Finding the Lovasz Number

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November 27, 2024

The Lovasz Number of a graph $\mathbf{G}$, denoted $\vartheta(\mathbf{G})$, is the upper bound on the Shannon capacity of the graph ([1]). For an adjacency matrix $\mathbf{B} = [B_{ij}]$ the problem of finding the Lovasz number is given by the following primal SQLP problem

$$\begin{array}{lll} \underset{\mathbf{X}}{\text{minimize}} & tr(\mathbf{CX}) & \\ \text{subject to} & & \\ & tr(\mathbf{X}) = 1 & \\ & X_{ij} = 0 & \text{if } B_{ij} = 1 \\ & \mathbf{X} \in \mathcal{S}^n & \end{array}$$

The function `lovasz` takes as input an adjacency matrix `B`, and returns the the optimal Lovasz number using `sqlp`.

```
R> out <- lovasz(B)
```

Numerical Example

To compute the Lovasz number using `sqlp`, we need only the (weighted) adjacency matrix representing a graph object.

```
R> data(Glovasz)
```

```
      V1 V2 V3 V4 V5 V6 V7 V8 V9 V10
 [1,]  0  0  0  1  0  0  1  1  0   0
 [2,]  0  0  0  1  0  0  1  0  1   1
 [3,]  0  0  0  0  0  0  0  1  0   0
 [4,]  1  1  0  0  0  0  0  1  0   1
 [5,]  0  0  0  0  0  0  1  1  1   1
 [6,]  0  0  0  0  0  0  0  0  1   0
 [7,]  1  1  0  0  1  0  0  1  1   1
 [8,]  1  0  1  1  1  0  1  0  0   0
 [9,]  0  1  0  0  1  1  1  0  0   1
[10,]  0  1  0  1  1  0  1  0  1   0
```

The Lovasz number for the associated graph is the value of the primal objective function. Again, since the objective function was negated to make the primal problem a minimization, we negate the value of the objective function.

```
R> out <- lovasz(Glovasz)

R> -out$pobj

[1] 5
```

References

[1] László Lovász. On the shannon capacity of a graph. *IEEE Transactions on Information theory*, 25(1):1–7, 1979.