# Overview of the package BuyseTest

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This vignette describes the main functionalities of the **BuyseTest** package. This package implements the Generalized Pairwise Comparisons (GPC) as defined in ? for complete observations, and extended in ? to deal with right-censoring. When considering a single endpoint, the GPC procedure can be summarized as follow. Denote the endpoint by Y in the treatment group and by X in the control group. Given a threshold of clinical relevance  $\tau$ , the aim of GPC is to estimate the proportion in favor of treatment  $\mathbb{P}[Y \geq X + \tau]$  and the proportion in favor of control  $\mathbb{P}[X \geq Y + \tau]$ . Their difference  $\mathbb{P}[Y \geq X + \tau] - \mathbb{P}[X \geq Y + \tau]$  leads to the net benefit and their ratio  $\frac{\mathbb{P}[Y \geq X + \tau]}{\mathbb{P}[X \geq Y + \tau]}$  to the win ratio (in absence of ties). The vignette is written for readers familiar with the GPC framework  $\mathbb{P}[X \geq X + \tau]$ , e.g. prioritized endpoints, pair, net benefit, win ratio, threshold of clinical relevance, . . . , since it focuses on the software aspect of the **BuyseTest** package (not on the underlying statistical model):

- the function BuyseTest performs the GPC procedure and is the main function of the package. The user can interact with its output via various methods:
  - summary to obtain an overview of the results, including the estimated net benefit. The result table at the end of the output can be directly access using model.tables.
  - coef to extract the estimates.
  - confint to extract estimates, confidence intervals, and p.values.
  - plot for a graphical display of the scoring of the pair per endpoint.
  - sensitivity to perform a sensitivity analysis on the choice of the threshold(s).
  - nobs to extract the number of observations and pairs.
  - getIid to extract the iid decomposition of the estimator.
  - getPairScore to extract the contribution of each pair to the net benefit/win ratio.
  - getSurvival to extract the estimates of the survival used for right-censored endpoints.
  - BuyseMultComp to adjust p-values and confidence intervals for multiple comparisons.
- the powerBuyseTest function performs simulation studies, e.g. to estimate the statistical power or assess the bias / type 1 error rate of a test for a specific design. The simBuyseTest function can facilitate the definition of the data generating mechanism.
- the BuyseTest.options function enables the user to access the default values used in the BuyseTest package. The function can also change the default values to better match the user needs.

<sup>&</sup>lt;sup>1</sup>in absence of ties this equals the Wilcoxon-Mann-Whitney parameter

<sup>&</sup>lt;sup>2</sup>if not, ? is a good place to start.

Another vignette, "Wilcoxon test via GPC", details connexions between GPC and the Wilcoxon rank sum test.

Before going further we need to load the **BuyseTest** package in the R session:

```
library(BuyseTest)
library(data.table)
```

To illustrate the functionalities of the package, we will used the **veteran** dataset from the **survival** package:

```
data(cancer, package = "survival")
veteran <- cbind(id = 1:NROW(veteran), veteran)
veteran$trt <- factor(veteran$trt,1:2,c("Pl","Exp"))
head(veteran)</pre>
```

```
id trt celltype time status karno diagtime age prior
1
  1 Pl squamous
                   72
                           1
                                60
                                          7
                                             69
                                                    0
2
  2 Pl squamous 411
                                70
                                             64
                                                   10
                           1
                                          5
  3 Pl squamous 228
                                60
                                          3
                                             38
                                                    0
  4 Pl squamous 126
4
                           1
                                60
                                          9
                                             63
                                                   10
  5 Pl squamous 118
                                70
5
                                             65
                                                   10
                           1
                                         11
  6 Pl squamous
                  10
                           1
                                20
                                                    0
                                          5
                                             49
```

See ?veteran for a presentation of the database.

<u>Note</u>: the **BuyseTest** package is under active development. Newer package versions may include additional functionalities and fix previous bugs. The version of the package that is being is:

```
utils::packageVersion("BuyseTest")
```

## [1] '3.0.4'

For completness, the details of the R session used to generate this document are:

## sessionInfo()

time zone: Europe/Copenhagen

```
R version 4.3.3 (2024-02-29)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 22.04.4 LTS
Matrix products: default
        /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.10.0
LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.10.0
locale:
                                                            LC_TIME=en_US.UTF-8
 [1] LC_CTYPE=en_US.UTF-8
                                LC_NUMERIC=C
 [4] LC_COLLATE=en_US.UTF-8
                                LC_MONETARY=en_US.UTF-8
                                                            LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8
                                LC_NAME=C
                                                            LC_ADDRESS=C
[10] LC_TELEPHONE=C
                                LC MEASUREMENT=en US.UTF-8 LC IDENTIFICATION=C
```

tzcode source: system (glibc)

# attached base packages:

[1] stats graphics grDevices utils datasets methods base

# other attached packages:

[1] data.table\_1.15.4 BuyseTest\_3.0.4 Rcpp\_1.0.12 prodlim\_2023.08.28

[5] ggplot2\_3.5.1 survival\_3.5-8

# loaded via a namespace (and not attached):

[1]	Matrix_1.6-5	gtable_0.3.5	<pre>future.apply_1.11.2</pre>	dplyr_1.1.4
[5]	compiler_4.3.3	tidyselect_1.2.1	MatrixModels_0.5-3	parallel_4.3.3
[9]	globals_0.16.3	splines_4.3.3	scales_1.3.0	lattice_0.22-5
[13]	R6_2.5.1	generics_0.1.3	future_1.33.2	tibble_3.2.1
[17]	munsell_0.5.1	pillar_1.9.0	rlang_1.1.3	utf8_1.2.4
[21]	cli_3.6.2	withr_3.0.0	magrittr_2.0.3	digest_0.6.35
[25]	grid_4.3.3	lifecycle_1.0.4	lava_1.8.0	vctrs_0.6.5
[29]	SparseM_1.81	glue_1.7.0	listenv_0.9.1	codetools_0.2-19
[33]	stats4_4.3.3	parallelly_1.37.1	fansi_1.0.6	<pre>colorspace_2.1-0</pre>
[37]	tools_4.3.3	pkgconfig_2.0.3		

# 1 Performing generalized pairwise comparisons (GPC) using the BuyseTest function

To perform generalized pairwise comparisons, the BuyseTest function needs:

• where the data are stored - argument data

• the name of the endpoints - argument endpoint

• the type of each endpoint - argument type

• the variable defining the two treatment groups - argument treatment

The BuyseTest function has many optional arguments. For example:

- the threshold of clinical relevance associated to each endpoint argument threshold
- the censoring associated to each endpoint (for time to event endpoints) argument status

There are two equivalent ways to define the GPC:

• using a separate argument for each element:

### Generalized Pairwise Comparisons

#### Settings

Point estimation and calculation of the iid decomposition

```
Estimation of the estimator's distribution - method: moments of the U-statistic
```

Gather the results in a S4BuyseTest object

- or via a formula interface. In the formula interface endpoint are wrapped by parentheses. The parentheses must be preceded by their type:
  - binary (b, bin, or binary)
  - continuous (c, cont, or continuous)
  - time to event (t, tte, or timetoevent)

Here we also set the argument trace to FALSE to execute silently the function:

```
BT.f <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status"),
data = veteran, trace = FALSE)
```

We can check that the two approaches are equivalent:

```
BT.f@call <- list(); BT@call <- list();
testthat::expect_equal(BT.f,BT)</pre>
```

# 1.1 Displaying the results

The results of the GPC can be displayed using the summary method:

```
summary(BT)
```

Generalized pairwise comparisons with 1 endpoint

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
                  : H-projection of order 1 after atanh transformation
- inference
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                              Delta
                                   37.78
                                                  46.54
                20
                        100
                                                             15.68
                                                                          0 -0.0877
CI [2.5%; 97.5%] p.value
 [-0.2735;0.1045] 0.37162
```

It displays information about each endpoint, percentage of pairs classified as favorable, unfavorable, neutral, and uninformative, as well as the estimated net benefit (column Delta), its confidence interval, and the corresponding p-value testing the absence of a group difference. Other To display the number of pairs instead of the percentage of pairs that are favorable/unfavorable/neutral/uniformative, set the argument percentage to FALSE. See help(S4BuyseTest-summary) for more details about the summary method, its input and output. For a more concise display of the results, consider using the print method:

```
print(BT, percentage = FALSE)
```

```
endpoint threshold total favorable unfavorable neutral uninf Delta CI [2.5%; 97.5%] time 20 4692 1772.59 2183.89 735.52 0 -0.0877 [-0.2735;0.1045] p.value 0.37162
```

To access these values, we recommand using the model.tables method that outputs the information from the previous table in a data.frame format:

```
model.tables(BT, percentage = FALSE)
```

An even more concise output can be obtained via the confint method:

```
confint(BT)
```

```
estimate se lower.ci upper.ci null p.value time_t20 -0.08765836 0.09760901 -0.2735301 0.1045245 0 0.371617
```

or coef method:

coef(BT)

[1] -0.08765836

# 1.2 Stratified GPC

GPC can be performed for subgroups of a categorical variable

- argument strata

For instance, the celltype may have huge influence on the survival time and the investigator would like to only compare patients that have the same celltype. In the formula interface this is achieved by adding a single variable in the right hand side of the formula:

```
ffstrata <- trt \sim tte(time, threshold = 20, status = "status") + celltype BTstrata <- BuyseTest(ffstrata, data = veteran, trace = 0)
```

Not being wrapped by bin, cont or tte differentiates it from endpoint variables. When doing a stratified analysis, the summary method displays strata-specific and global results<sup>3</sup>:

Generalized pairwise comparisons with 1 endpoint and 4 strata

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : H-projection of order 1 after atanh transformation
- treatment groups: Exp (treatment) vs. Pl (control)
- strata weights : 26.38%, 34.63%, 18.47%, 20.52%
- censored pairs : probabilistic score based on the survival curves
- uninformative pairs: no contribution
- results
            strata total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
endpoint
                                                                                delta
                                                                                         Delta
                                                   45.77
                                                               17.33
                                                                         0.85 -0.0997 -0.0997
                     100.00
                                    36.06
    time
            global
                                    14.33
                                                    8.77
                                                                         0.00 0.2193
          squamous
                      25.38
                                                                2.28
                      45.69
                                    12.69
                                                   20.88
                                                               11.27
                                                                         0.85 - 0.1792
         smallcell
                      13.71
                                     4.74
                                                                         0.00 -0.1034
             adeno
                                                    6.15
                                                                2.81
                      15.23
                                     4.30
                                                                0.96
                                                                         0.00 - 0.3722
             large
                                                    9.97
```

The percentage of pairs in the total/favorable/unfavorable/neutral/uninf columns are relative to the overall number of pairs whereas the column delta presents the strata-specific net benefits (in the last 4 lines). The last column (Delta) displays the overall net benefit.

With this weighting scheme the proportion of favorable pairs minus the proportion of unfavorable pairs does not equal the stata-specific nor the global net benefit. To retrieve the net benefits, we first extract the number of pairs per strata using the method nobs:

```
strata.obs <- as.data.frame(nobs(BTstrata, strata = TRUE))
strata.obs</pre>
```

<sup>&</sup>lt;sup>3</sup>the strata-specific results can be removed by setting the argument strata to "global" when calling summary.

```
Pl Exp pairs
squamous
          15
               20
                    300
smallcell 30
               18
                    540
           9
                    162
adeno
              18
                    180
large
          15
              12
```

and use the method model.tables to extract the number of favorable and unfavorable pairs per strata:

```
strata total favorable unfavorable
2 squamous 300 169.40260 103.6104
3 smallcell 540 150.00000 246.7778
4 adeno 162 56.00000 72.7500
5 large 180 50.83333 117.8333
```

We retrieve the strata-specific net benefits by comparing, in each strata, the number of favorable and unfavorable pairs relative to the number of pairs<sup>4</sup>:

```
delta <- (dfStrata$favorable - dfStrata$unfavorable)/strata.obs$pairs delta
```

### [1] 0.2193074 -0.1792181 -0.1033951 -0.3722222

The global net benefit is then the sum of the strata-specific net benefits weighted by the strata weights:

```
weightCMH <- strata.obs$pairs/(strata.obs$Pl + strata.obs$Exp)
list(estimate = sum(delta * weightCMH/sum(weightCMH)),
    weight = 100*weightCMH/sum(weightCMH))</pre>
```

#### \$estimate

[1] -0.09967584

### \$weight

```
[1] 26.38329 34.62807 18.46830 20.52034
```

The default weighting scheme is CMH, standing for Cochran-Mantel-Haenszel, which has been recommaned in the litterature (?). It is efficient under the assumption of a common multiplicative effect (across strata) on the odds ratio scale. If the effect is thought additive, one should instead weight proportionnaly to the number of pairs. This can be achieved by specifying the argument pool.strata:

```
BTstrata2 <- BuyseTest(ffstrata, data = veteran, trace = 0, pool.strata = "buyse")
summary(BTstrata2, type.display = keep.colStrata)
```

<sup>&</sup>lt;sup>4</sup>Alernatively one could compute, from the summary, the difference between the percentage of favorable and unfavorable pairs relative to the percentage of pairs in the strata, e.g.  $(14.33\% - 8.77\%)/25.38\% \approx 21.93\%$ 

### Generalized pairwise comparisons with 1 endpoint and 4 strata

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : H-projection of order 1 after atanh transformation
- treatment groups: Exp (treatment) vs. Pl (control)
- strata weights : 25.38%, 45.69%, 13.71%, 15.23%
- censored pairs : probabilistic score based on the survival curves
- uninformative pairs: no contribution
- results
            strata total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
endpoint
                                                                               delta
                                                                                       Delta
                                   36.06
                                                  45.77
    time
            global
                     100.00
                                                              17.33
                                                                        0.85 -0.0971 -0.0971
                                                   8.77
                                                               2.28
                                                                        0.00 0.2193
          squamous
                      25.38
                                   14.33
         smallcell
                      45.69
                                   12.69
                                                  20.88
                                                              11.27
                                                                        0.85 -0.1792
                      13.71
                                    4.74
                                                                        0.00 -0.1034
             adeno
                                                    6.15
                                                               2.81
                                                                        0.00 - 0.3722
             large
                      15.23
                                    4.30
                                                    9.97
                                                               0.96
```

The weighting scheme only affect the evaluation of the overall net benefit, which now equals the difference between the overall proportion of favorable vs. unfavorable pairs. As before the strata-specific net benefits are still not equal to the difference between the proportion of favorable and unfavorable pairs. While extractors will by default output global estimates (i.e. after pooling the results over strata)

```
confint(BTstrata2)
```

```
estimate se lower.ci upper.ci null p.value time_t20 -0.09706901 0.0977929 -0.2829348 0.09582321 0 0.323961
```

one can specify the argument strata to extract strata-specific estimates:

```
confint(BTstrata, strata = TRUE)
```

```
estimate se lower.ci upper.ci null p.value time_t20.squamous 0.2193074 0.1911515 -0.1690137 0.5486919 0 0.2669352 time_t20.smallcell -0.1792181 0.1540933 -0.4567640 0.1301230 0 0.2551275 time_t20.adeno -0.1033951 0.2465197 -0.5314450 0.3667172 0 0.6771002 time_t20.large -0.3722222 0.2190018 -0.7110335 0.1068610 0 0.1240457
```

# 1.3 Using multiple endpoints

More than one endpoint can be considered by indicating a vector of endpoints, types, and thresholds. In the formula interface, the different endpoints must be separated with a "+" on the right hand side of the formula:

```
ff2 <- trt ~ tte(time, threshold = 20, status = "status") + cont(karno, threshold = 0)
BT.H <- BuyseTest(ff2, data = veteran, trace = 0)
summary(BT.H)</pre>
```

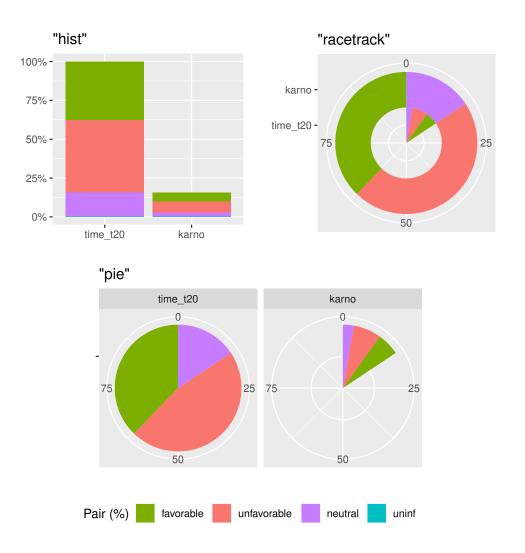
### Generalized pairwise comparisons with 2 prioritized endpoints

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
                  : H-projection of order 1 after atanh transformation
- inference
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
                  : re-analyzed using lower priority endpoints
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                                       Delta
                                                                               delta
                20
                     100.00
                                   37.78
                                                  46.54
                                                              15.68
                                                                           0 -0.0877 -0.0877
                      15.68
                                    5.78
                                                   7.11
                                                               2.78
                                                                           0 -0.0133 -0.1009
  karno
CI [2.5%; 97.5%] p.value
 [-0.2735;0.1045] 0.37162
 [-0.2901;0.0959] 0.31478
```

The hierarchy of the endpoint is defined from left (most important endpoint, here time) to right (least important endpoint, here karno). In the summary output, the confidence intervals and p.values are computed for the column Delta, i.e. here -8.77% is the net benefit for the first endpoint (line 1) and -10.09% is the net benefit for the first and second endpoint (line 2). In other words, the last confidence interval and p-value is the one for the analysis over all endpoints (generally the one to report).

A graphical representation of the GPC procedure can be obtained by the plot method. It will display the percentage of favorable, unfavorable, neutral, and uninformative pairs per endpoint. Three (equivalent) graphical display are possible, the first one ("hist") being the recommanded one:

```
plot(BT.H, type = "hist")
plot(BT.H, type = "pie")
plot(BT.H, type = "racetrack")
```



It is also possible to perform the comparisons on all pairs for all endpoints by setting the argument hierarchical to FALSE:

```
BT.nH <- BuyseTest(ff2, hierarchical = FALSE, data = veteran, trace = 0) summary(BT.nH)
```

### Generalized pairwise comparisons with 2 endpoints

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : H-projection of order 1 after atanh transformation
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold weight total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                                      delta
                20
                      0.5
                               100
                                          37.78
                                                         46.54
                                                                     15.68
                                                                                  0 -0.0877
   time
                               100
  karno
                      0.5
                                          41.82
                                                         44.95
                                                                     13.24
                                                                                  0 -0.0313
 Delta CI [2.5%; 97.5%] p.value
-0.0438 [-0.1388;0.0519] 0.36977
-0.0595 [-0.2267;0.1111] 0.49514
```

In that case the score of a pair is the weighted sum of the score relative to each endpoint. By default, the weights are all set to the same value but this behavior can be changed by setting the argument weight when calling BuyseTest, e.g.:

```
ff2w <- trt \sim tte(time, threshold = 20, status = "status", weight = 0.8) ff2w <- update.formula(ff2w, . \sim . + cont(karno, threshold = 0, weight = 0.2)) BT.nHw <- BuyseTest(ff2w, hierarchical = FALSE, data = veteran, trace = 0) model.tables(BT.nHw)
```

```
endpoint threshold weight total favorable unfavorable neutral uninf
                                                                              delta
               2e+01
                        0.8
                                                46.54489 15.67606
1
      time
                              100
                                  37.77905
                                                                      0 -0.08765836
3
     karno
               1e-12
                        0.2
                              100 41.81586
                                                44.94885 13.23529
                                                                      0 -0.03132992
        Delta
                lower.ci
                           upper.ci
                                      p.value
1 -0.07012668 -0.2203714 0.08336855 0.3707289
3 -0.07639267 -0.2503756 0.10237001 0.4026905
```

This has been referred as the O'Brien test in the litterature (?, section 3.2). Alternatively, one may be interested in the endpoint specific results. This can be performed apply the BuyseTest function separately to each endpoint, e.g.:

```
confint(BuyseTest(trt \sim cont(karno, threshold = 0), data = veteran, trace = 0))
```

```
estimate se lower.ci upper.ci null p.value karno -0.03132992 0.09787113 -0.2197111 0.1593037 0 0.7490407
```

or setting the argument cumulative to FALSE when calling the confint function:

```
confint(BT.nHw, cumulative = FALSE)
```

```
estimate se lower.ci upper.ci null p.value
time_t20 -0.08765836 0.09760901 -0.2735301 0.1045245 0 0.3716170
karno -0.03132992 0.09787113 -0.2197111 0.1593037 0 0.7490407
```

# 1.4 Statistical inference

Several methods are available in BuyseTest to quantify uncertainty about the estimates:

• permutation test setting the argument method.inference to "permutation". Assuming exchangeability under the null hypothesis, this approach gives valid p-values (regardless to the sample size) for testing the absence of a difference between the groups.

```
BT.perm <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status"),
data = veteran, trace = 0, method.inference = "permutation",
seed = 10)
summary(BT.perm)
```

Generalized pairwise comparisons with 1 endpoint

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : permutation test with 1000 samples
                    p-value computed using the permutation distribution
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta p.value
                                   37.78
                                                                          0 -0.0877 0.35265
   time
                20
                        100
                                                  46.54
                                                             15.68
```

• bootstrap resampling setting the argument method.inference to "bootstrap". In large enough samples, this approach gives valid p-values and confidence intervals.

Generalized pairwise comparisons with 1 endpoint

```
- statistic
                  : net treatment benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : bootstrap resampling with 1000 samples
                    CI computed using the percentile method; p-value by test inversion
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                   37.78
                                                  46.54
                                                             15.68
                20
                        100
                                                                          0 -0.0877
CI [2.5%; 97.5%] p.value
 [-0.2721;0.1034]
                  0.383
```

• asymptotic distribution setting the argument method.inference to "u-statistic". In large enough samples, this approach gives valid p-values and confidence intervals (?).

```
BT.ustat <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status"),

data = veteran, trace = 0, method.inference = "u-statistic")
summary(BT.ustat)
```

### Generalized pairwise comparisons with 1 endpoint

```
: net treatment benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- confidence level: 0.95
                  : H-projection of order 1 after atanh transformation
- inference
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                              Delta
                20
                        100
                                   37.78
                                                  46.54
                                                             15.68
                                                                          0 -0.0877
CI [2.5%; 97.5%] p.value
 [-0.2735;0.1045] 0.37162
```

The first two approaches require simulating a large number of samples and applying the GPC to each of these samples. The seed argument is used to generate a seed for each sample. The number of samples is set using the arugment n.resampling and it should large enough to limit the Monte Carlo error when estimating the p-value. Typically should be at least 10000 to get, roughtly, 2-digit precision, as examplified below:

```
set.seed(10)
sapply(1:10, function(i){mean(rbinom(1e4, size = 1, prob = 0.05))})
```

### [1] 0.0511 0.0491 0.0489 0.0454 0.0516 0.0522 0.0468 0.0483 0.0491 0.0508

Indeed, here we get a reasonnable approximation of 0.05 (if we round and only keep 2 digits). Note that to get 3 digits precision we would need more samples. The last method does not rely on resampling but on the computation of the influence function of the estimator. Fortunately, when using the Gehan's scoring rule, this does not really involve any extra-calculations and this is therefore very fast to perform. When using the Peron's scoring rule, more serious extra-calculations are involved so the computation time is expected to increase by a factor 5 to 10 compared to the point estimate alone (i.e. method.inference equal to "none").

Note: it is possible to relax the exchangeability assumption using a studentized permutation. A studentized bootstrap is also possible to improve on the better small samples properties of the bootstrap confidence intervals. Both rely on the asymptotic approach to estimate standard errors and are more numerically intensive.

## 1.5 What if smaller is better?

By default BuyseTest will always assume that higher values of an endpoint are favorable. This behavior can be changed by specifying operator = "<0" for an endpoint:

```
ffop <- trt ~ tte(time, status = "status", threshold = 20, operator = "<0")
BTinv <- BuyseTest(ffop, data = veteran, trace = 0)
summary(BTinv)</pre>
```

#### Generalized pairwise comparisons with 1 endpoint

```
: net treatment benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- confidence level: 0.95
                 : H-projection of order 1 after atanh transformation
- inference
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
                                                                         0 0.0877
                                  46.54
                                                 37.78
               20 100
                                                            15.68
CI [2.5%; 97.5%] p.value
 [-0.1045;0.2735] 0.37162
```

Internally BuyseTest will compute the favorable and unfavorable score as usual and then switch them around if the operator equals "<0".

# 1.6 Stopping comparison for neutral pairs

In presence of neutral pairs, BuyseTest will, by default, continue the comparison on the endpoints with lower priority. For instance let consider a dataset with one observation in each treatment arm:

```
Id treatment tumor size
1: 1 Yes Yes 15
2: 2 No Yes 20
```

If we use the GPC with tumor as the first endpoint and size as the second endpoint:

Generalized pairwise comparisons with 2 prioritized endpoints

```
: net treatment benefit (delta: endpoint specific, Delta: global)
- treatment groups: Yes (treatment) vs. No (control)
                  : re-analyzed using lower priority endpoints
- neutral pairs
- results
endpoint total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) delta Delta
  tumor
              100
                             0
                                             0
                                                      100
                                                                 0
                                                                       0
                                             0
                                                                 0
    size
              100
                           100
                                                        0
                                                                       1
                                                                              1
```

the outcome of the comparison is neutral for the first priority, but favorable for the second. Setting the argument neutral.as.uninf to FALSE will stop the comparison when a pair is classified as neutral:

```
BT.pair2 <- BuyseTest(treatment ~ bin(tumor) + cont(size, operator = "<0"), data = dt.sim, trace = 0, method.inference = "none", neutral.as.uninf = FALSE) summary(BT.pair2)
```

Generalized pairwise comparisons with 2 prioritized endpoints

```
: net treatment benefit (delta: endpoint specific, Delta: global)
- treatment groups: Yes (treatment) vs. No (control)
- neutral pairs : ignored at lower priority endpoints
- results
endpoint total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) delta Delta
              100
                             0
                                                      100
                                                                 0
  tumor
                                             0
                                                                        0
                0
                             0
                                             0
                                                        0
                                                                 0
                                                                        0
                                                                              0
    size
```

So in this case no pair is analyzed at second priority.

# 1.7 What about other summary statistics than the net benefit?

By default methods such as summary display results relative to the net benefit. Three other summary statistics are accessible: proportion in favor of treatment  $\mathbb{P}[Y \geq X + \tau]$ , proportion in favor of control  $\mathbb{P}[X \geq Y + \tau]$ , and their ratio  $\mathbb{P}[X \geq Y + \tau]$ . They can be access via the argument statistic, e.g.:

```
summary(BT, statistic = "winRatio")
```

Generalized pairwise comparisons with 1 endpoint

```
: win ratio (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 1
- confidence level: 0.95
                  : H-projection of order 1 after log transformation
- inference
- treatment groups: Exp (treatment) vs. Pl (control)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
                                   37.78
                                                  46.54
                                                             15.68
                        100
                                                                          0 0.8117
CI [2.5%; 97.5%] p.value
  [0.5134;1.2833] 0.37195
```

In presence of ties, it is recommanded to use another definition for the proportion in favorable of treatment/control as their null distribution depends on the data generative mechanism and the threshold of clinical relevance. This is why the confint method will not produce any p.value:

```
confint(BT, statistic = "favorable")
```

```
estimate se lower.ci upper.ci null p.value time t20 0.3777905 0.04902199 0.2874747 0.477467 NA NA
```

unless the argument null is provided by the user. A permutation test may be used to empirically estimate a value for the null hypothesis:

```
estimate se lower.ci upper.ci null p.value time_t20 0.3777905 0.04770182 NA NA 0.4205855 0.3636364
```

which, in this example, is around 0.42. It worth noting that testing an inadequate null hypothesis can have dramatic consequences on the p-value:

```
estimate se lower.ci upper.ci null p.value
time_t20 0.3777905 0.04902199 0.2874747 0.477467 0.42 0.39826735
time_t201 0.3777905 0.04902199 0.2874747 0.477467 0.50 0.01673643
```

For statistical testing we therefore recommand defining the proportion in favor of treatment as  $\mathbb{P}[Y \geq X + \tau] + 0.5\mathbb{P}[|Y - X| < \tau]$  which matches the definition of the Wilcoxon-Mann-Whitney parameter. To do so, one should set the argument add.halfNeutral to TRUE when calling BuyseTest:

```
BT.half <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status"),
data = veteran, trace = FALSE, add.halfNeutral = TRUE)
confint(BT.half, statistic = "favorable")
```

```
estimate se lower.ci upper.ci null p.value time_t20 0.4561708 0.04880921 0.3632263 0.5522714 0.5 0.3716632
```

Note that the win ratio is also often defined that way, i.e. as  $\frac{\mathbb{P}[Y \ge X + \tau] + 0.5\mathbb{P}[|Y - X| < \tau]}{\mathbb{P}[X \ge Y + \tau] + 0.5\mathbb{P}[|Y - X| < \tau]}$ , leading to:

```
confint(BT.half, statistic = "winRatio")
```

```
estimate se lower.ci upper.ci null p.value time_t20 0.8388127 0.1650208 0.5704361 1.233454 1 0.3716211
```

Testing a Net Benefit of 0, a Win Ratio of 1, and a Wilcoxon-Mann-Whitney parameter of 0.5 corresponds to the same hypothesis and therefore the same p-value should be obtained. The (small) discrepancy in p-values observed in this example (0.371617 vs. 0.3716211 vs. 0.3716632) are due to small sample approximation. Such discrepancies will not arise when using non-parametric bootstrap or permutation tests using quantiles of the bootstrap or permutation distribution, e.g.:

```
estimate se lower.ci upper.ci null p.value netBenefit -0.08765836 0.10021632 -0.2720510 0.1033974 0.0 0.383 winRatio 0.83881270 0.17440155 0.5722640 1.2306429 1.0 0.383 favorable 0.45617082 0.05010816 0.3639745 0.5516987 0.5 0.383
```

# 1.8 Is multiple testing a concern with GPC?

Yes, as with any other statistical method. Having a pre-defined statistical plan (i.e. written before looking at the data) specifying the hierarchy of endpoints, their threshold of clinical relevance is recommanded. When planning multiple GPC, summarize the results can be done via one of two principles:

• intersection union principle: one rejects the (global) null hypothesis if there is evidence for an effect in all the GPC analyses. This is typically a sensitivity analysis: checking that the results are not too sensitive to the choice of an hyperparameter. No multiplicity adjustment is needed other than considering the largest p-value among all tests. For instance, when checking whether the estimated net benefit is similar across a range of threshold of clincial relevance, we would obtain a p-value of 0.76

```
BTse <- sensitivity(BT.ustat, threshold = seq(0,500, length.out=10),
trace = FALSE)
BTse
```

```
time
                estimate
                                 se
                                       lower.ci
                                                  upper.ci null
                                                                   p.value
     0.00000 -0.08752774 0.10041203 -0.27851884 0.11012263
1
                                                               0 0.3858177
2
    55.5556 -0.08095829 0.08957699 -0.25229456 0.09530004
                                                               0 0.3682107
   111.11111 -0.03170177 0.07463991 -0.17629003 0.11422560
3
                                                               0 0.6712414
              0.01896964 0.06452954 -0.10713643 0.14447503
4
   166.66667
                                                               0 0.7688360
              0.03315614 0.05523512 -0.07506821 0.14060850
5
  222.22222
                                                               0 0.5486177
             0.04217485 0.04654025 -0.04914025 0.13279075
6
  277.77778
                                                               0 0.3653982
7
             0.04112991 0.03946828 -0.03631838 0.11808708
  333.33333
                                                               0 0.2979105
  388.88889
              0.04075638 0.03300933 -0.02402114 0.10519310
                                                               0 0.2174545
8
  444.4444
              0.04097871 0.03027888 -0.01844156 0.10011054
                                                               0 0.1764199
10 500.00000
             0.03517173 0.02769280 -0.01915553 0.08929191
                                                               0 0.2044340
```

• union intersection principle: one rejects the (global) null hypothesis if there is evidence for an effect for at least on of the GPC analyses. This is a typical exploratory analysis where one look for the most promising outcome. Adjustment for multiplicity is needed. Since estimates from GPC procedure are typically highly correlated, one can improve on bonferroni adjustment using a max-test adjustment. This is what is performed via the <code>BuyseMultComp</code> function:

```
BuyseMultComp(BT.H, endpoint = 1:2)
```

```
- Univariate tests:

estimate se lower.ci upper.ci null p.value lower.band upper.band time_t20 -0.08765836 0.09760901 -0.2735301 0.10452446 0 0.371617 -0.2798817 0.1113226 karno -0.10092285 0.09971277 -0.2901336 0.09588144 0 0.314777 -0.2965716 0.1028561
```

adj.p.value time\_t20 0.4117239 karno 0.3508339 Here we look at whether there is a benefit in survival alone (first priority time\_t20) or a benefit over both endpoint (second priority karno). Setting the argument cumulative to FALSE when considering nonhierarchical GPC analyses enables to efficiently adjust endpoint-specific GPC for multiple comparisons:

```
BuyseMultComp(BT.nH, cumulative = FALSE, endpoint = 1:2)
```

#### - Univariate tests:

```
estimate se lower.ci upper.ci null p.value lower.band upper.band time_t20 -0.08765836 0.09760901 -0.2735301 0.1045245 0 0.3716170 -0.2953329 0.1279261 karno -0.03132992 0.09787113 -0.2197111 0.1593037 0 0.7490407 -0.2420777 0.1822409 adj.p.value time_t20 0.5597555 karno 0.9236602
```

One can also consider the global endpoint of two different GPC analyses:

```
BuyseMultComp(list(hierarchical = BT.H, Obrien = BT.nH), cluster = "id")
```

```
- Univariate tests:
```

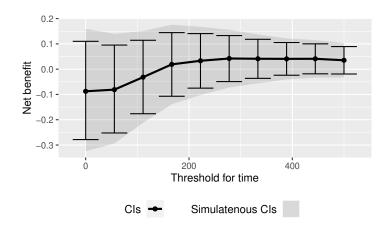
```
estimate
                                 se
                                      lower.ci
                                                  upper.ci null
                                                                  p.value lower.band
hierarchical -0.10092285 0.09971277 -0.2901336 0.09588144
                                                              0 0.3147770 -0.3014645
             -0.05949414 0.08700807 -0.2266953 0.11111326
                                                              0 0.4951361 -0.2368800
Obrien
             upper.band adj.p.value
hierarchical 0.1081696
                          0.3831444
Obrien
              0.1217304
                          0.5851872
```

Finally the **sensitivity** method can also be used to adjust for multiple comparisons over multiple thresholds:

```
time
            estimate
                     lower.ci
                             upper.ci
                                     p.value lower.band upper.band adj.p.value
1
    0.00000 \ -0.08752774 \ -0.27851884 \ 0.11012263 \ 0.3858177 \ -0.32447773 
                                                     0.1597587
                                                              0.7745625
2
  55.55556 -0.08095829 -0.25229456 0.09530004 0.3682107 -0.29398532
                                                     0.1397311
                                                              0.7528122
  111.11111 -0.03170177 -0.17629003 0.11422560 0.6712414 -0.21221507
3
                                                     0.1509036
                                                              0.9810274
          0.01896964 \ -0.10713643 \ 0.14447503 \ 0.7688360 \ -0.13890539
4
  166.66667
                                                     0.1759043
                                                              0.9969926
  5
                                                     0.1675845
                                                              0.9257172
  6
                                                     0.1556050
                                                              0.7492675
  7
                                                     0.1375213
                                                              0.6545176
  388.88889 0.04075638 -0.02402114 0.10519310 0.2174545 -0.04052732
8
                                                     0.1215042
                                                              0.5203739
  444.4444 0.04097871 -0.01844156 0.10011054 0.1764199 -0.03358825
                                                     0.1150920
                                                              0.4429140
0.1030201
                                                              0.4967546
```

Here by setting the argument band to TRUE (and adj.p.value to TRUE), we obtain confidence intervals (and p-values) adjusted for multiple comparisons. Said otherwise, the columns lower.ci and upper.ci provide a (pointwise) confidence interval with 95% coverage for a given threshold while the columns lower.band and upper.band provide a (simutaneous) confidence interval with 95% coverage across all given thresholds. The difference can be visualized using the autoplot method:

```
library(ggplot2)
autoplot(BTse.ustat)
```



Simultaneous and pointwise confidence intervals are here of similar width due to the very high correlation between estimates across thresholds:

```
BTse.cor <- cor(lava::iid(BTse.ustat))
range(BTse.cor[lower.tri(BTse.cor)])</pre>
```

#### [1] 0.3716902 0.9848999

Note that with multiple endpoints, the thresholds can be specified using a list:

```
BTse.H <- sensitivity(BT.H, trace = FALSE,
threshold = list(time = seq(0,500,length = 10), karno = c(0,40,80)))
head(BTse.H)
```

```
upper.ci null
       time karno
                     estimate
                                       se
                                            lower.ci
                                                                        p.value
                0 -0.08754474 0.10044847 -0.2786016 0.11017738
    0.00000
                                                                    0 0.3858987
  55.55556
                0 -0.11177487 0.09915501 -0.2995661 0.08435417
                                                                   0 0.2636263
3 111.11111
                0 -0.08618872 0.09822940 -0.2732475 0.10715096
                                                                    0 0.3826244
4 166.66667
                0 -0.05180121 0.09818252 -0.2400240 0.14017526
                                                                   0 0.5984319
 222.22222
                0 -0.03668720 0.09810141 -0.2253052 0.15458146
                                                                    0 0.7086747
6 277.77778
                0 -0.02906324 0.09773146 -0.2172647 0.16122161
                                                                    0 0.7663054
```

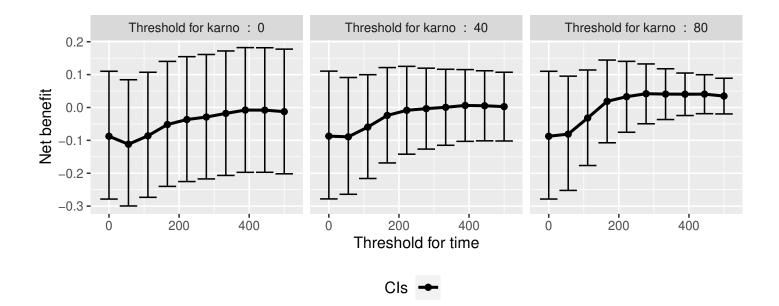
or a matrix:

```
grid <- expand.grid(list("time_t20" = seq(0,500,length = 10), "karno" = c(0,40,80)))
cbind(head(grid)," " = " ... ",tail(grid))
BTse.H2 <-sensitivity(BT.H, threshold = grid, trace = FALSE)
range(BTse.H-BTse.H2)</pre>
```

```
time_t20 karno
                             time_t20 karno
    0.00000
                 0
                             222.2222
                                          80
1
2
  55.55556
                 0
                             277.7778
                                          80
3 111.11111
                 0
                             333.3333
                                          80
4 166.66667
                 0
                             388.8889
                                          80
 222.22222
                 0
                             444.4444
                                          80
6 277.77778
                             500.0000
                                          80
[1] 0 0
```

The latter should be used when the same endpoint is used at different priorities (each column correspond to the threshold that should be used at a priority). As before we can display the results using the autoplot function:

```
autoplot(BTse.H, col = NA)
## alternative display:
## autoplot(BTse.H, position = position_dodge(width = 15))
```



The autoplot function can only be used when 1 or 2 thresholds are varied at the same time.

# 2 Getting additional inside: looking at the pair level

So far we have looked at the overall score and probabilities. But it is also possible to extract the score relative to each pair, as well as to "manually" compute this score. This can give further inside on what the software is actually doing and what is the contribution of each individual on the evaluation of the treatment.

# 2.1 Extracting the contribution of each pair to the statistic

The net benefit or the win ratio statistics can be expressed as a sum of a score over all pairs of patients. The argument keep.pairScore enables to export the score relative to each pair in the output of BuyseTest:

The method **getPairScore** can then be used to extract the contribution of each pair. For instance the following code extracts the contribution for the first endpoint:

```
getPairScore(BT.keep, endpoint = 1)
```

```
Key: <index.Exp, index.Pl>
       index.Pl index.Exp favorable unfavorable neutral uninf weight
   1:
                1
                          70
                                        1
                                                       0
                                                                 0
                                                                        0
   2:
                2
                          70
                                        1
                                                       0
                                                                 0
                                                                        0
                                                                                 1
                3
                           70
                                        1
                                                       0
                                                                 0
                                                                        0
                                                                                 1
   3:
   4:
                4
                          70
                                        1
                                                       0
                                                                 0
                                                                        0
                                                                                 1
                5
                          70
                                        1
                                                       0
                                                                 0
                                                                        0
                                                                                 1
   5:
4688:
              65
                          137
                                        0
                                                       1
                                                                 0
                                                                        0
                                                                                 1
4689:
               66
                          137
                                        0
                                                       1
                                                                 0
                                                                                 1
                                        0
4690:
               67
                          137
                                                       1
                                                                 0
                                                                        0
                                                                                 1
4691:
               68
                          137
                                        0
                                                       1
                                                                 0
                                                                        0
                                                                                 1
4692:
               69
                          137
                                                       1
                                                                 0
                                                                                 1
```

Each line corresponds to different comparison between a pair from the control arm and the treatment arm. The column strata store to which strata the pair belongs (first, second, ...). The columns favorable, unfavorable, neutral, uninformative contains the result of the comparison, e.g. the first pair was classified as favorable while the last was classified as favorable with a weight of 1. The second and third columns indicates the rows in the original dataset corresponding to the pair:

```
veteran[c(70,1),]
```

For the first pair, the event was observed for both observations and since 999 > 72 + 20 the pair is rated favorable. Substracting the average probability of the pair being favorable minus the average probability of the pair being unfavorable:

```
getPairScore(BT.keep, endpoint = 1)[, mean(favorable) - mean(unfavorable)]
```

### [1] -0.08765836

gives the net benefit in favor of the treatment for the first endpoint:

```
BT.keep
```

```
endpoint threshold delta Delta
time 20 -0.0877 -0.0877
karno -0.0133 -0.1009
```

More examples and explanation can be found in the documentation of the method getPairScore.

# 2.2 Extracting the survival probabilities

When using scoring.rule equals "Peron", survival probabilities at event time, and event times +/threshold in the control and treatment arms are used to score the pair. Setting keep.survival to TRUE
and precompute to FALSE in BuyseTest.options enables to export the survival probabilities in the output
of BuyseTest:

```
BuyseTest.options(keep.survival = TRUE, precompute = FALSE)

BT.keep2 <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status") + cont(karno),

data = veteran, keep.pairScore = TRUE, scoring.rule = "Peron",

trace = 0, method.inference = "none")
```

The method **getSurvival** can then be used to extract these survival probabilities. For instance the following code extracts the survival for the first endpoint:

```
outSurv <- getSurvival(BT.keep2, endpoint = 1, strata = 1)
str(outSurv)</pre>
```

```
List of 5

$ survTimeC: num [1:69, 1:13] 72 411 228 126 118 10 82 110 314 100 ...
..- attr(*, "dimnames")=List of 2
....$: NULL
....$: chr [1:13] "time" "survivalC-threshold" "survivalC_0" "survivalC+threshold" ...
$ survTimeT: num [1:68, 1:13] 999 112 87 231 242 991 111 1 587 389 ...
..- attr(*, "dimnames")=List of 2
....$: NULL
....$: chr [1:13] "time" "survivalC-threshold" "survivalC_0" "survivalC+threshold" ...
$ survJumpC: num [1:57, 1:6] 3 4 7 8 10 11 12 13 16 18 ...
..- attr(*, "dimnames")=List of 2
....$: NULL
....$: chr [1:6] "time" "survival" "dSurvival" "index.survival" ...
```

```
$ survJumpT: num [1:51, 1:6] 1 2 7 8 13 15 18 19 20 21 ...
..- attr(*, "dimnames")=List of 2
....$ : NULL
....$ : chr [1:6] "time" "survival" "dSurvival" "index.survival" ...
$ lastSurv : num [1:2] 0 0
```

### 2.2.1 Computation of the score with only one censored event

Let's look at pair 91:

```
getPairScore(BT.keep2, endpoint = 1, rm.withinStrata = FALSE)[91]
```

```
Key: <index.Exp, index.Pl>
  index.Pl index.Exp indexWithinStrata.Pl indexWithinStrata.Exp favorable unfavorable
1: 22 71 22 2 0 0.6950827
  neutral uninf weight
1: 0.3049173 0 1
```

In the dataset this corresponds to:

```
veteran[c(22,71),]
```

```
id trt celltype time status karno diagtime age prior 22 22 Pl smallcell 97 0 60 5 67 0 71 71 Exp squamous 112 1 80 6 60 0
```

The observation from the control group is censored at 97 while the observation from the treatment group has an event at 112. Since the threshold is 20, and (112-20)<97, we know that the pair is not in favor of the treatment. The formula for probability in favor of the control is  $\frac{S_c(97)}{S_c(112+20)}$ . The survival at the event time in the censoring group is stored in survTimeC. Since observation 22 is the 22th observation in the control group:

```
iSurv <- outSurv$survTimeC[22,]
iSurv</pre>
```

```
time
                                 survivalC-threshold
                                                                    survivalC_0
               97.000000
                                           0.5615232
                                                                      0.5171924
      survivalC+threshold
                                 survivalT-threshold
                                                                    survivalT_0
                0.4235463
                                           0.4558824
                                                                      0.3643277
      survivalT+threshold index.survivalC-threshold
                                                              index.survivalC_0
                                                                     28.0000000
                0.2827500
                                          25.0000000
index.survivalC+threshold index.survivalT-threshold
                                                              index.survivalT_0
                                          27.0000000
                                                                     32.0000000
               33.0000000
index.survivalT+threshold
               35.0000000
```

Since we are interested in the survival in the control arm exactly at the event time:

```
Sc97 <- iSurv["survivalC_0"]
Sc97
```

## survivalC\_0 0.5171924

The survival at the event time in the treatment group is stored in survTimeC. Since observation 71 is the 2nd observation in the treatment group:

```
iSurv <- outSurv$survTimeT[2,] ## survival at time 112+20
iSurv</pre>
```

```
survivalC-threshold
                                                                   survivalC_0
                     time
              112.0000000
                                                                     0.4549201
                                           0.5319693
      survivalC+threshold
                                survivalT-threshold
                                                                   survivalT_0
                0.3594915
                                           0.3801681
                                                                     0.2827500
      survivalT+threshold index.survivalC-threshold
                                                             index.survivalC_0
                0.2827500
                                                                    32.0000000
index.survivalC+threshold index.survivalT-threshold
                                                             index.survivalT_0
               37.0000000
                                          31.0000000
                                                                    35.0000000
index.survivalT+threshold
               35,0000000
```

Since we are interested in the survival in the control arm at the event time plus threshold:

```
Sc132 <- iSurv["survivalC+threshold"]
Sc132
```

```
survivalC+threshold 0.3594915
```

The probability in favor of the control is then:

```
Sc132/Sc97
```

```
survivalC+threshold
0.6950827
```

### 2.2.2 Computation of the score with two censored events

When both observations are censored, the formula for computing the probability in favor of treatment or control involves an integral. This integral can be computed using the function calcIntegralSurv\\_cpp that takes as argument a matrix containing the survival and the jumps in survival, e.g.:

```
head(outSurv$survJumpT)
```

time survival dSurvival index.survival index.dsurvival1 index.dsurvival2 [1,] 1 0.7681159 -0.02941176 12 [2,] 2 2 0.7536232 -0.01470588 13 [3,] 7 0.7388463 -0.02941176 14 2 3 [4,]8 0.7388463 -0.02941176 14 3 4 [5,] 13 0.7092924 -0.01470588 16 4 5 [6,]15 0.6945155 -0.02941176

and the starting time of the integration time. For instance, let's look at pair 148:

```
getPairScore(BT.keep2, endpoint = 1, rm.withinStrata = FALSE)[148]
```

which corresponds to the observations:

```
veteran[c(10,72),]
```

```
id trt celltype time status karno diagtime age prior
10 10 Pl squamous 100 0 70 6 70 0
72 72 Exp squamous 87 0 80 3 48 0
```

The probability in favor of the treatment  $(p_F)$  and control  $(p_{UF})$  can be computed as:

$$p_{F} = -\frac{1}{S_{T}(x)S_{C}(y)} \int_{t>y} S_{T}(t+\tau)dS_{C}(t)$$

$$p_{UF} = -\frac{1}{S_{T}(x)S_{C}(y)} \int_{t>x} S_{C}(t+\tau)dS_{T}(t)$$

where x = 87 and y = 100. To ease the call of calcIntegralScore\_cpp we create a warper:

and then call it to compute the probabilities:

```
lastdSurv = outSurv$lastSurv[2])/denom)
rownames(M) <- c("lowerBound","upperBound")
M</pre>
```

```
favorable unfavorable lowerBound 0.5058685 0.3770426 upperBound 0.5058685 0.3770426
```

Note: the lower bound is identical to the upper bound as we could estimate the full survival curve:

outSurv\$lastSurv

[1] 0 0

# 3 Dealing with missing values or/and right censoring

In presence of censoring or missing values, it is often not be possible to classify all pairs without a model for the censoring mechanism. The unclassified pairs, called uninformative, have a score of 0 which will typically bias the estimate of the net net benefit towards 0 <sup>5</sup>. Consider the following dataset:

```
id treatment eventtimeUncensored eventtime status toxicity eta_toxicity status1
               \mathsf{C}
                             0.2135567 0.2135567
                                                                      -0.07945702
1:
   1
                                                         1
                                                                 yes
                                                                                          1
               C
                             0.3422379 0.3422379
2:
    2
                                                                       1.18175155
                                                                                          1
                                                         1
                                                                 no
3:
    3
               C
                             1.3933222 1.3933222
                                                                       2.18614406
                                                                                          1
                                                         1
                                                                 no
    4
               \mathsf{C}
                             0.6737702 0.1961599
                                                         0
                                                                       0.40617493
4:
                                                                 no
                                                                                          1
               C
                             0.5642992 0.5642992
    5
5:
                                                         1
                                                                 yes -0.73835910
                                                                                          1
               C
                             1.1039218 0.1764950
6:
    6
                                                         0
                                                                      -1.95648670
                                                                                          1
                                                                 yes
```

where we have the uncensored event times (eventtimeUncensored) as well as the censored event times (eventtime). The percentage of censored observations is:

```
100*dt[,mean(status==0)]
```

## [1] 44

We would like to be able to recover the net benefit estimated with the uncensored event times:

```
endpoint threshold Delta eventtimeUncensored 0.5 -0.271
```

using the censored survival times.

<sup>&</sup>lt;sup>5</sup>While the power is typically reduced, the type 1 error will still be controlled if censoring is at random

The BuyseTest function handles missing values via two arguments:

- scoring.rule indicates how pairs involving missing data are compared.
  - the Gehan's scoring rule compares the observed values. If it is not possible to decide whether one observation has a better endpoint than the other (e.g. because both are right-censoring) then the paired is scored uninformative.
  - the Peron's scoring rule compares the probability of one observation having a better endpoint
    than the other given the observed values. This require a model for the censoring distribution.
    If the full survival curve can be identified then all pairs can be fully classified otherwise some
    of the pair will be partially uninformative.
- correction.uninf indicates what to do with the uninformative scores. Setting this argument to TRUE will re-distribute this score to favorable/unfavorable/neutral scores.

When the survival curve can be fully identified, the default (and recommanded) approach is to use the Peron's scoring rule where the censoring model rely on Kaplan Meier curve is fitted in each treatment group. When the last observation are censored, then part of the survival curve is unknown and there is no perfect solution. One can:

- only use the Peron's scoring rule, which will lead to a non-0 uninformative score and therefore a "conservative" estimate of the net benefit.
- use the Peron's scoring rule in conjonction with the correction which will led to an unbiased estimator if certain assumption are met.
- only use the Peron's scoring rule with a parametric model which, if appropriate, will lead to an unbiased (and rather efficient) estimator.

# 3.1 Gehan's scoring rule

In the example, Gehan's scoring rule:

leads to many uninformative pairs (about 60%) and an estimate much closer to 0 than the truth.

# 3.2 Peron's scoring rule

In the example, Peron's scoring rule:

```
endpoint threshold total favorable unfavorable neutral uninf Delta lower.ci

1 eventtime 0.5 100 11.1737 43.33707 44.12373 1.365504 -0.3216337 -0.4584262
upper.ci p.value

1 -0.1699543 5.385074e-05
```

leads to no uninformative pairs. Indeed the last observation in each group is an (uncensored) event:

```
dt[,.SD[which.max(eventtime)],by="treatment"]
```

so the full survival curve could be identified. As a result the estimate is very close to the truth.

<u>Note 1:</u> the censoring model can be specified by first fitting a Kaplan Meier model for the survival time:

```
library(prodlim)
e.prodlim <- prodlim(Hist(eventtime, status) ~ treatment, data = dt)</pre>
```

Then passing the model to the BuyseTest via the model.tte argument:

Note that the CI/p-value have changed since, unless stated otherwise, BuyseTest assumes no uncertainty about the survival model when using model.tte. One can force it to account for the uncertainty adding an attribute:

```
endpoint threshold total favorable unfavorable neutral uninf Delta lower.ci

1 eventtime 0.5 100 11.1737 43.33707 44.12373 1.365504 -0.3216337 -0.4584262
upper.ci p.value

1 -0.1699543 5.385074e-05
```

Note 2: it is possible to use a parametric model via the survreg function:

Then passing the model to the BuyseTest via the model.tte argument:

Internally the survival curve is discretized using 1000 points starting from survival = 1 to survival = 0.001 (this is why there is a non-0 but small percentage of uninformative pairs). This is performed internally by applying the BuyseTTEM method. Another discretisation can be obtained by calling BuyseTTEM with another value for the n.grid argument:

```
e.TTEM <- BuyseTTEM(e.survreg, treatment = "treatment", iid = TRUE, n.grid = 2500)
attr(e.TTEM, "iidNuisance") <- TRUE
str(e.TTEM$peron$jumpSurvHaz[[1]][[1]])
```

```
'data.frame': 2500 obs. of 3 variables:

$ index.jump: logi NA NA NA NA NA NA ...

$ time.jump: num 0 0.000307 0.000632 0.000964 0.001301 ...

$ survival : num 1 1 0.999 0.999 0.998 ...
```

and then passing to BuyseTest:

It is therefore possible to extend the approach to other model by defining an appropriate BuyseTTEM method. Looking at the code use for defining BuyseTTEM.survreg can be helpful.

# 3.3 Correction via inverse probability-of-censoring weights (IPCW)

With IPCW, the weights of the non-informative pairs is redistributed to the informative pairs. This is only a good strategy when there are no neutral pairs or there are no lower priority endpoints. This gives an estimate much closer to the true net benefit:

```
BT <- BuyseTest(treatment ~ tte(eventtime, status, threshold = 0.5),
data = dt, keep.pairScore = TRUE, trace = 0,
scoring.rule = "Gehan", method.inference = "none", correction.uninf = 2)
summary(BT)
```

### Generalized pairwise comparisons with 1 endpoint

```
- statistic : net treatment benefit (delta: endpoint specific, Delta: global)
- treatment groups: T (treatment) vs. C (control)
- censored pairs : deterministic score or uninformative
- uninformative pairs: no contribution, their weight is passed to the informative pairs using IPCW
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
eventtime 0.5 100 11.82 36.43 51.75 0 -0.2461
```

We can also see that no pair is finally classified as non informative. To get some inside about the correction we can look at the scores of the pairs:

```
iScore <- getPairScore(BT, endpoint = 1)
```

To get a synthetic view, we only look at the unique favorable/unfavorable/neutral/uniformative results:

```
favorable unfavorable neutral uninf favorableC unfavorableC neutralC uninfC
1:
                        0
                                 1
                                       0
                                            0.000000
                                                          0.000000 2.531646
2:
           0
                        1
                                 0
                                       0
                                            0.000000
                                                          2.531646 0.000000
                                                                                   0
           0
                        0
                                 0
                                       1
                                            0.000000
                                                          0.000000 0.000000
                                                                                   0
3:
4:
            1
                        0
                                 0
                                       0
                                            2.531646
                                                          0.000000 0.000000
                                                                                   0
```

We can see that the favorable/unfavorable/neutral pairs have seen their contribution multiplied by:

```
iScore[,1/mean(favorable + unfavorable + neutral)]
```

#### [1] 2.531646

i.e. the inverse probability of being informative.

# 3.4 Correction at the pair level

Another possible correction is to distribute the non-informative weight of a pair to the average favor-able/unfavorable/neutral probability observed on the sample:

```
BT <- BuyseTest(treatment ~ tte(eventtime, status, threshold = 0.5),
data = dt, keep.pairScore = TRUE, trace = 0,
scoring.rule = "Gehan", method.inference = "none", correction.uninf = TRUE)
summary(BT)
```

Generalized pairwise comparisons with 1 endpoint

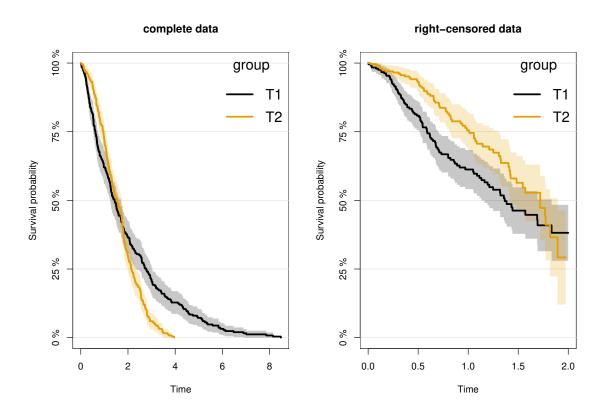
Looking at the scores of the pairs:

```
favorable unfavorable neutral uninf favorableC unfavorableC neutralC uninfC
                                      0.0000000
                                                      0.0000000 1.0000000
           0
                       0
                                1
                                                                                0
1:
                                      0 0.0000000
                                                      1.0000000 0.0000000
2:
           0
                               0
                                                                                0
                       1
                                      1 0.1182278
                                                      0.3643038 0.5174684
3:
           0
                       0
                                0
                                                                                0
                                         1.0000000
                                                      0.0000000 0.0000000
4:
                                                                                0
```

we can see that the corrected probability have not changed for the informative pairs, but for the non-informative they have been set to:

## 3.5 Note on the use of the corrections

As mentioned in ?, the corrections (at the pair level or IPCW) are assumes that uninformative pairs would on average behave like informative pairs. This is typically the case under the proportional hazard assumption. However that may not be the case with other distributions, e.g.:



Here the net benefit that we would have estimated with complete data:

```
BuyseTest.options(method.inference = "none")
e.ref <- BuyseTest(group ~ tte(time,status), data = df, trace = FALSE)
s.ref <- model.tables(e.ref, column = c("favorable","unfavorable","neutral","uninf","Delta"))
s.ref
```

```
favorable unfavorable neutral uninf Delta 1 50.2048 49.7952 0 0 0.004096
```

can be taken as a reference. Violation of the assumption will in this example have a substantial impact and lead to a worse estimate with the correction:

```
e.correction <- BuyseTest(group ~ tte(timeC,statusC), data = df, trace = FALSE, correction.
    uninf = TRUE)
s.correction <- model.tables(e.correction, column = c("favorable","unfavorable","neutral","
    uninf","Delta"))</pre>
```

### Warning message:

```
In .BuyseTest(envir = envirBT, iid = outArgs$iid, method.inference = "none", :

Some of the survival curves for endpoint(s) "timeC" are unknown beyond a survival of 0.25.

The correction of uninformative pairs assume that uninformative pairs would on average behave like

This can be a strong assumption and have substantial impact when the tail of the survival curve is
```

### than without:

```
e.Peron <- BuyseTest(group ~ tte(timeC,statusC), data = df, trace = FALSE)
s.Peron <- model.tables(e.Peron, column = c("favorable","unfavorable","neutral","uninf","Delta
    "))
rbind("reference" = s.ref,
    "no correction" = s.Peron,
    "correction" = s.correction)</pre>
```

```
favorable unfavorable neutral uninf Delta reference 50.20480 49.79520 0 0.00000 0.00409600 no correction 49.09253 39.74775 0 11.15972 0.09344778 correction 55.25931 44.74069 0 0.00000 0.10518628
```

# 4 Simulating data using simBuyseTest

You can simulate data with the simBuyseTest function. For instance the following code simulates data for 5 individuals in the treatment arm and 5 individuals in the control arm:

```
set.seed(10)
simBuyseTest(n.T = 5, n.C = 5)
```

```
id treatment eventtime status toxicity
                                                    score
               C 0.60539304
1:
    1
                                  0
                                          yes -1.85374045
               C 0.31328027
2:
    2
                                  1
                                          yes -0.07794607
               C 0.03946623
3:
    3
                                  0
                                          yes 0.96856634
                                          yes 0.18492596
    4
               C 0.32147489
4:
                                  1
               C 1.57044952
    5
                                  0
                                          yes -1.37994358
5:
               T 0.29069131
    6
                                  0
                                              1.10177950
    7
               T 0.19522131
                                              0.75578151
7:
                                  0
                                          yes
               T 0.04640668
                                          yes -0.23823356
    8
                                  0
8:
               T 0.05277335
     9
                                  1
                                          yes 0.98744470
9:
10: 10
               T 0.43062009
                                          yes 0.74139013
```

By default a categorical, continuous and time to event outcome are generated independently. You can modify their distribution via the arguments argsBin, argsCont, argsTTE. For instance the following code simulates two continuous variables with mean 5 in the treatment arm and 10 in the control arm all with variance 1:

```
id treatment eventtime status toxicity tumorSize
                                                           score
               C 0.1805891
                                 0
                                        yes 11.086551
                                                       8.564486
     1
 1:
     2
               C 0.1702538
 2:
                                 1
                                             9.237455 10.362087
               C 0.2621793
     3
                                 1
                                             9.171337 8.240913
 3:
               C 0.2959301
     4
                                 0
                                         no 10.834474 9.675456
 4:
     5
               C 0.4816549
                                             9.032348
                                                       9.348437
                                 1
               T 0.6446131
                                             5.089347
                                                       6.101780
 6:
     6
                                 1
                                         no
 7:
    7
               T 0.7372264
                                        yes 4.045056
                                                       5.755782
                                 1
               T 0.7213402
     8
                                 0
                                             4.804850
                                                       4.761766
 8:
                                        yes
 9:
     9
               T 0.1580651
                                 1
                                             5.925521
                                                       5.987445
                                        yes
10: 10
               T 0.2212117
                                 0
                                        yes 5.482979
                                                       5.741390
```

This functionality is based on the sim function of the lava package.

# 5 Power calculation using powerBuyseTest

The function powerBuyseTest can be used to perform power calculation, i.e., estimate the probability of rejecting a null hypothesis under a specific generative mechanism. The user therefore need to specify:

- the generative mechanism via a function argument sim
- the null hypothesis argument null
- the sample size(s) for the which the power should be computed argument sample.size

Consider the following generative mechanism where the outcome follows a Student's t-distribution in the treatment and control group, with same variance and degrees of freedom but different mean:

```
Y group
     0.02241932
                      0
  2: -1.07273566
      0.76072274
                      0
  4: -0.25812356
                      0
      0.97207866
                      0
      1.82349375
198:
199: -0.98560076
200:
      1.48143637
201: 3.69314316
                      1
202:
      0.96244416
                      1
```

We then define the null hypothesis:

```
null <- c("netBenefit" = 0)</pre>
```

Naming the value is important since that will indicate which statistic should be used (here the net benefit). We can assess the power of a test based on the net benefit using the following syntax:

```
summary(powerW)
```

Simulation study with Generalized pairwise comparison with 1000 samples  $\,$ 

```
- net benefit statistic (null hypothesis Delta=0)
endpoint threshold n.T n.C mean.estimate sd.estimate mean.se rejection.rate
             1e-12
       Y
                     5
                                  0.2484
                                               0.359 0.3395
                        5
                                                                      0.069
                    10 10
                                  0.2471
                                              0.2551 0.2464
                                                                      0.137
                    20 20
                                  0.2444
                                              0.1746 0.1757
                                                                      0.221
                                              0.1436 0.1437
                    30 30
                                   0.243
                                                                      0.365
                    50 50
                                  0.2438
                                              0.1114 0.1113
                                                                      0.557
                   100 100
                                  0.2458
                                              0.0804 0.0787
                                                                      0.865
```

```
n.T : number of observations in the treatment group

n.C : number of observations in the control group

mean.estimate: average estimate over simulations

sd.estimate : standard deviation of the estimate over simulations

mean.se : average estimated standard error of the estimate over simulations

rejection : frequency of the rejection of the null hypothesis over simulations
(standard error: H-projection of order 1| p-value: after transformation)
```

It is also possibly to use an asymptotic approximation to derive a approximate sample size satisfying a specific type 1 and type 2 error rate:

This procedure is inspired from the procedure presented by ? in section 3.8.2.2. In short, several 'large' datasets are generated and analyzed using GPC to approximate the statistic of interest ( $\Delta$ ) and its asymptotic variance ( $\sigma^2$ ). The sample size needed to achieve the requested power  $(1-\beta)$  and the requested type 1 error ( $\alpha$ ) is then deduce, give a dataset, according to the equation  $N = \sigma^2 \frac{\left(u_{1-\alpha/2} + u_{1-\beta}\right)^2}{\Delta^2}$  where  $u_x$  denotes the x-quantile of the normal distribution. The estimated sample size is then the average calculated sample size across dataset. The argument max.sample.size specifies the number of observation per group in the 'large' dataset (here 1000 per group) and the second element of the argument n.rep specifies the number of datasets (here 10). The quality of the approximation, as well as the computation time, thus improves when increasing max.sample.size and n.rep[2]. The achieved power with the estimated sample size can be output as usual using the summary method:

```
summary(nW)
```

```
Sample size calculation with Generalized pairwise comparison for a power of 0.8 and type 1 error rate of 0.05
```

- estimated sample size (mean [min;max]): 89 [60;145] controls 89 [60;145] treated

- net benefit statistic (null hypothesis Delta=0)

endpoint threshold n.T n.C mean.estimate sd.estimate mean.se rejection.rate Y 1e-12 89 89 0.2452 0.0854 0.0834 0.806

n.T : number of observations in the treatment groupn.C : number of observations in the control group

mean.estimate: average estimate over simulations

sd.estimate : standard deviation of the estimate over simulations

mean.se : average estimated standard error of the estimate over simulations rejection : frequency of the rejection of the null hypothesis over simulations

(standard error: H-projection of order 1| p-value: after transformation)

# 6 Modifying default options

The BuyseTest.options method enable to get and set the default options of the BuyseTest function. For instance, the default option for trace is:

```
BuyseTest.options("trace")
```

#### \$trace

[1] 2

To change the default option to 0 (i.e. no output) use:

```
BuyseTest.options(trace = 0)
```

To change what the results output by the summary function use:

Generalized pairwise comparisons with 1 endpoint

To restore the original default options do:

```
BuyseTest.options(reinitialise = TRUE)
```

# References

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