exp(w_0)
$$

Note that the posterior algorithm is based on computing a discrete version of the likelihood over a fine mesh on X .

Value

An object of class bsad representing the Bayesian spectral analysis density estimation model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Jo, S., Choi, T., Park, B. and Lenk, P. (2019). bsamGP: An R Package for Bayesian Spectral Analysis Models Using Gaussian Process Priors. *Journal of Statistical Software*, 90, 310-320.

Lenk, P. (2003) Bayesian semiparametric density estimation and model verification using a logistic Gaussian process. *Journal of Computational and Graphical Statistics*, 12, 548-565.

Examples

```
## Not run:
############################
# Old Faithful geyser data #
############################
data(faithful)
attach(faithful)
# mcmc parameters
mcmc \leq - list(nblow = 10000,
            smcmc = 1000,
            nskip = 10,ndisp = 1000,
```
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```
kappaloop = 5)
# fits BSAD model
fout \le bsad(x = eruptions, xmin = 0, xmax = 8, nint = 501, mcmc = mcmc,
             smoother = 'geometric', parametric = 'gamma')
# Summary
print(fout); summary(fout)
# fitted values
fit <- fitted(fout)
# predictive density plot
plot(fit, ask = TRUE)
detach(faithful)
## End(Not run)
```
bsaq *Bayesian Shape-Restricted Spectral Analysis Quantile Regression*

Description

This function fits a Bayesian semiparametric quantile regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors.

Usage

```
bsaq(formula, xmin, xmax, p, nbasis, nint, mcmc = list(), prior = list(),
shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
'IncreasingRotatedS','DecreasingRotatedS','InvertedU','Ushape',
'IncMultExtreme','DecMultExtreme'), nExtreme = NULL,
marginal.likelihood = TRUE, spm.adequacy = FALSE, verbose = FALSE)
```
Arguments

prior a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): iflagprior choosing a smoothing prior for spectral coefficients (iflagprior=0 assigns T-Smoother prior (default), iflagprior=1 chooses Lasso-Smoother prior), theta0_m0 and theta0_s0 giving the hyperparameters for prior distribution of the spectral coefficients (theta0_m0 and theta0_s0 are used when the functions have shape-restriction), tau2_m0, tau2_s0 and w0 giving the prior mean and standard deviation of smoothing prior (When iflagprior=1, tau2_m0 is only used as the hyperparameter), beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response, alpha_m0 and alpha_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, iflagpsi determining the prior of slope for logisitic function in S or U shaped (iflagpsi=1 (default), slope ψ is sampled and iflagpsi=0, ψ is fixed), psifixed giving initial value (iflagpsi=1) or fixed value (iflagpsi=0) of slope, omega_m0 and omega_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function. shape a vector giving types of shape restriction. nExtreme a vector of extreme points for 'IncMultExtreme', 'DecMultExtreme' shape restrictions. marginal.likelihood

> a logical variable indicating whether the log marginal likelihood is calculated. The methods of Gelfand and Dey (1994) and Newton and Raftery (1994) are used.

spm.adequacy a logical variable indicating whether the log marginal likelihood of linear model is calculated. The marginal likelihood gives the values of the linear regression model excluding the nonlinear parts.

verbose a logical variable. If TRUE, the iteration number and the Metropolis acceptance rate are printed to the screen.

Details

This generic function fits a Bayesian spectral analysis quantile regression model for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, the model assumed that the derivatives of the functions are squares of Gaussian processes.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. Further, let $x_{i,k}$ be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$
y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \ i = 1, ..., n,
$$

where f_k is an unknown shape-restricted function of the scalar $x_{i,k} \in [0,1]$ and the error terms $\{\epsilon_i\}$ are a random sample from an asymmetric Laplace distribution, $ALD_p(0, \sigma^2)$, which has the following probability density function:

$$
ALD_p(\epsilon; \mu, \sigma^2) = \frac{p(1-p)}{\sigma^2} \exp\left(-\frac{(x-\mu)[p-I(x \le \mu)]}{\sigma^2}\right),\,
$$

where $0 < p < 1$ is the skew parameter, $\sigma^2 > 0$ is the scale parameter, $-\infty < \mu < \infty$ is the location parameter, and $I(\cdot)$ is the indication function.

The prior of function without shape restriction is:

$$
f(x) = Z(x),
$$

where Z is a second-order Gaussian process with mean function equal to zero and covariance function $\nu(s,t) = E[Z(s)Z(t)]$ for $s, t \in [0,1]$. The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$
Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)
$$

$$
\varphi_0(x) = 1 \text{ and } \varphi_j(x) = \sqrt{2} \cos(\pi jx), \ j \ge 1, \ 0 \le x \le 1
$$

The shape-restricted functions are modeled by assuming the qth derivatives of f are squares of Gaussian processes:

$$
f^{(q)}(x) = \delta Z^2(x)h(x), \ \delta \in \{1, -1\}, \ q \in \{1, 2\},\
$$

where h is the squish function. For monotonic, monotonic convex, and concave functions, $h(x) =$ 1, while for S and U shaped functions, h is defined by

$$
h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \ \psi > 0, \ 0 < \omega < 1
$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in β):

$$
\theta_j|\sigma, \tau, \gamma \sim N(0, \sigma^2 \tau^2 \exp[-j\gamma]), j \ge 1
$$

The priors for the spectral coefficients of shape restricted functions are:

$$
\theta_0 | \sigma \sim N(m_{\theta_0}, \sigma v_{\theta_0}^2), \quad \theta_j | \sigma, \tau, \gamma \sim N(m_{\theta_j}, \sigma \tau^2 \exp[-j\gamma]), \ j \ge 1
$$

To complete the model specification, the conjugate priors are assumed for β and σ :

$$
\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)
$$

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Value

An object of class bsam representing the Bayesian spectral analysis model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Jo, S., Choi, T., Park, B. and Lenk, P. (2019). bsamGP: An R Package for Bayesian Spectral Analysis Models Using Gaussian Process Priors. *Journal of Statistical Software*, 90, 310-320.

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, 27: 43-69.

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, 56, 501-514.

Kozumi, H. and Kobayashi, G. (2011) Gibbs sampling methods for Bayesian quantile regression. *Journal of Statistical Computation and Simulation*, 81(11), 1565-1578.

Newton, M. A. and Raftery, A. E. (1994) Approximate Bayesian inference with the weighted likelihood bootstrap (with discussion). *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, 56, 3-48.

See Also

[bsar](#page-16-1), [gbsar](#page-36-1)

Examples

```
## Not run:
######################
# Increasing-concave #
######################
# Simulate data
set.seed(1)
```
n <- 200

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```
x \leftarrow runif(n)y \le -\log(1 + 10*x) + \text{rald}(n, \text{ scale} = 0.5, p = 0.5)# Number of cosine basis functions
nbasis <- 50
# Fit the model with default priors and mcmc parameters
fout1 <- bsaq(y \sim fs(x), p = 0.25, nbasis = nbasis,
              shape = 'IncreasingConcave')
fout2 <- bsaq(y \sim fs(x), p = 0.5, nbasis = nbasis,
              shape = 'IncreasingConcave')
fout3 <- bsaq(y \sim fs(x), p = 0.75, nbasis = nbasis,
               shape = 'IncreasingConcave')
# fitted values
fit1 <- fitted(fout1)
fit2 <- fitted(fout2)
fit3 <- fitted(fout3)
# plots
plot(x, y, lwd = 2, xlab = 'x', ylab = 'y')lines(fit1$xgrid, fit1$wbeta$mean[1] + fit1$fxgrid$mean, lwd=2, col=2)
lines(fit2$xgrid, fit2$wbeta$mean[1] + fit2$fxgrid$mean, lwd=2, col=3)
lines(fit3$xgrid, fit3$wbeta$mean[1] + fit3$fxgrid$mean, lwd=2, col=4)
legend('topleft', legend = c('1st Quartile', '2nd Quartile', '3rd Quartile'),
       1wd = 2, col = 2:4, 1ty = 1)
```
End(Not run)

bsaqdpm *Bayesian Shape-Restricted Spectral Analysis Quantile Regression with Dirichlet Process Mixture Errors*

Description

This function fits a Bayesian semiparametric quantile regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors. The model assumes that the errors follow a Dirichlet process mixture model.

Usage

```
bsaqdpm(formula, xmin, xmax, p, nbasis, nint,
mcmc = list(), prior = list(), egrid, ngrid = 500,
shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
'IncreasingRotatedS', 'DecreasingRotatedS', 'InvertedU', 'Ushape'),
verbose = FALSE)
```
Arguments

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Details

This generic function fits a Bayesian spectral analysis quantile regression model for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, the model assumes that the derivatives of the functions are squares of Gaussian processes. The model also assumes that the errors follow a Dirichlet process mixture model.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. Further, let $x_{i,k}$ be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$
y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \ i = 1, ..., n,
$$

where f_k is an unknown shape-restricted function of the scalar $x_{i,k} \in [0,1]$ and the error terms $\{\epsilon_i\}$ are a random sample from a Dirichlet process mixture of an asymmetric Laplace distribution, $ALD_p(0, \sigma^2)$, which has the following probability density function:

$$
\epsilon_i \sim f(\epsilon) = \int ALD_p(\epsilon; 0, \sigma^2) dG(\sigma^2),
$$

$$
G \sim DP(M, G0), \quad G0 = Ga\left(\sigma^{-2}; \frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)
$$

.

The prior of function without shape restriction is:

$$
f(x) = Z(x),
$$

where Z is a second-order Gaussian process with mean function equal to zero and covariance function $\nu(s,t) = E[Z(s)Z(t)]$ for $s, t \in [0,1]$. The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$
Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)
$$

$$
\varphi_0(x) = 1 \text{ and } \varphi_j(x) = \sqrt{2} \cos(\pi jx), \ j \ge 1, \ 0 \le x \le 1
$$

The shape-restricted functions are modeled by assuming the qth derivatives of f are squares of Gaussian processes:

$$
f^{(q)}(x) = \delta Z^2(x)h(x), \ \delta \in \{1, -1\}, \ q \in \{1, 2\},\
$$

where h is the squish function. For monotonic, monotonic convex, and concave functions, $h(x) =$ 1, while for S and U shaped functions, h is defined by

$$
h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \ \psi > 0, \ 0 < \omega < 1
$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in β):

$$
\theta_j|\tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), j \ge 1
$$

The priors for the spectral coefficients of shape restricted functions are:

$$
\theta_0 \sim N(m_{\theta_0}, v_{\theta_0}^2), \quad \theta_j | \tau, \gamma \sim N(m_{\theta_j}, \tau^2 \exp[-j\gamma]), \ j \geq 1
$$

To complete the model specification, the popular normal prior is assumed for β :

$$
\beta \sim N(m_{0,\beta}, V_{0,\beta})
$$

Value

An object of class bsam representing the Bayesian spectral analysis model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list $loglik.draws$. The output list also includes the following objects:

References

Jo, S., Choi, T., Park, B. and Lenk, P. (2019). bsamGP: An R Package for Bayesian Spectral Analysis Models Using Gaussian Process Priors. *Journal of Statistical Software*, 90, 310-320.

Kozumi, H. and Kobayashi, G. (2011) Gibbs sampling methods for Bayesian quantile regression. *Journal of Statistical Computation and Simulation*, 81(11), 1565-1578.

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, 27, 43-69.

MacEachern, S. N. and Müller, P. (1998) Estimating mixture of Dirichlet process models. *Journal of Computational and Graphical Statistics*, 7, 223-238.

Mukhopadhyay, S. and Gelfand, A. E. (1997) Dirichlet process mixed generalized linear models. *Journal of the American Statistical Association*, 92, 633-639.

Neal, R. M. (2000) Markov chain sampling methods for Dirichlet process mixture models. *Journal of Computational and Graphical Statistics*, 9, 249-265.

See Also

[bsaq](#page-8-1), [bsardpm](#page-23-1)

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Examples

```
## Not run:
######################
# Increasing-concave #
######################
# Simulate data
set.seed(1)
n < -500x \leq runif(n)e \leq c (rald(n/2, scale = 0.5, p = 0.5),
       rald(n/2, scale = 3, p = 0.5)y \le -\log(1 + 10*x) + e# Number of cosine basis functions
nbasis <- 50
# Fit the model with default priors and mcmc parameters
fout1 <- bsaqdpm(y \sim fs(x), p = 0.25, nbasis = nbasis,
                 shape = 'IncreasingConcave')
fout2 <- bsaqdpm(y \sim fs(x), p = 0.5, nbasis = nbasis,
                 shape = 'IncreasingConcave')
fout3 <- bsaqdpm(y ~ fs(x), p = 0.75, nbasis = nbasis,
                 shape = 'IncreasingConcave')
# fitted values
fit1 <- fitted(fout1)
fit2 <- fitted(fout2)
fit3 <- fitted(fout3)
# plots
plot(x, y, lwd = 2, xlab = 'x', ylab = 'y')lines(fit1$xgrid, fit1$wbeta$mean[1] + fit1$fxgrid$mean, lwd=2, col=2)
lines(fit2$xgrid, fit2$wbeta$mean[1] + fit2$fxgrid$mean, lwd=2, col=3)
lines(fit3$xgrid, fit3$wbeta$mean[1] + fit3$fxgrid$mean, lwd=2, col=4)
legend('topleft',legend=c('1st Quartile','2nd Quartile','3rd Quartile'),
       lwd=2, col=2:4, lty=1)
```
End(Not run)

bsar *Bayesian Shape-Restricted Spectral Analysis Regression*

Description

This function fits a Bayesian semiparametric regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors.

Usage

```
bsar(formula, xmin, xmax, nbasis, nint, mcmc = list(), prior = list(),
shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
'IncreasingRotatedS','DecreasingRotatedS','InvertedU','Ushape',
'IncMultExtreme','DecMultExtreme'), nExtreme = NULL,
marginal.likelihood = TRUE, spm.adequacy = FALSE, verbose = FALSE)
```
Arguments

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Details

This generic function fits a Bayesian spectral analysis regression model (Lenk and Choi, 2015) for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, they assumed that the derivatives of the functions are squares of Gaussian processes.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. Further, let $x_{i,k}$ be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$
y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \ i = 1, ..., n,
$$

where f_k is an unknown shape-restricted function of the scalar $x_{i,k} \in [0,1]$ and the error terms $\{\epsilon_i\}$ are a random sample from a normal distribution, $N(0, \sigma^2)$.

The prior of function without shape restriction is:

$$
f(x) = Z(x),
$$

where Z is a second-order Gaussian process with mean function equal to zero and covariance function $\nu(s,t) = E[Z(s)Z(t)]$ for $s, t \in [0,1]$. The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$
Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)
$$

$$
\varphi_0(x) = 1 \text{ and } \varphi_j(x) = \sqrt{2} \cos(\pi jx), \ j \ge 1, \ 0 \le x \le 1
$$

The shape-restricted functions are modeled by assuming the qth derivatives of f are squares of Gaussian processes:

$$
f^{(q)}(x) = \delta Z^2(x)h(x), \ \ \delta \in \{1, -1\}, \ \ q \in \{1, 2\},
$$

where h is the squish function. For monotonic, monotonic convex, and concave functions, $h(x) =$ 1, while for S and U shaped functions, h is defined by

$$
h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \ \psi > 0, \ 0 < \omega < 1
$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in β):

$$
\theta_j|\sigma, \tau, \gamma \sim N(0, \sigma^2 \tau^2 \exp[-j\gamma]), j \ge 1
$$

The priors for the spectral coefficients of shape restricted functions are:

$$
\theta_0 | \sigma \sim N(m_{\theta_0}, \sigma v_{\theta_0}^2), \quad \theta_j | \sigma, \tau, \gamma \sim N(m_{\theta_j}, \sigma \tau^2 \exp[-j\gamma]), \ j \ge 1
$$

To complete the model specification, the conjugate priors are assumed for β and σ :

$$
\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)
$$

Value

An object of class bsam representing the Bayesian spectral analysis model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Jo, S., Choi, T., Park, B. and Lenk, P. (2019). bsamGP: An R Package for Bayesian Spectral Analysis Models Using Gaussian Process Priors. *Journal of Statistical Software*, 90, 310-320.

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, 27, 43-69.

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, 56, 501-514.

Newton, M. A. and Raftery, A. E. (1994) Approximate Bayesian inference with the weighted likelihood bootstrap (with discussion). *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, 56, 3-48.

See Also

[bsardpm](#page-23-1)

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Examples

```
## Not run:
##########################################
# Increasing Convex to Concave (S-shape) #
##########################################
# simulate data
f <- function(x) 5*exp(-10*(x - 1)^4) + 5*x^2set.seed(1)
n < -100x \leftarrow runif(n)y \leftarrow f(x) + \text{norm}(n, sd = 1)# Number of cosine basis functions
nbasis <- 50
# Fit the model with default priors and mcmc parameters
fout \leq bsar(y \sim fs(x), nbasis = nbasis, shape = 'IncreasingConvex',
             spm.adequacy = TRUE)
# Summary
print(fout); summary(fout)
# Trace plots
plot(fout)
# fitted values
fit <- fitted(fout)
# Plot
plot(fit, ask = TRUE)
#############################################
# Additive Model #
# Monotone-Increasing and Increasing-Convex #
#############################################
# Simulate data
f1 <- function(x) 2*pi*x + sin(2*pi*x)f2 <- function(x) exp(6*x - 3)n < - 200x1 \leftarrow runif(n)x2 \le- runif(n)
x \leftarrow \text{cbind}(x1, x2)y \le -5 + f1(x1) + f2(x2) + rnorm(n, sd = 0.5)
```

```
# Number of cosine basis functions
nbasis <- 50
# MCMC parameters
mcmc <- list(nblow0 = 1000, nblow = 10000, nskip = 10,
             smcmc = 5000, ndisp = 1000, maxmodmet = 10)
# Prior information
xmin \leftarrow apply(x, 2, min)xmax \leftarrow apply(x, 2, max)
xrange <- xmax - xmin
prior \le list(iflagprior = 0, theta0_m0 = 0, theta0_s0 = 100,
              tau2_m0 = 1, tau2_v0 = 100, w0 = 2,
              beta_m0 = numeric(1), beta_v0 = diag(100, 1),
              sigma2_m0 = 1, sigma2_v0 = 1000,
              alpha_m0 = 3, alpha_s0 = 50, iflagpsi = 1,
              psifixed = 1000, omega_m0 = (xmin + xmax)/2,
              omega_S0 = (xrange)/8)# Fit the model with user specific priors and mcmc parameters
fout \leq bsar(y \leq fs(x1) + fs(x2), nbasis = nbasis, mcmc = mcmc, prior = prior,
             shape = c('Increasing', 'IncreasingS'))
# Summary
print(fout); summary(fout)
## End(Not run)
```


```
bsarBig Bayesian Spectral Analysis Regression for Big data
```
Description

This function fits a Bayesian spectral analysis regression model for Big data.

Usage

```
bsarBig(formula, nbasis, nint, mcmc = list(), prior = list(), verbose = FALSE)
```
Arguments

Value

The MCMC samples of the parameters in the model are stored in the list mcmc.draws and the posterior samples of the fitted values are stored in the list fit.draws. The output list also includes the following objects:

See Also

[bsar](#page-16-1)

Examples

```
# Ttrue function
ftrue <- function(x){
  ft <- 7*exp(-3*x) + 2*exp(-70*(x-.6)^2) - 2 + 5*x
  return(ft)
}
# Generate data
set.seed(1)
nobs <- 100000 # Number of observations
sigmat <- .5 # True sigma
nxgrid <- 500 # number of grid points: approximate likelihood & plots
xdata <- runif(nobs) # Generate x values
fobst <- ftrue(xdata) # True f at observations
ydata <- fobst + sigmat*rnorm(nobs)
# Compute grid on 0 to 1
xdelta <- 1/nxgrid
xgrid <- seq(xdelta/2, 1-xdelta/2, xdelta)
xgrid <- matrix(xgrid,nxgrid)
fxgridt <- ftrue(xgrid) # True f on xgrid
# Fit data
```

```
fout \leq bsarBig(ydata \sim xdata, nbasis = 50, nint = nxgrid, verbose = TRUE)
# Plots
smcmc <- fout$mcmc$smcmc
t < -1:smcmc
par(mfrow=c(2,2))
matplot(t, fout$mcmc.draws$theta, type = "l", main = "Theta", xlab = "Iteration", ylab = "Draw")
plot(t, fout$mcmc.draws$sigma, type = "l", main = "Sigma", xlab = "Iteration", ylab = "Draw")
matplot(t, fout$mcmc.draws$tau, type = "l", main = "Tau", xlab = "Iteration", ylab = "Draw")
matplot(t, fout$mcmc.draws$gamma, type = "l", main = "Gamma", xlab = "Iteration", ylab = "Draw")
dev.new()
matplot(fout$fit.draws$xgrid, cbind(fxgridt, fout$post.est$fhatm, fout$post.est$fhatq),
        type = "1", main = "Regression Function", xlab = "X", ylab = "Y")
# Compute RMISE for regression function
sse <- (fout$post.est$fhatm - fxgridt)^2
rmise <- intgrat(sse, 1/nxgrid)
rmise <- sqrt(rmise)
rmise
```
bsardpm *Bayesian Shape-Restricted Spectral Analysis Regression with Dirichlet Process Mixture Errors*

Description

This function fits a Bayesian semiparametric regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors. The model assumes that the errors follow a Dirichlet process mixture model.

Usage

```
bsardpm(formula, xmin, xmax, nbasis, nint,
mcmc = list(), prior = list(), egrid, ngrid, location = TRUE,
shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
'IncreasingRotatedS','DecreasingRotatedS','InvertedU','Ushape'),
verbose = FALSE)
```
Arguments

mcmc a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): nblow0 (1000) giving the number of initialization period for adaptive metropolis, maxmodmet (5) giving the maximum number of times to modify metropolis, nblow (10000) giving the number of MCMC in transition period, nskip (10) giving the thinning interval, smcmc (1000) giving the number of MCMC for analysis, and ndisp (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every ndisp iterations have been carried out).

- prior a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): iflagprior choosing a smoothing prior for spectral coefficients (iflagprior=0 assigns T-Smoother prior (default), iflagprior=1 chooses Lasso-Smoother prior), theta0_m0 and theta0_s0 giving the hyperparameters for prior distribution of the spectral coefficients (theta0_m0 and theta0_s0 are used when the functions have shape-restriction), tau2_m0, tau2_s0 and w0 giving the prior mean and standard deviation of smoothing prior (When iflagprior=1, tau2_m0 is only used as the hyperparameter), beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response, alpha_m0 and alpha_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, iflagpsi determining the prior of slope for logisitic function in S or U shaped (iflagpsi=1 (default), slope ψ is sampled and iflagpsi=0, ψ is fixed), psifixed giving initial value (iflagpsi=1) or fixed value (iflagpsi=0) of slope, omega_m0 and omega_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function. egrid a vector giving grid points where the residual density estimate is evaluated. The default range is from -10 to 10. ngrid a vector giving number of grid points where the residual density estimate is evaluated. The default value is 500.
- location a logical value. If it is true, error density is modelled using location-scale mixture.
- shape a vector giving types of shape restriction.
- verbose a logical variable. If TRUE, the iteration number and the Metropolis acceptance rate are printed to the screen.

Details

This generic function fits a Bayesian spectral analysis regression model for estimating shaperestricted functions using Gaussian process priors. For enforcing shape-restrictions, the model assumes that the derivatives of the functions are squares of Gaussian processes. The model also assumes that the errors follow a Dirichlet process mixture model.

.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. Further, let $x_{i,k}$ be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$
y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \ i = 1, ..., n,
$$

where f_k is an unknown shape-restricted function of the scalar $x_{i,k} \in [0,1]$ and the error terms $\{\epsilon_i\}$ are a random sample from a Dirichlet process mixture model,

1. scale mixture :

$$
\epsilon_i \sim f(\epsilon) = \int N(\epsilon; 0, \sigma^2) dG(\sigma^2),
$$

$$
G \sim DP(M, G0), \quad G0 = Ga\left(\sigma^{-2}; \frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right).
$$

2. location-scale mixture :

$$
\epsilon_i \sim f(\epsilon) = \int N(\epsilon; \mu, \sigma^2) dG(\mu, \sigma^2),
$$

$$
G \sim DP(M, G0), \quad G0 = N(\mu; \mu_0, \kappa \sigma^2) \, Ga\left(\sigma^{-2}; \frac{r_{0, \sigma}}{2}, \frac{s_{0, \sigma}}{2}\right)
$$

The prior of function without shape restriction is:

$$
f(x) = Z(x),
$$

where Z is a second-order Gaussian process with mean function equal to zero and covariance function $\nu(s,t) = E[Z(s)Z(t)]$ for $s, t \in [0,1]$. The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$
Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)
$$

$$
\varphi_0(x) = 1 \text{ and } \varphi_j(x) = \sqrt{2} \cos(\pi jx), \ j \ge 1, \ 0 \le x \le 1
$$

The shape-restricted functions are modeled by assuming the q th derivatives of f are squares of Gaussian processes:

$$
f^{(q)}(x) = \delta Z^2(x)h(x), \ \delta \in \{1, -1\}, \ q \in \{1, 2\},\
$$

where h is the squish function. For monotonic, monotonic convex, and concave functions, $h(x) =$ 1, while for S and U shaped functions, h is defined by

$$
h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \ \psi > 0, \ 0 < \omega < 1
$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in β):

$$
\theta_j|\tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), j \ge 1
$$

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The priors for the spectral coefficients of shape restricted functions are:

$$
\theta_0 \sim N(m_{\theta_0}, v_{\theta_0}^2), \quad \theta_j | \tau, \gamma \sim N(m_{\theta_j}, \tau^2 \exp[-j\gamma]), \ j \ge 1
$$

To complete the model specification, the popular normal prior is assumed for β :

$$
\beta \vert \sim N(m_{0,\beta}, V_{0,\beta})
$$

Value

An object of class bsam representing the Bayesian spectral analysis model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Jo, S., Choi, T., Park, B. and Lenk, P. (2019). bsamGP: An R Package for Bayesian Spectral Analysis Models Using Gaussian Process Priors. *Journal of Statistical Software*, 90, 310-320.

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, 27, 43-69.

MacEachern, S. N. and Müller, P. (1998) Estimating mixture of Dirichlet process models. *Journal of Computational and Graphical Statistics*, 7, 223-238.

Mukhopadhyay, S. and Gelfand, A. E. (1997) Dirichlet process mixed generalized linear models. *Journal of the American Statistical Association*, 92, 633-639.

Neal, R. M. (2000) Markov chain sampling methods for Dirichlet process mixture models. *Journal of Computational and Graphical Statistics*, 9, 249-265.

See Also

[bsar](#page-16-1), [bsaqdpm](#page-12-1)

Examples

```
## Not run:
#####################
# Increasing-convex #
#####################
# Simulate data
set.seed(1)
n <- 200
x \leftarrow runif(n)e \leq c(\text{norm}(n/2, sd = 0.5), \text{norm}(n/2, sd = 3))y \le - \exp(6*x - 3) + e# Number of cosine basis functions
nbasis <- 50
# Fit the model with default priors and mcmc parameters
fout \leq bsardpm(y \sim fs(x), nbasis = nbasis, shape = 'IncreasingConvex')
# Summary
print(fout); summary(fout)
# fitted values
fit <- fitted(fout)
# Plot
plot(fit, ask = TRUE)
## End(Not run)
```
cadmium *Cadmium dose-response meta data*

Description

This dataset includes minimal information of NCC-2012 meta data.

Usage

data("cadmium")

Format

A data frame with 190 observations on the following 5 variables.

gender a numeric vector with 1 : Female, 0 : Male, 0.5 : Unknown or both

ethnicity a integer vector with 1 : Asian, 2 : Caucasian

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Ucd_GM a numeric vector of Geometric means of urinary cadmium

b2_GM a numeric vector of Geometric means of Beta2-Microglobulin

isOld a logical vector whether the observation is older than 50

References

Lee, Minjea, Choi, Taeryon; Kim, Jeongseon; Woo, Hae Dong (2013) Bayesian Analysis of Dose-Effect Relationship of Cadmium for Benchmark Dose Evaluation. *Korean Journal of Applied Statistics*, 26(3), 453–470.

Examples

Not run: data(cadmium)

End(Not run)

Elec.demand *Electricity demand data*

Description

The Elec.demand data consists of 288 quarterly observations in Ontario from 1971 to 1994.

Usage

data(Elec.demand)

Format

A data frame with 288 observations on the following 7 variables.

quarter date (yyyy-mm) from 1971 to 1994

enerm electricity demand.

gdp gross domestic product.

pelec price of electricity.

pgas price of natural gas.

hddqm the number of heating degree days relative to a reference temperature.

cddqm the number of cooling degree days relative to a reference temperature.

Source

Yatchew, A. (2003). *Semiparametric Regression for the Applied Econometrician*. Cambridge University Press.

References

Engle, R. F., Granger, C. W. J., Rice, J. and Weiss, A. (1986). Semiparametric estimates of the relation between weather and electricity sales. *Journal of the American Statistical Association*, 81, 310-320.

Lenk, P. and Choi, T. (2017). Bayesian analysis of shape-restricted functions using Gaussian process priors. *Statistica Sinica*, 27, 43-69.

Examples

```
## Not run:
data(Elec.demand)
plot(Elec.demand)
```
End(Not run)

fitted.blm *Compute fitted values for a blm object*

Description

Computes pointwise posterior means and 95% credible intervals of the fitted Bayesian linear models.

Usage

S3 method for class 'blm' fitted(object, alpha = 0.05 , HPD = TRUE, ...)

Arguments

Details

None.

Value

A list containing posterior means and 95% credible intervals. The output list includes the following objects:

wbeta posterior estimates for regression function. yhat posterior estimates for generalised regression function.

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References

Chen, M., Shao, Q. and Ibrahim, J. (2000) *Monte Carlo Methods in Bayesian computation*. Springer-Verlag New York, Inc.

See Also

[blq](#page-1-1), [blr](#page-3-1), [gblr](#page-33-1)

Examples

See examples for blq and blr

fitted.bsad *Compute fitted values for a bsad object*

Description

Computes pointwise posterior means and $100(1 - \alpha)\%$ credible intervals of the fitted Bayesian spectral analysis density estimation model.

Usage

S3 method for class 'bsad' fitted(object, alpha = 0.05 , HPD = TRUE, ...)

Arguments

Details

None.

Value

A list object of class fitted.bsad containing posterior means and $100(1 - \alpha)\%$ credible intervals. Generic function plot displays the results of the fit.

The output list includes the following objects:

See Also

[bsad](#page-5-1)

Examples

See examples for bsad

fitted.bsam *Compute fitted values for a bsam object*

Description

Computes pointwise posterior means and $100(1 - \alpha)\%$ credible intervals of the fitted Bayesian spectral analysis models.

Usage

S3 method for class 'bsam' fitted(object, alpha = 0.05 , HPD = TRUE, ...)

Arguments

Details

None.

Value

A list object of class fitted.bsam containing posterior means and $100(1 - \alpha)\%$ credible intervals. Generic function plot displays the results of the fit.

The output list includes the following objects:

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See Also

[bsaq](#page-8-1), [bsaqdpm](#page-12-1), [bsar](#page-16-1), [bsardpm](#page-23-1)

Examples

See examples for bsaq, bsaqdpm, bsar, and bsardpm

fitted.bsamdpm *Compute fitted values for a bsamdpm object*

Description

Computes pointwise posterior means and $100(1 - \alpha)\%$ credible intervals of the fitted Bayesian spectral analysis models with Dirichlet process mixture error.

Usage

S3 method for class 'bsamdpm' fitted(object, alpha = 0.05 , HPD = TRUE, ...)

Arguments

Details

None.

Value

A list object of class fitted.bsamdpm containing posterior means and 95% credible intervals. Generic function plot displays the results of the fit.

The output list includes the following objects:

See Also

[bsaqdpm](#page-12-1), [bsardpm](#page-23-1)

Examples

See examples for bsaqdpm and bsardpm

fs *Specify a Fourier Basis Fit in a BSAM Formula*

Description

A symbolic wrapper to indicate a nonparametric term in a formula argument to bsaq, bsaqdpm, bsar, bsardpm, and gbsar.

Usage

 $fs(x)$

Arguments

x a vector of the univariate covariate for nonparametric component

Examples

Not run:

fit x using a Fourier basis $y \sim w + fs(x)$ # fit x1 and x2 using a Fourier basis $y ~ 5(s(1) + fs(x2)$

End(Not run)

gblr *Generalized Bayesian Linear Models*

Description

This function fits a Bayesian generalized linear regression model.

Usage

```
gblr(formula, data = NULL, family, link, mcmc = list(), prior = list(),
marginal.likelihood = TRUE, algorithm = c('AM', 'KS'), verbose = FALSE)
```
Arguments

Details

This generic function fits a Bayesian generalized linear regression models.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. The model is as follows.

$$
y_i | \mu_i \sim F(\mu_i),
$$

$$
g(\mu_i) = w_i^T \beta, \ i = 1, \dots, n,
$$

where $g(\cdot)$ is a link function and $F(\cdot)$ is a distribution of an exponential family.

For unknown coefficients, the following prior is assumed for β :

$$
\beta \sim N(m_{0,\beta}, V_{0,\beta})
$$

The prior for the dispersion parameter of negative-binomial regression is

$$
\kappa \sim Ga(r_0,s_0)
$$

Value

An object of class blm representing the generalized Bayesian linear model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Albert, J. H. and Chib, S. (1993) Bayesian Analysis of Binary and Polychotomous Response Data. *Journal of the American Statistical Association*, 88, 669-679.

Holmes, C. C. and Held, L. (2006) Bayesian Auxiliary Variables Models for Binary and Multinomial Regression. *Bayesian Analysis*, 1, 145-168.

Gelfand, A. E. and Dey, K. K. (1994) Bayesian Model Choice: Asymptotics and Exact Calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, 56, 501-514.

Roberts, G. O. and Rosenthal, J. S. (2009) Examples of Adaptive MCMC. *Journal of Computational and Graphical Statistics*, 18, 349-367.

See Also

[blr](#page-3-1), [blq](#page-1-1)

Examples

```
############################
# Poisson Regression Model #
############################
```

```
# Simulate data
set.seed(1)
```

```
n < - 100x \leftarrow runif(n)y <- rpois(n, exp(0.5 + x*0.4))
```

```
# Fit the model with default priors and mcmc parameters
fout \leq gblr(y \sim x, family = 'poisson', link = 'log')
```


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```
# Summary
print(fout); summary(fout)
# Plot
plot(fout)
# fitted values
fitf <- fitted(fout)
```
gbsar *Bayesian Shape-Restricted Spectral Analysis for Generalized Partial Linear Models*

Description

This function fits a Bayesian generalized partial linear regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors.

Usage

```
gbsar(formula, xmin, xmax, family, link, nbasis, nint, mcmc = list(), prior = list(),
shape = c('Free','Increasing','Decreasing','IncreasingConvex','DecreasingConcave',
          'IncreasingConcave','DecreasingConvex','IncreasingS','DecreasingS',
          'IncreasingRotatedS','DecreasingRotatedS','InvertedU','Ushape'),
marginal.likelihood = TRUE, algorithm = c('AM', 'KS'), verbose = FALSE)
```
Arguments

the number of saved draws to be displayed on screen (the function reports on the screen when every ndisp iterations have been carried out).

verbose a logical variable. If TRUE, the iteration number and the Metropolis acceptance rate are printed to the screen.

Details

This generic function fits a Bayesian generalized partial linear regression models for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, they assumed that the derivatives of the functions are squares of Gaussian processes.

Let y_i and w_i be the response and the vector of parametric predictors, respectively. Further, let $x_{i,k}$ be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$
y_i | \mu_i \sim F(\mu_i),
$$

$$
g(\mu_i) = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}), i = 1, \dots, n,
$$

where $g(\cdot)$ is a link function and f_k is an unknown nonlinear function of the scalar $x_{i,k} \in [0,1]$.

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The prior of function without shape restriction is:

$$
f(x) = Z(x),
$$

where Z is a second-order Gaussian process with mean function equal to zero and covariance function $\nu(s,t) = E[Z(s)Z(t)]$ for $s, t \in [0,1]$. The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$
Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)
$$

$$
\varphi_0(x) = 1 \text{ and } \varphi_j(x) = \sqrt{2} \cos(\pi jx), \ j \ge 1, \ 0 \le x \le 1
$$

The shape-restricted functions are modeled by assuming the q th derivatives of f are squares of Gaussian processes:

$$
f^{(q)}(x) = \delta Z^2(x)h(x), \ \ \delta \in \{1, -1\}, \ \ q \in \{1, 2\},
$$

where h is the squish function. For monotonic, monotonic convex, and concave functions, $h(x) =$ 1, while for S and U shaped functions, h is defined by

$$
h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \ \psi > 0, \ 0 < \omega < 1
$$

For the spectral coefficients of functions without shape constraints, the following prior is used (The intercept is included in β):

$$
\theta_j|\tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), \ j \ge 1
$$

The priors for the spectral coefficients of shape restricted functions are:

$$
\theta_0 \vert \sim N(m_{\theta_0}, v_{\theta_0}^2), \quad \theta_j \vert \tau, \gamma \sim N(m_{\theta_j}, \tau^2 \exp[-j\gamma]), \ j \ge 1
$$

To complete the model specification, the following prior is assumed for β :

$$
\beta \sim N(m_{0,\beta}, V_{0,\beta})
$$

Value

An object of class bsam representing the Bayesian spectral analysis model fit. Generic functions such as print, fitted and plot have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list mcmc.draws, the posterior samples of the fitted values are stored in the list fit.draws, and the MCMC samples for the log marginal likelihood are saved in the list loglik.draws. The output list also includes the following objects:

References

Jo, S., Choi, T., Park, B. and Lenk, P. (2019). bsamGP: An R Package for Bayesian Spectral Analysis Models Using Gaussian Process Priors. *Journal of Statistical Software*, 90, 310-320.

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, 27, 43-69.

Roberts, G. O. and Rosenthal, J. S. (2009) Examples of Adaptive MCMC. *Journal of Computational and Graphical Statistics*, 18, 349-367.

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Albert, J. H. and Chib, S. (1993) Bayesian Analysis of Binary and Polychotomous Response Data. *Journal of the American Statistical Association*, 88, 669-679.

See Also

[bsaq](#page-8-1), [bsar](#page-16-1)

Examples

```
## Not run:
###########################
# Probit Regression Model #
###########################
# Simulate data
set.seed(1)
f \le function(x) 1.5 \star sin(pi \star x)
n < -1000b \leq c(1,-1)rho <-0.7u \le - runif(n, min = -1, max = 1)
x \le - runif(n, min = -1, max = 1)
w1 \le runif(n, min = -1, max = 1)
w2 \leftarrow \text{round}(f(\text{rho} * x + (1 - \text{rho}) * u))w <- cbind(w1, w2)
y \le -w %*% b + f(x) + rnorm(n)
y \leftarrow (y > 0)# Number of cosine basis functions
nbasis <- 50
# Fit the model with default priors and mcmc parameters
```
intgrat 41

```
fout \leq gbsar(y \sim w1 + w2 + fs(x), family = "bernoulli", link = "probit",
              nbasis = nbasis, shape = 'Free')
# Summary
print(fout); summary(fout)
# fitted values
fit <- fitted(fout)
# Plot
plot(fit, ask = TRUE)
######################################
# Logistic Additive Regression Model #
######################################
# Wage-Union data
data(wage.union); attach(wage.union)
race[race==1 | race==2]=0
race[race==3]=1
y <- union
w <- cbind(race,sex,south)
x <- cbind(wage,education,age)
# mcmc parameters
mcmc <- list(nblow0 = 10000,
             nblow = 10000,
             nskip = 10,
             smcmc = 1000,
             ndisp = 1000,
             maxmodmet = 10foutGBSAR <- gbsar(y \sim race + sex + south + fs(wage) + fs(education) + fs(age),
                   family = 'bernoulli', link = 'logit', nbasis = 50, mcmc = mcmc,
                   shape = c('Free','Decreasing','Increasing'))
# fitted values
fitGBSAR <- fitted(foutGBSAR)
# Plot
plot(fitGBSAR, ask = TRUE)
## End(Not run)
```
Description

Trapezoidal rule is a technique for approximating the definite integral.

Usage

```
intgrat(f, delta)
```
Arguments

Value

intgrat returns the value of the intergral.

intsim *Numerical integration using Simpson's rule*

Description

Simpson's rule is a method for numerical integration.

Usage

intsim(f, delta)

Arguments

Value

intsim returns the value of the intergral.

Description

The London.Mortality data consists of daily death occurrences from Jan. 1st, 1993 to Dec. 31st, 2006 and corresponding weather observations including temperature and humidity in London.

Usage

```
data(London.Mortality)
```
Format

A data frame with 5113 observations on the following 7 variables.

date date in YYYY-MM-DD.

tmean Mean temperature.

tmin Minimum dry-bulb temperature.

tmax Maximum dry-bulb temperature.

dewp Dew point.

rh Relative humidity.

death the number of death occurences.

Source

Office for National Statistics

British Atmospheric Data Centre

https://github.com/gasparrini/2015_gasparrini_Lancet_Rcodedata

References

Armstrong BG, Chalabi Z, Fenn B, Hajat S, Kovats S, Milojevic A, Wilkinson P (2011). Association of mortality with high temperatures in a temperate climate: England and Wales. *Journal of Epidemiology & Community Health*, 65(4), 340–345.

Gasparrini A, Armstrong B, Kovats S, Wilkinson P (2012). The effect of high temperatures on cause-specific mortality in England and Wales. *Occupational and Environmental Medicine*, 69(1), 56–61.

Gasparrini A, Guo Y, Hashizume M, Lavigne E, Zanobetti A, Schwartz J, Tobias A, Tong S, Rocklöv J, Forsberg B, et al.(2015). Mortality risk attributable to high and low ambient temperature: a multicountry observational study. *The Lancet*, 386(9991), 369-375.

Examples

```
## Not run:
data(London.Mortality)
```
End(Not run)

plasma *A Data Set for Plasma Levels of Retinol and Beta-Carotene*

Description

This data set contains 314 observations on 14 variables.

Usage

data(plasma)

Format

age Age (years). sex Sex (1=Male, 2=Female). smoke Smoking status (1=Never, 2=Former, 3=Current Smoker). vmi BMI values (weight/(height^2)). vitas Vitamin use (1=Yes,fairly often, 2=Yes, not often, 3=No). calories Number of calories consumed per day. fat Grams of fat consumed per day. fiber Grams of fiber consumed per day. alcohol Number of alcoholic drinks consumed per week. cholesterol Cholesterol consumed (mg per day). beta diet Dietary beta-carotene consumed (mcg per day). reedit Dietary retinol consumed (mcg per day). betaplasma Plasma beta-carotene (ng/ml). retplasma Plasma Retinol (ng/ml).

Source

https://lib.stat.cmu.edu/datasets/Plasma_Retinol

References

Nierenberg, D. W., Stukel, T. A., Baron, J. A., Dain, B. J., and Greenberg, E. R. (1989). Determinants of plasma levels of beta-carotene and retinol. *American Journal of Epidemiology*, 130, 511-521.

Meyer, M. C., Hackstadt, A. J., and Hoeting, J. A. (2011). Bayesian estimation and inference for generalized partial linear models using shape-restricted splines. *Journal of Nonparametric Statistics*, 23(4), 867-884.

plot.blm and the contract of t

Examples

Not run: data(plasma)

End(Not run)

plot.blm *Plot a blm object*

Description

Plots the posterior samples for Bayesian linear models

Usage

S3 method for class 'blm' plot(x, ...)

Arguments

x a blm object ... other options to pass to the plotting functions

Value

Returns a plot.

See Also

[blq](#page-1-1), [blr](#page-3-1)

Examples

See examples for blq and blr

Description

Plots the posterior samples for Bayesian semiparametric density estimation using a logistic Gaussian process.

Usage

S3 method for class 'bsad' plot(x, ...)

Arguments

Value

Returns a plot.

See Also

[bsad](#page-5-1)

Examples

See examples for bsad

plot.bsam *Plot a bsam object*

Description

Plots the posterior samples for Bayesian spectral analysis models.

Usage

S3 method for class 'bsam' $plot(x, \ldots)$

Arguments

plot.bsamdpm 47

Value

Returns a plot.

See Also

[bsaq](#page-8-1), [bsaqdpm](#page-12-1), [bsar](#page-16-1), [bsardpm](#page-23-1)

Examples

See examples for bsaq, bsaqdpm, bsar, and bsardpm

plot.bsamdpm *Plot a bsamdpm object*

Description

Plots the posterior samples for Bayesian spectral analysis models with Dirichlet process mixture error.

Usage

S3 method for class 'bsamdpm' plot(x, ...)

Arguments

Value

Returns a plot.

See Also

[bsaqdpm](#page-12-1), [bsardpm](#page-23-1)

Examples

See examples for bsaqdpm and bsardpm

Description

Plots the predictive density for Bayesian density estimation model using logistic Gaussian process

Usage

S3 method for class 'fitted.bsad' plot(x, ggplot2, legend.position, nbins, ...)

Arguments

Value

Returns a plot.

See Also

[bsad](#page-5-1), [fitted.bsad](#page-30-1)

Examples

See example for bsad

plot.fitted.bsam *Plot a fitted.bsam object*

Description

Plots the data and the fit for Bayesian spectral analysis models.

Usage

```
## S3 method for class 'fitted.bsam'
plot(x, type, ask, ggplot2, legend.position, ...)
```
Arguments

Value

Returns a plot.

See Also

[bsaq](#page-8-1), [bsaqdpm](#page-12-1), [bsar](#page-16-1), [bsardpm](#page-23-1), [fitted.bsam](#page-31-1)

Examples

See examples for bsaq, bsaqdpm, bsar, and bsardpm

plot.fitted.bsamdpm *Plot a fitted.bsamdpm object*

Description

Plots the data and the fit for Bayesian spectral analysis models with Dirichlet process mixture error.

Usage

```
## S3 method for class 'fitted.bsamdpm'
plot(x, ask, ggplot2, legend.position, ...)
```
Arguments

Value

Returns a plot.

See Also

[bsaqdpm](#page-12-1), [bsardpm](#page-23-1), [fitted.bsamdpm](#page-32-1)

Examples

See examples for bsaqdpm and bsardpm

predict.blm *Predict method for a blm object*

Description

Computes predicted values of Bayesian linear models.

Usage

```
## S3 method for class 'blm'
predict(object, newdata, alpha = 0.05, HPD = TRUE, ...)
```
Arguments

Details

None.

Value

A list containing posterior means and 95% credible intervals.

The output list includes the following objects:

wbeta posterior estimates for regression function. yhat posterior estimates for generalised regression function.

predict.bsam 51

References

Chen, M., Shao, Q. and Ibrahim, J. (2000) *Monte Carlo Methods in Bayesian computation*. Springer-Verlag New York, Inc.

See Also

[blq](#page-1-1), [blr](#page-3-1), [gblr](#page-33-1)

Examples

```
## Not run:
#####################
# Simulated example #
#####################
# Simulate data
   set.seed(1)
   n < -100w \leftarrow runif(n)y \le -3 + 2 \times w + \text{rnorm}(n, \text{sd} = 0.8)# Fit the model with default priors and mcmc parameters
   fout \leftarrow blr(y \sim w)
   # Predict
   new <- rnorm(n)
   predict(fout, newdata = new)
```
End(Not run)

predict.bsam *Predict method for a bsam object*

Description

Computes the predicted values of Bayesian spectral analysis models.

Usage

```
## S3 method for class 'bsam'
predict(object, newp, newnp, alpha = 0.05, HPD = TRUE, type = "response", ...)
```
Arguments

Details

None.

Value

A list object of class predict.bsam containing posterior means and $100(1-\alpha)\%$ credible intervals. The output list includes the following objects:

See Also

[bsaq](#page-8-1), [bsar](#page-16-1), [gbsar](#page-36-1)

Examples

Not run:

```
##########################################
# Increasing Convex to Concave (S-shape) #
##########################################
# simulate data
f <- function(x) 5*exp(-10*(x - 1)^4) + 5*x^2set.seed(1)
n < - 100x \leftarrow runif(n)y \leftarrow f(x) + \text{norm}(n, sd = 1)# Number of cosine basis functions
```
predict.bsamdpm 53

```
nbasis <- 50
# Fit the model with default priors and mcmc parameters
fout \leq bsar(y \leq fs(x), nbasis = nbasis, shape = 'IncreasingConvex',
             spm.adequacy = TRUE)
# Prediction
xnew <- runif(n)
predict(fout, newnp = xnew)
## End(Not run)
```
predict.bsamdpm *Predict method for a bsamdpm object*

Description

Computes the predicted values of Bayesian spectral analysis models with Dirichlet process mixture errors.

Usage

S3 method for class 'bsamdpm' $predict(object, newp, newnp, alpha = 0.05, HPD = TRUE, ...)$

Arguments

Details

None.

Value

A list object of class predict.bsamdpm containing posterior means and $100(1 - \alpha)\%$ credible intervals.

The output list includes the following objects:

See Also

[bsaqdpm](#page-12-1), [bsardpm](#page-23-1)

Examples

Not run:

```
#####################
# Increasing-convex #
#####################
# Simulate data
set.seed(1)
n < -200x \leftarrow runif(n)e \leq c(\text{rnorm}(n/2, sd = 0.5), \text{rnorm}(n/2, sd = 3))y \le - \exp(6*x - 3) + e# Number of cosine basis functions
nbasis <- 50
# Fit the model with default priors and mcmc parameters
fout \leq bsardpm(y \leq fs(x), nbasis = nbasis, shape = 'IncreasingConvex')
# Prediction
xnew < -runif(n)predict(fout, newnp = xnew)
```
End(Not run)

rald *The asymmetric Laplace distribution*

Description

Density for and random values from a three-parameter asymmetric Laplace distribution.

traffic 55

Usage

rald(n, location=0, scale=1, p=0.5)

Arguments

Details

This generic function generates a random variable from an asymmetric Laplace distribution (ALD). The ALD has the following probability density function:

$$
ALD_p(x; \mu, \sigma) = \frac{p(1-p)}{\sigma} \exp\left(-\frac{(x-\mu)[p-I(x \leq \mu)]}{\sigma}\right),\,
$$

where $0 < p < 1$ is the skew parameter, $\sigma > 0$ is the scale parameter, $-\infty < \mu < \infty$ is the location parameter, and $I(\cdot)$ is the indication function. The range of x is $(-\infty, \infty)$.

Value

rald gives out a vector of random numbers generated by the asymmetric Laplace distribution.

References

Koenker, R. and Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. *Journal of the American Statistical Association*, 94(3), 1296-1309.

Yu, K. and Zhang, J. (2005). A Three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics - Theory and Methods*, 34, 1867-1879.

traffic *Monthly traffic accidents data*

Description

This data set contains 108 observations on 6 variables.

Usage

data(traffic)

Format

ln_number logarithm of the number of monthly automobile accidents in the state of Michigan. month months from January 1st, 1979 to Decembe 31st, 1987.

ln_unemp logarithm of unemployment rate

spring indicator for spring season.

summer indicator for summer season.

fall indicator for fall season.

References

Lenk (1999) Bayesian inference for semiparametric regression using a Fourier representation. *Journal of the Royal Statistical Society: Series B*, 61(4), 863-879.

Examples

```
## Not run:
data(traffic)
pairs(traffic)
```
End(Not run)

wage.union *Wage-Union data*

Description

This data set contains 534 observations on 11 variables.

Usage

data(wage.union)

Format

education number of years of education. south indicator of living in southern region of U.S.A. sex gender indicator: 0=male,1=female. experience number of years of work experience. union indicator of trade union membership: 0=non-member, 1=member. wage wages in dollars per hour. age age in years. race 1=black, 2=Hispanic, 3=white. occupation 1=management, 2=sales, 3=clerical, 4=service, 5=professional, 6=other. sector 0=other, 1=manufacturing, 2=construction. married indicator of being married: 0=unmarried, 1=married.

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References

Berndt, E.R. (1991) *The Practice of Econometrics*. New York: Addison-Wesley.

Ruppert, D., Wand, M.P. and Carroll, R.J. (2003) *Semiparametric Regression*. Cambridge University Press.

Examples

```
## Not run:
data(wage.union)
 pairs(wage.union)
```
End(Not run)

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