Reproducing Harnau and Nielsen (2016) using the apc package

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Contents

1 Introduction

The purpose of this vignette is to use the `apc` package version 1.2.3 to reproduce some the result in Harnau and Nielsen (2016): *Asymptotic theory for over-dispersed age-period-cohort and extended chain-ladder models*. This is generalises the theory presented in Martínez Miranda, Nielsen and Nielsen (2015), from a Poisson model to an over-dispersed Poisson model. There is also a vignette available for that paper. The `apc` package builds on the identification analysis and the forecast theory in Kuang, Nielsen and Nielsen (2008a,b), the development of deviance analysis for general data arrays in Nielsen (2014). The package is discussed in Nielsen (2015).

2 Table 1: The data

The data set is taken from Table 1 of Verrall (1991), who attributes the data to Taylor and Ashe (1983). The data consists of a reserving triangle.

The data are available in the `apc` package. They can be called with the command

```
> library(apc)
> data <- data.loss.TA()
> data$response
```

```
        1       2       3       4      5      6      7      8      9    10
1  357848  766940  610542  482940 527326 574398 146342 139950 227229 67948
2  352118  884021  933894 1183289 445745 320996 527804 266172 425046    NA
3  290507 1001799  926219 1016654 750816 146923 495992 280405     NA    NA
4  310608 1108250  776189 1562400 272482 352053 206286     NA     NA    NA
5  443160  693190  991983  769488 504851 470639     NA     NA     NA    NA
6  396132  937085  847498  805037 705960     NA     NA     NA     NA    NA
7  440832  847631 1131398 1063269     NA     NA     NA     NA     NA    NA
8  359480 1061648 1443370      NA     NA     NA     NA     NA     NA    NA
9  376686  986608      NA      NA     NA     NA     NA     NA     NA    NA
10 344014      NA      NA      NA     NA     NA     NA     NA     NA    NA
```

3 Table 2: Deviance analysis

The deviance table can be reproduced by the following commands. The first call has the APC model as reference. The second call has the AC model as reference. The third call has the Ad model as reference. For an overview of the models, see Nielsen (2014). The output is wide, so only selected columns are shown.

```
> apc.fit.table(data,"od.poisson.response")[,c(1,2,4,6)]
```

```
    deviance df.residual F vs.APC prob(>F)
APC  1395518          28      NaN      NaN
AP   1780577          36    0.966    0.482
```

```
AC   1903014          36    1.273    0.297
PC   6862733          36   13.712    0.000
Ad   2269756          44    1.096    0.403
Pd   7990746          44    8.271    0.000
Cd   7807867          44    8.041    0.000
A    2474053          45    1.273    0.278
P    9765797          45    9.879    0.000
C    8597579          45    8.500    0.000
t    8897725          52    6.272    0.000
tA   9096181          53    6.180    0.000
tP  10655658          53    7.432    0.000
tC   9674925          53    6.645    0.000
1   10699464          54    7.180    0.000
```

```
> apc.fit.table(data,"od.poisson.response","AC")[,c(1,2,4,6)]
```

```
   deviance df.residual F vs.AC prob(>F)
AC  1903014          36     NaN      NaN
Ad  2269756          44   0.867    0.552
Cd  7807867          44  13.963    0.000
A   2474053          45   1.200    0.325
C   8597579          45  14.071    0.000
t   8897725          52   8.270    0.000
tA  9096181          53   8.004    0.000
tC  9674925          53   8.648    0.000
1  10699464          54   9.245    0.000
```

```
> apc.fit.table(data,"od.poisson.response","Ad")[,c(1,2,4,6)]
```

```
   deviance df.residual F vs.Ad prob(>F)
Ad  2269756          44     NaN      NaN
A   2474053          45   3.960    0.053
t   8897725          52  16.061    0.000
tA  9096181          53  14.704    0.000
1  10699464          54  16.341    0.000
```

Thus, Table 2 in the paper is constructed as follows.

```
> Table2  <- apc.fit.table(data,"od.poisson.response")[c(1:3,5,8),c(2,1,4,6)]
> Table2  <- cbind(Table2,rbind(matrix(nrow=3,ncol=2),apc.fit.table(data,"od.poisson.
> Table2  <- cbind(Table2,rbind(matrix(nrow=4,ncol=2),apc.fit.table(data,"od.poisson.
> colnames(Table2)<-c("df","Dsub","F_sub,apc","p","F_sub,ac","p","F_sub,ad","p")
> Table2
```

```
    df    Dsub F_sub,apc     p F_sub,ac     p F_sub,ad     p
APC 28 1395518       NaN   NaN       NA    NA       NA    NA
```

```
AP   36 1780577      0.966 0.482        NA    NA        NA    NA
AC   36 1903014      1.273 0.297        NA    NA        NA    NA
Ad   44 2269756      1.096 0.403     0.867 0.552        NA    NA
A    45 2474053      1.273 0.278     1.200 0.325      3.96 0.053
```

4 Table 3: Estimates

The table of estimates can be reproduced by the following commands. The two first calls are for an APC design, the last two calls are for an AC call.

The first and the third call are for a Poisson response model, which is inappropriate here. The second and the fourth call are for an overdispersed response model. The point estimates are the same, but the standard deviations and p-values differ.

There was previously a bug in the calculation of the standard errors. This bug is corrected in version 1.3.2. Thus, the standard errors reported in the published paper are incorrect as they are based on version 1.2.3. The following code gives both the incorrect numbers in the published paper and the correct values.

```
> apc.fit.model(data,"poisson.response","APC")$coefficients.canonical
```

```
                 Estimate   Std. Error     z value      Pr(>|z|)
level         12.787863594           NA          NA            NA
age slope      0.697763921 0.0019497418  357.875035  0.000000e+00
cohort slope   0.111481669 0.0020135573   55.365530  0.000000e+00
DD_age_3      -0.895632465 0.0009858174 -908.517576  0.000000e+00
DD_age_4       0.013570514 0.0009117808   14.883528  4.216670e-50
DD_age_5      -0.642054188 0.0010298524 -623.442928  0.000000e+00
DD_age_6       0.258903877 0.0014025258  184.598298  0.000000e+00
DD_age_7       0.256459103 0.0017973054  142.690889  0.000000e+00
DD_age_8      -0.294147370 0.0022282682 -132.007165  0.000000e+00
DD_age_9       0.705787632 0.0028645524  246.386703  0.000000e+00
DD_age_10     -1.759462290 0.0047534360 -370.145359  0.000000e+00
DD_period_3    0.046442718 0.0026689076   17.401396  8.051629e-68
DD_period_4    0.213821686 0.0018875907  113.277570  0.000000e+00
DD_period_5    0.211836483 0.0015128207  140.027484  0.000000e+00
DD_period_6   -0.405308334 0.0012639987 -320.655662  0.000000e+00
DD_period_7    0.354415338 0.0012145608  291.805356  0.000000e+00
DD_period_8   -0.559003985 0.0011424331 -489.310018  0.000000e+00
DD_period_9    0.556712364 0.0011954761  465.682542  0.000000e+00
DD_period_10  -0.075721211 0.0011024668  -68.683437  0.000000e+00
DD_cohort_3   -0.365436915 0.0011273315 -324.161004  0.000000e+00
DD_cohort_4   -0.025435276 0.0011204241  -22.701472 4.332418e-114
DD_cohort_5   -0.009240882 0.0011665838   -7.921318  2.350053e-15
DD_cohort_6    0.114695160 0.0012454096   92.094327  0.000000e+00
DD_cohort_7    0.053026763 0.0012911471   41.069499  0.000000e+00
DD_cohort_8    0.050815892 0.0013498121   37.646640  0.000000e+00
```

```
DD_cohort_9  -0.408218405 0.0015891640 -256.876190  0.000000e+00
DD_cohort_10  0.101509160 0.0025485309   39.830460  0.000000e+00
```

> *apc.fit.model(data,"od.poisson.response","APC")$coefficients.canonical*

```
                 Estimate Std. Error     t value     Pr(>|t|)
level        12.787863594         NA          NA           NA
age slope     0.697763921  0.4352771  1.60303368 0.1201497769
cohort slope  0.111481669  0.4495239  0.24799944 0.8059446052
DD_age_3     -0.895632465  0.2200824 -4.06953304 0.0003486379
DD_age_4      0.013570514  0.2035538  0.06666795 0.9473198145
DD_age_5     -0.642054188  0.2299131 -2.79259495 0.0093198443
DD_age_6      0.258903877  0.3131119  0.82687324 0.4153002025
DD_age_7      0.256459103  0.4012459  0.63915691 0.5279155530
DD_age_8     -0.294147370  0.4974578 -0.59130118 0.5590624974
DD_age_9      0.705787632  0.6395073  1.10364274 0.2791431307
DD_age_10    -1.759462290  1.0611980 -1.65799628 0.1084815489
DD_period_3   0.046442718  0.5958299  0.07794627 0.9384251530
DD_period_4   0.213821686  0.4214020  0.50740550 0.6158438289
DD_period_5   0.211836483  0.3377351  0.62722669 0.5355923324
DD_period_6  -0.405308334  0.2821859 -1.43631653 0.1619920281
DD_period_7   0.354415338  0.2711490  1.30708703 0.2018161859
DD_period_8  -0.559003985  0.2550466 -2.19177189 0.0368752879
DD_period_9   0.556712364  0.2668884  2.08593707 0.0462147549
DD_period_10 -0.075721211  0.2461242 -0.30765449 0.7606221130
DD_cohort_3  -0.365436915  0.2516752 -1.45201805 0.1576107937
DD_cohort_4  -0.025435276  0.2501331 -0.10168696 0.9197298671
DD_cohort_5  -0.009240882  0.2604382 -0.03548205 0.9719471107
DD_cohort_6   0.114695160  0.2780360  0.41251916 0.6831005515
DD_cohort_7   0.053026763  0.2882468  0.18396307 0.8553682435
DD_cohort_8   0.050815892  0.3013437  0.16863102 0.8672991385
DD_cohort_9  -0.408218405  0.3547787 -1.15062842 0.2596172382
DD_cohort_10  0.101509160  0.5689560  0.17841303 0.8596832346
```

> *apc.fit.model(data,"poisson.response","AC")$coefficients.canonical*

```
                 Estimate   Std. Error     z value      Pr(>|z|)
level        12.506404677           NA          NA            NA
age slope     0.912526274 0.0006490038 1406.041577  0.000000e+00
cohort slope  0.331272153 0.0006694427  494.847656  0.000000e+00
DD_age_3     -0.866221921 0.0009618684 -900.561768  0.000000e+00
DD_age_4      0.020862021 0.0009000823   23.177904 7.607769e-119
DD_age_5     -0.657887194 0.0010211274 -644.275319  0.000000e+00
DD_age_6      0.235501183 0.0013952621  168.786338  0.000000e+00
DD_age_7      0.268781621 0.0017901786  150.142348  0.000000e+00
DD_age_8     -0.301632720 0.0022207760 -135.823119  0.000000e+00
```

```
DD_age_9      0.791901153 0.0028547701  277.395766  0.000000e+00
DD_age_10    -1.793115320 0.0047435076 -378.014643  0.000000e+00
DD_cohort_3  -0.341425729 0.0011054800 -308.848408  0.000000e+00
DD_cohort_4  -0.005004999 0.0011101242   -4.508503  6.528651e-06
DD_cohort_5  -0.071485115 0.0011587242  -61.692949  0.000000e+00
DD_cohort_6   0.137404391 0.0012376357  111.021676  0.000000e+00
DD_cohort_7   0.051370708 0.0012814238   40.088772  0.000000e+00
DD_cohort_8   0.078993227 0.0013406177   58.923006  0.000000e+00
DD_cohort_9  -0.365523501 0.0015746744 -232.126403  0.000000e+00
DD_cohort_10  0.057497628 0.0025389348   22.646359 1.515286e-113
```

```
> apc.fit.model(data,"od.poisson.response","AC")$coefficients.canonical
```

```
                 Estimate Std. Error     t value     Pr(>|t|)
level        12.506404677         NA          NA           NA
age slope     0.912526274  0.1492165  6.11545002 4.873380e-07
cohort slope  0.331272153  0.1539158  2.15229489 3.815328e-02
DD_age_3     -0.866221921  0.2211492 -3.91691154 3.839396e-04
DD_age_4      0.020862021  0.2069436  0.10081019 9.202605e-01
DD_age_5     -0.657887194  0.2347738 -2.80221693 8.119824e-03
DD_age_6      0.235501183  0.3207935  0.73412084 4.676299e-01
DD_age_7      0.268781621  0.4115912  0.65303050 5.178857e-01
DD_age_8     -0.301632720  0.5105926 -0.59075031 5.583786e-01
DD_age_9      0.791901153  0.6563582  1.20650767 2.354907e-01
DD_age_10    -1.793115320  1.0906097 -1.64414033 1.088534e-01
DD_cohort_3  -0.341425729  0.2541679 -1.34330808 1.875795e-01
DD_cohort_4  -0.005004999  0.2552356 -0.01960933 9.844633e-01
DD_cohort_5  -0.071485115  0.2664096 -0.26832787 7.899786e-01
DD_cohort_6   0.137404391  0.2845526  0.48287869 6.321055e-01
DD_cohort_7   0.051370708  0.2946202  0.17436247 8.625575e-01
DD_cohort_8   0.078993227  0.3082299  0.25628026 7.991933e-01
DD_cohort_9  -0.365523501  0.3620433 -1.00961269 3.194190e-01
DD_cohort_10  0.057497628  0.5837425  0.09849828 9.220831e-01
```

Thus, Table 3 with the correct standard errors is constructed as follows.

```
> Table3  <- apc.fit.model(data,"poisson.response","APC")$coefficients.canonical[,c(1
> Table3  <- cbind(Table3,apc.fit.model(data,"od.poisson.response","APC")$coefficient
> Tab3  <- apc.fit.model(data,"poisson.response","AC")$coefficients.canonical[,c(1,2)
> Tab3  <- cbind(Tab3,apc.fit.model(data,"od.poisson.response","AC")$coefficients.can
> Tab3  <- rbind(Tab3[1:11,],matrix(nrow=8,ncol=3),Tab3[12:19,])
> Table3  <- cbind(Table3,Tab3)
> colnames(Table3) <- c("apc est","apc se N","apc se t","ac est","ac se N","ac se t")
> Table3
```

```
                  apc est     apc se N  apc se t       ac est      ac se N
level        12.787863594           NA        NA 12.506404677           NA
```

```
age slope      0.697763921 0.0019497418 0.4352771  0.912526274 0.0006490038
cohort slope   0.111481669 0.0020135573 0.4495239  0.331272153 0.0006694427
DD_age_3      -0.895632465 0.0009858174 0.2200824 -0.866221921 0.0009618684
DD_age_4       0.013570514 0.0009117808 0.2035538  0.020862021 0.0009000823
DD_age_5      -0.642054188 0.0010298524 0.2299131 -0.657887194 0.0010211274
DD_age_6       0.258903877 0.0014025258 0.3131119  0.235501183 0.0013952621
DD_age_7       0.256459103 0.0017973054 0.4012459  0.268781621 0.0017901786
DD_age_8      -0.294147370 0.0022282682 0.4974578 -0.301632720 0.0022207760
DD_age_9       0.705787632 0.0028645524 0.6395073  0.791901153 0.0028547701
DD_age_10     -1.759462290 0.0047534360 1.0611980 -1.793115320 0.0047435076
DD_period_3    0.046442718 0.0026689076 0.5958299           NA           NA
DD_period_4    0.213821686 0.0018875907 0.4214020           NA           NA
DD_period_5    0.211836483 0.0015128207 0.3377351           NA           NA
DD_period_6   -0.405308334 0.0012639987 0.2821859           NA           NA
DD_period_7    0.354415338 0.0012145608 0.2711490           NA           NA
DD_period_8   -0.559003985 0.0011424331 0.2550466           NA           NA
DD_period_9    0.556712364 0.0011954761 0.2668884           NA           NA
DD_period_10 -0.075721211 0.0011024668 0.2461242           NA           NA
DD_cohort_3   -0.365436915 0.0011273315 0.2516752 -0.341425729 0.0011054800
DD_cohort_4   -0.025435276 0.0011204241 0.2501331 -0.005004999 0.0011101242
DD_cohort_5   -0.009240882 0.0011665838 0.2604382 -0.071485115 0.0011587242
DD_cohort_6    0.114695160 0.0012454096 0.2780360  0.137404391 0.0012376357
DD_cohort_7    0.053026763 0.0012911471 0.2882468  0.051370708 0.0012814238
DD_cohort_8    0.050815892 0.0013498121 0.3013437  0.078993227 0.0013406177
DD_cohort_9   -0.408218405 0.0015891640 0.3547787 -0.365523501 0.0015746744
DD_cohort_10   0.101509160 0.0025485309 0.5689560  0.057497628 0.0025389348
               ac se t
level                NA
age slope     0.1492165
cohort slope  0.1539158
DD_age_3      0.2211492
DD_age_4      0.2069436
DD_age_5      0.2347738
DD_age_6      0.3207935
DD_age_7      0.4115912
DD_age_8      0.5105926
DD_age_9      0.6563582
DD_age_10     1.0906097
DD_period_3          NA
DD_period_4          NA
DD_period_5          NA
DD_period_6          NA
DD_period_7          NA
DD_period_8          NA
DD_period_9          NA
```

```
DD_period_10        NA
DD_cohort_3  0.2541679
DD_cohort_4  0.2552356
DD_cohort_5  0.2664096
DD_cohort_6  0.2845526
DD_cohort_7  0.2946202
DD_cohort_8  0.3082299
DD_cohort_9  0.3620433
DD_cohort_10 0.5837425
```

Thus, Table 3 in the paper with the incorrect standard errors is constructed as follows.

```
> Table3  <- apc.fit.model(data,"poisson.response","APC",replicate.version.1.3.1=TRUE
> Table3  <- cbind(Table3,apc.fit.model(data,"od.poisson.response","APC",replicate.ve
> Tab3  <- apc.fit.model(data,"poisson.response","AC",replicate.version.1.3.1=TRUE)$c
> Tab3  <- cbind(Tab3,apc.fit.model(data,"od.poisson.response","AC",replicate.version
> Tab3  <- rbind(Tab3[1:11,],matrix(nrow=8,ncol=3),Tab3[12:19,])
> Table3  <- cbind(Table3,Tab3)
> colnames(Table3) <- c("apc est","apc se N","apc se t","ac est","ac se N","ac se t")
> Table3
```

```
                  apc est     apc se N  apc se t       ac est      ac se N
level        12.787863594           NA        NA 12.506404677           NA
age slope     0.697763921 0.0010034992 0.2240298  0.912526274 0.0005121711
cohort slope  0.111481669 0.0011224663 0.2505890  0.331272153 0.0005731092
DD_age_3     -0.895632465 0.0009858174 0.2200824 -0.866221921 0.0008821453
DD_age_4      0.013570514 0.0009117808 0.2035538  0.020862021 0.0009000816
DD_age_5     -0.642054188 0.0010298524 0.2299131 -0.657887194 0.0010211239
DD_age_6      0.258903877 0.0014025258 0.3131119  0.235501183 0.0013952616
DD_age_7      0.256459103 0.0017973054 0.4012459  0.268781621 0.0017901749
DD_age_8     -0.294147370 0.0022282682 0.4974578 -0.301632720 0.0022207471
DD_age_9      0.705787632 0.0028645524 0.6395073  0.791901153 0.0028546193
DD_age_10    -1.759462290 0.0047534360 1.0611980 -1.793115320 0.0047413068
DD_period_3   0.046442718 0.0020805252 0.4644743           NA           NA
DD_period_4   0.213821686 0.0018875907 0.4214020           NA           NA
DD_period_5   0.211836483 0.0015128207 0.3377351           NA           NA
DD_period_6  -0.405308334 0.0012639987 0.2821859           NA           NA
DD_period_7   0.354415338 0.0012145608 0.2711490           NA           NA
DD_period_8  -0.559003985 0.0011424331 0.2550466           NA           NA
DD_period_9   0.556712364 0.0011954761 0.2668884           NA           NA
DD_period_10 -0.075721211 0.0011024668 0.2461242           NA           NA
DD_cohort_3  -0.365436915 0.0011273315 0.2516752 -0.341425729 0.0010535291
DD_cohort_4  -0.025435276 0.0011204241 0.2501331 -0.005004999 0.0011101093
DD_cohort_5  -0.009240882 0.0011665838 0.2604382 -0.071485115 0.0011587235
DD_cohort_6   0.114695160 0.0012454096 0.2780360  0.137404391 0.0012376357
```

```
DD_cohort_7   0.053026763 0.0012911471 0.2882468  0.051370708 0.0012814185
DD_cohort_8   0.050815892 0.0013498121 0.3013437  0.078993227 0.0013405098
DD_cohort_9  -0.408218405 0.0015891640 0.3547787 -0.365523501 0.0015744312
DD_cohort_10  0.101509160 0.0025485309 0.5689560  0.057497628 0.0025285794
               ac se t
level               NA
age slope    0.1177565
cohort slope 0.1317671
DD_age_3     0.2028196
DD_age_4     0.2069434
DD_age_5     0.2347730
DD_age_6     0.3207934
DD_age_7     0.4115904
DD_age_8     0.5105859
DD_age_9     0.6563235
DD_age_10    1.0901037
DD_period_3         NA
DD_period_4         NA
DD_period_5         NA
DD_period_6         NA
DD_period_7         NA
DD_period_8         NA
DD_period_9         NA
DD_period_10        NA
DD_cohort_3  0.2422235
DD_cohort_4  0.2552322
DD_cohort_5  0.2664094
DD_cohort_6  0.2845526
DD_cohort_7  0.2946190
DD_cohort_8  0.3082050
DD_cohort_9  0.3619874
DD_cohort_10 0.5813616
```

5 Table 4: Forecasts

Table 4 with the correct standard errors is reproduced as follows.

```
> ac.fit <- apc.fit.model(data,"od.poisson.response","AC")
> ac.forecast <- apc.forecast.ac(ac.fit,quantiles=0.95)
> Table4  <- ac.forecast$response.forecast.per[,c(1,6)]
> Table4  <- rbind(Table4,ac.forecast$response.forecast.coh[,c(1,6)])
> Table4  <- rbind(Table4,ac.forecast$response.forecast.all[,c(1,6)])
> rownames(Table4)[19] <- "all"
> Table4
```

```
        forecast   t-0.950
per_11  5226535.83  6491431.2
per_12  4179394.44  5381287.5
per_13  3131667.52  4221849.2
per_14  2127271.92  2938174.2
per_15  1561878.91  2247271.8
per_16  1177743.69  1794299.1
per_17   744287.39  1242589.9
per_18   445521.29   870306.7
per_19    86554.62   269795.5
coh_2     94633.81   280973.2
coh_3    469511.29   835155.7
coh_4    709637.82  1151152.9
coh_5    984888.64  1498634.3
coh_6   1419459.46  2054154.9
coh_7   2177640.62  3016047.4
coh_8   3920301.01  5257277.2
coh_9   4278972.26  6050153.2
coh_10  4625810.69  7977049.4
all    18680855.61 23666264.4
```

Table 4 in the paper with the incorrect standard errors is reproduced as follows.

```
> ac.fit <- apc.fit.model(data,"od.poisson.response","AC",replicate.version.1.3.1=TRU
> ac.forecast <- apc.forecast.ac(ac.fit,quantiles=0.95)
> Table4  <- ac.forecast$response.forecast.per[,c(1,6)]
> Table4  <- rbind(Table4,ac.forecast$response.forecast.coh[,c(1,6)])
> Table4  <- rbind(Table4,ac.forecast$response.forecast.all[,c(1,6)])
> rownames(Table4)[19] <- "all"
> Table4
```

```
        forecast   t-0.950
per_11  5226535.83  6429497.8
per_12  4179394.44  5323703.3
per_13  3131667.52  4171983.7
per_14  2127271.92  2906977.7
per_15  1561878.91  2224013.7
per_16  1177743.69  1774416.8
per_17   744287.39  1228946.0
per_18   445521.29   858592.0
per_19    86554.62   267850.3
coh_2     94633.81   280437.0
coh_3    469511.29   832650.6
coh_4    709637.82  1147493.1
coh_5    984888.64  1493914.5
coh_6   1419459.46  2047360.8
```

```
coh_7    2177640.62  3005099.3
coh_8    3920301.01  5232860.0
coh_9    4278972.26  6015633.6
coh_10   4625810.69  7862685.3
all     18680855.61 23297767.3
```

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