# Package: Rlgt (via r-universe)

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Type Package

Title Bayesian Exponential Smoothing Models with Trend Modifications

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URL https://github.com/cbergmeir/Rlgt

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Description An implementation of a number of Global Trend models for time series forecasting that are Bayesian generalizations and extensions of some Exponential Smoothing models. The main differences/additions include 1) nonlinear global trend, 2) Student-t error distribution, and 3) a function for the error size, so heteroscedasticity. The methods are particularly useful for short time series. When tested on the well-known M3 dataset, they are able to outperform all classical time series algorithms. The models are fitted with MCMC using the 'rstan' package.

License GPL-3

**Encoding** UTF-8

LazyData true

ByteCompile true

**Depends** R (>= 3.4.0), Rcpp (>= 0.12.0), methods, rstantools, forecast, truncnorm

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SystemRequirements GNU make

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RoxygenNote 7.3.2

Suggests doParallel, foreach, knitr, rmarkdown

VignetteBuilder knitr

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# Description

An implementation of Bayesian ETS models named LGT (for non-seasonal time series data) and SGT (for seasonal time series data). These models have been tested on the M3 competition dataset in which they outperform all of the models originally participating in the competition.

# **Getting started**

The best way to get started with the package is to have a look at the vignettes and the various demos that ship with the package. There is a vignette with examples of how to use the various methods included in the package, and a vignette that discusses some of the theoretical background.

As to the demos, you can find their source code in the "demo" subfolder in the package sources (available on CRAN). There are some basic demos and other more advanced ones that run on subsets of the M3 dataset and run potentially for hours.

The package contains models for seasonal and non-seasonal data, allows for external regressors, and different error distributions. In the following, we briefly also present some of the theoretical background of the methods.

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## LGT (Local and Global Trend)

The LGT model is constructed based on Holt's linear trend method. The model is designed to allow for a more general term of error by allowing for heteroscedasticity and an addition of constant "global" trend in the model.

## **Model Equations:**

In terms of mathematical notation, the model can be fully represented as follows:

$$y_{t+1} \sim Student(\nu, y_{t+1}, \sigma_{t+1}) \quad (eq.1.1)$$

$$\widehat{y}_{t+1} = l_t + \gamma l_t^{\rho} + \lambda b_t \quad (eq.1.2)$$

$$l_{t+1} = \alpha y_{t+1} + (1 - \alpha) (l_t) \quad (eq.1.3)$$

$$b_{t+1} = \beta (l_{t+1} - l_t) + (1 - \beta) b_t \quad (eq.1.4)$$

$$\widehat{\sigma}_{t+1} = \sigma l_t^{\tau} + \xi \quad (eq.1.5)$$

#### **Notations:**

 $y_t$  value of the dependent variable of interest at time t

 $\hat{y}_{t+1}$  forecasted value of y at time t+1 given information up to time t

 $\hat{\sigma}_{t+1}$  forecasted deviation at time t+1 given information up to time t

 $l_t$  level at time t

 $b_t$  local trend at time t

## **Parameters:**

- $\nu$  degrees of freedom of the t-distribution
- $\gamma$  coefficient of the global trend
- $\rho$  power coefficient of the global trend
- $\lambda$  damping coefficient of the local trend
- $\alpha$  smoothing parameter for the level term
- $\beta$  smoothing parameter for the local trend term
- $\sigma$  coefficient of the size of error function
- $\tau$  power coefficient of the size of error function
- $\xi$  base or minimum value of the size of error function

# SGT (Seasonal, Global Trend)

The SGT model was designed as a seasonal counterpart to the LGT model. Similar to LGT, this model is devised to allow for a global trend term and heteroscedastic error.

# **Model Equations:**

$$y_{t+1} \sim Student (\nu, \widehat{y}_{t+1}, \sigma_{t+1}) \quad (eq.2.1)$$

$$\widehat{y}_{t+1} = (l_t + \gamma l_t^{\rho}) s_{t+1} \quad (eq.2.2)$$

$$l_{t+1} = \alpha \frac{y_{t+1}}{s_{t+1}} + (1 - \alpha) (l_t) \quad (eq.2.3)$$

$$s_{t+m+1} = \zeta \frac{y_{t+1}}{l_{t+1}} + (1 - \zeta) s_{t+1} \quad (eq.2.4)$$

$$\widehat{\sigma}_{t+1} = \sigma \widehat{y}_{t+1}^{\tau} + \xi \quad (eq.2.5)$$

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## **Additional Notations:**

 $s_t$  seasonality factor at time t

m number of seasons in the data (e.g. 12 for monthly, 4 for quarterly)

## **Additional Parameters:**

 $\zeta$  smoothing parameter for the seasonality terms

## S2GT (Double Seasonal, Global Trend)

S2GT is designed as an extension to SGT for time series data which exhibit two seasonality patterns.

## **Model Equations:**

$$y_{t+1} \sim Student (\nu, \widehat{y}_{t+1}, \sigma_{t+1}) \quad (eq.3.1)$$

$$\widehat{y}_{t+1} = (l_t + \gamma l_t^{\rho}) s_{t+1} w_{t+1} \quad (eq.3.2)$$

$$l_t = \alpha \frac{y_t}{s_t w_t} + (1 - \alpha) (l_{t-1}) \quad (eq.3.3)$$

$$s_{t+m} = \zeta \frac{y_t}{l_t w_t} + (1 - \zeta) s_t \quad (eq.3.4)$$

$$w_{t+d} = \delta \frac{y_t}{l_t s_t} + (1 - \delta) w_t \quad (eq.3.5)$$

$$\widehat{\sigma}_{t+1} = \sigma y_{t+1}^{\tau} + \xi \quad (eq.3.6)$$

## **Additional Notations:**

 $w_t$  second seasonality factor prevailing at time t

d number of (second) seasons in a complete period (e.g. 12 for monthly, 4 for quarterly)

## **Additional Parameters:**

 $\delta$  smoothing parameters for the second seasonality factors

# NA

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As to the demos, you can find their source code in the "demo" subfolder in the package sources (available on CRAN). There are some basic demos and other more advanced ones that run on subsets of the M3 dataset and run potentially for hours.

The package contains models for seasonal and non-seasonal data, allows for external regressors, and different error distributions. In the following, we briefly also present some of the theoretical background of the methods.

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# See Also

Useful links:

• https://github.com/cbergmeir/Rlgt

blgt.multi.forecast Rlgt LSGT Gibbs run in parallel

# **Description**

Fit a list of series and produce forecast, then calculate the accuracy.

# Usage

```
blgt.multi.forecast(
    train,
    future,
    n.samples = 20000,
    burnin = 10000,
    parallel = T,
    m = 1,
    homoscedastic = F
)
```

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## **Arguments**

train A list of training series.

future A list of corresponding future values of the series.

n. samples The number of samples to sample from the posterior (the default is 2e4).

burnin The number of burn in samples (the default is 1e4).

parallel Whether run in parallel or not (Boolean value only, default TRUE).

m The seasonality period, with default m=1, i.e., no seasonality specified.

homoscedastic Run with homoscedastic or heteroscedastic version of the Gibbs sampler ver-

sion. By default it is set to FALSE, i.e., run a heteroscedastic model.

## Value

returns a forecast object compatible with the forecast package in R

## **Examples**

```
## Not run: demo(exampleScript)
## Not run:
## Build data and test
library(Mcomp)
M3.data <- subset(M3, "yearly")
train.data = list()
future.data = list()
for (i in 1:645)
  train.data[[i]] = as.numeric(M3.data[[i]]$x)
  future.data[[i]] = as.numeric(M3.data[[i]]$xx)
}
## Test -- change below to test more series
w.series = 1:20
# w.series = 1:645
                          # uncomment to test all series
# use 10,000 posterior samples; change n.samples to 20,000 to test that as well if you want
s = system.time({
  rv=blgt.multi.forecast(train.data[w.series], future.data[w.series], n.samples=1e4)
})
                          # overall timing info
s[[3]] / length(w.series) # per series time
mean(rv$sMAPE)
                          # performance in terms of mean sMAPE
                          # coverage of prediction intervals -- should be close to 95%
mean(rv$InCI)/6
## End(Not run)
```

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forecast.rlgtfit Rlgtforecast

## **Description**

produce forecasts from an rlgtfit object

# Usage

```
## S3 method for class 'rlgtfit'
forecast(
   object,
   xreg = NULL,
   h = ifelse(frequency(object$x) > 1, 2 * frequency(object$x), 10),
   level = c(80, 95),
   NUM_OF_TRIALS = 2000,
   ...
)
```

# **Arguments**

object rlgtfit object

xreg input regression matrix

h forecasting horizon (the default is 10 for annual and 2\*periods otherwise)

confidence levels for prediction intervals a.k.a. coverage percentiles. Musat be between 0 and 100.

NUM\_OF\_TRIALS number of simulations to run. Suggested range is between (1000,5000), but it needs to be higher for good coverage for very high levels, e.g. 99.8.

... currently not used

## Value

returns a forecast object compatible with the forecast package in R

# **Examples**

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```
# Produce Forecasts for the next 10 years
forecast_result <- forecast(rlgt_model, h = 10, level=c(80, 95, 98))
plot(forecast_result,main="Forecasting lynx dataset with LGT model")</pre>
```

iclaims.example

Weekly Initial Claims of US Unemployment Benefits & Google Trends Queries

## Description

A dataset containing the weekly initial claims for US unemployment benefits against a few related Google trend queries from Jan 2010 - June 2018. This aims to mimick the dataset from Scott and Varian (2014).

## Usage

```
data("iclaims.example")
```

## **Format**

A data frame with 443 rows and 5 variables with log-transformation

week date of records starting by Mondays with US calendar format

claims weekly initial claims of unemployment benefits in thousands

**trend.unemploy** normalized trend queries retreived from gtrendsR API

trend.filling normalized trend queries retreived from gtrendsR API

trend.job normalized trend queries retreived from gtrendsR API

### References

U.S. Employment and Training Administration, Initial Claims [ICNSA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/ICNSA, October 27, 2018.

Trend queries from Google search engine. https://trends.google.com/trends/?geo=US

An interface for retrieving and displaying the information returned online by Google Trends is provided. Trends (number of hits) over the time as well as geographic representation of the results can be displayed. https://CRAN.R-project.org/package=gtrendsR

Scott, S. L. and Varian, H. R. (2014). Predicting the Present with Bayesian Structural Time Series. International Journal of Mathematical Modeling and Optimization 5 4–23.

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initModel

Initialize a model from the Rlgt family

## **Description**

This is an internal function that usually won't be called by users directly. It validates the model type and generates the corresponding list of parameters for the model.

# Usage

```
initModel(
  model.type = NULL,
  use.regression = FALSE,
  seasonalityMethodId = 0,
  levelMethodId = 0,
  useSmoothingMethodForError = FALSE
)
```

# **Arguments**

model.type type of the forecasting model selected, a character object

use.regression binary parameter indicating whether additional regressors will be used for forecasting in multivariate settings.

seasonalityMethodId Seasonality method Id (0- HW, 1- generalized).

levelMethodId Level method Id.
useSmoothingMethodForError

if the non-standard function for error size should be used, one based on smoothed innovations or surprises

## Value

an Rlgt skeleton model

# Description

This is a method of the link{rlgtfit} class to produce posterior intervals

#### Usage

```
## S3 method for class 'rlgtfit'
posterior_interval(object, prob = 0.9, type = "central", ...)
```

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## **Arguments**

object an object of class righti

prob percentile level to be generated (multiple values can be accepted as a vector)

type currently only central is available

... currently not in use

## Value

confidence interval

# **Examples**

print.rlgtfit

Generic print function for rlgtfit models

## **Description**

Print out some characteristics of an rlgtfit model.

# Usage

```
## S3 method for class 'rlgtfit'
print(x, ...)
```

## **Arguments**

x an rlgtfit object

. . . additional function parameters (currently not used)

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rlgt

Fit an Rlgt model

# **Description**

The main function to fit an rlgt model. It fits the parameter values with MCMC.

## Usage

```
rlgt(
   y,
   seasonality = 1,
   seasonality2 = 1,
   seasonality.type = c("multiplicative", "generalized"),
   error.size.method = c("std", "innov"),
   level.method = c("HW", "seasAvg", "HW_sAvg"),
   xreg = NULL,
   control = rlgt.control(),
   verbose = FALSE,
   method = "Stan",
   experimental = "",
   homoscedastic = F
```

## **Arguments**

-

time-series data for training (provided as a numeric vector, or a ts, or msts object).

seasonality

This specification of seasonality will be overridden by frequency of y, if y is of ts or msts class. 1 by default, i.e. no seasonality.

seasonality2

Second seasonality. If larger than 1, a dual seasonality model will be used. However, this is experimental. If not specified and multiple seasonality time series (of msts class) is used, a single seasonality model will be applied, one with seasonality equal to the largest of seasonalities of the time series. 1 by default, i.e. no seasonality or single seasonality.

seasonality.type

Either "multiplicative" (default) or "generalized". The latter seasonality generalizes additive and multiplicative seasonality types.

error.size.method

Function providing size of the error. Either "std" (monotonically, but slower than proportionally, growing with the series values) or "innov" (proportional to a smoothed abs size of innovations, i.e. surprises)

level.method

"HW", "seasAvg", "HW\_sAvg". Here, "HW" follows Holt-Winters approach. "seasAvg" calculates level as a smoothed average of the last seasonality number of points (or seasonality2 of them for the dual seasonality model), and HW\_sAvg is an weighted average of HW and seasAvg methods.

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xreg Optionally, a vector or matrix of external regressors, which must have the same

number of rows as y.

control list of control parameters, e.g. hyperparameter values for the model's prior dis-

tributions, number of fitting interations etc.

verbose whether verbose information should be printed (Boolean value only), default

FALSE.

method Sampling method, default Stan.

experimental Run different versions ("nostudent", "noglobal", "nohet", "ets") for ablation

studies

homoscedastic Run with homoscedastic or heteroscedastic version of the Gibbs sampler ver-

sion. By default it is set to FALSE, i.e., run a heteroscedastic model.

## Value

rlgtfit object

## **Examples**

rlgt.control

Sets and initializes the control parameters

## **Description**

This function initializes and sets the control parameters, i.e. hyperparameter values which control the prior distribution of the rlgtfit model. The purpose of this function is mainly to provide a default value for each of the hyperparameters. The function also accepts a customised set of values of the parameters as provided in the input of this function. This function is used in conjunction with the rlgt function.

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# Usage

```
rlgt.control(
 ADAPT_DELTA = 0.9,
 MAX\_TREE\_DEPTH = 12,
 NUM_OF_CHAINS = 4,
 NUM_OF_CORES = 4,
 ADD_JITTER = TRUE,
 CAUCHY\_SD\_DIV = 150,
 NUM_OF_ITER = 5000,
 MAX_NUM_OF_REPEATS = 2,
 MAX_RHAT_ALLOWED = 1.006,
 NUM_OF_SEASON_INIT_CYCLES = 3,
 MIN_NU = 2,
 MAX_NU = 20,
 MIN_POW_TREND = -0.5,
 MAX_POW_TREND = 1,
 POW_TREND_ALPHA = 1,
 POW_TREND_BETA = 1,
 POW\_SEASON\_ALPHA = 1,
 POW_SEASON_BETA = 1,
 MIN_SIGMA = 1e-10,
 MIN_VAL = 1e-30,
 MAX_VAL = 1e+38
)
```

# **Arguments**

ADAPT_DELTA	Target Metropolis acceptance rate. See Stan manual. Suggested range is between (0.85-0.97).
MAX_TREE_DEPTH	NUTS maximum tree depth. See Stan manual for more details. Suggested range is between (10-15), defaut is 12.
NUM_OF_CHAINS	Number of MCMC chains. Suggested range is 3 to 4. Default is 4.
NUM_OF_CORES	Number of cores used for calculations. It can be smaller than NUM_OF_CHAINS, but for best computational speed, it should be equal to NUM_OF_CHAINS. Default is 4.
ADD_JITTER	Whether to add a very small amount (sd=min(y)*0.0001) of jitter to the input series. It is sometimes useful in cases of series with some perfectly flat sections. Default is TRUE.
CAUCHY_SD_DIV	Cauchy distribution is used for some parameters with non-obvious range. The error size hyperparameter of this distribution is calculated by dividing the max value of the time series by this constant. Suggested range is between (100,300). Default 150.
NUM_OF_ITER	Starting number of iterations for each chain. Suggested range is between (2000,10000). Default is 5000. Generally, the longer the series, the smaller is the value to reach convergence. Some models e.g. those with "innov" error size method are more difficult to fit and require more iterations.

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#### MAX\_NUM\_OF\_REPEATS

Maximum number of the sampling procedure repeats if the fit is unsatisfactorily, i.e. avgRHat>MAX\_RHAT\_ALLOWED. Each round will double the number of iterations which could potentially double the running time. Suggested range is between (2,4). Default is 2.

## MAX\_RHAT\_ALLOWED

Maximum average value of Rhat's that suggests a good fit, i.e. the treshold below which the fit is considered as acceptable. Consult Stan's manual for more details on Rhat. Suggested range is between (1.005,1.02). Default is 1.006.

#### NUM\_OF\_SEASON\_INIT\_CYCLES

For seasonal models, number of seasonality periods used for establishing initial seasonality coefficients. Default is 3.

MIN\_NU Minimum degrees of freedom of the Student's distribution that is used in most models. Suggested range(1.2, 5). Default 2.

MAX\_NU Maximum degrees of freedom of the Student's distribution. Suggested range is between (15,30). Default 20.

MIN\_POW\_TREND Minimum value of the global trend power coefficient. Suggested range is between (-1,0). Default -.5

MAX\_POW\_TREND Maximum value of the global trend power coefficient. It should be 1 to allow the model to approach exponential growth when needed. Default is 1.

## POW\_TREND\_ALPHA

Alpha parameter of Beta prior distribution. To make the forecast more upward curved, so to nudge it towards larger values, make the parameter larger. Suggested range is between (1,6) Default 1.

POW\_TREND\_BETA Beta parameter of Beta prior distribution for the global trend power coefficient.

1 by default, see also above.

## POW\_SEASON\_ALPHA

Alpha parameter of Beta distribution that is the prior of the power coefficient in the formula of the generalized seasonality in gSGT model. 1 by default, increasing it (say, to 3 or 5) will push the seasonality towards multiplicative behavior.

### POW\_SEASON\_BETA

Beta parameter of Beta distribution that is the prior of the power coefficient in the formula of the generalized seasonality in gSGT model. 1 by default.

MIN\_SIGMA Minimum size of the fitted sigma, applied for numerical stability. Must be positive. 1e-10 by default.

MIN\_VAL Minimum value that forecast can take. Must be positive. 1e-30 by default.

MAX\_VAL Maximum value the forecast can take. 1e38 by default.

## Value

list of control parameters

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|--|

# Description

A constructor function for objects of class rlgtfit, the main class of the package. Objects of this class are output from the rlgt function. This constructor will usually not be called by users directly.

# Usage

```
rlgtfit(
   y,
   model.type,
   use.regression,
   seasonalityMethodId,
   levelMethodId,
   useSmoothingMethodForError = FALSE,
   seasonality,
   seasonality2,
   rlgtmodel,
   params,
   control,
   samples
)
```

# Arguments

У	time series data for training (provided as a vector or a ts object).								
model.type	the type of rlgt model, one of: "LGT", "SGT", "S2GT"								
use.regression	ther the data has any additional variables to be used with forecasting, i.e. tivariate time-series.								
seasonalityMeth	nodId								
	Seasonality method Id (0- HW, 1- generalized).								
levelMethodId useSmoothingMet	Level method Id. thodForError								
	if the non-standard function for error size should be used, one based on smoothed innovations or surprises								
seasonality	This specification of seasonality will be overridden by frequency of y, if y is of ts or msts class. 1 by default, i.e. no seasonality.								
seasonality2	Second seasonality. If larger than 1, a dual seasonality model will be used. This specification of seasonality will be overridden by the second seasonality of y, if y is of msts class. 1 by default, i.e. no seasonality or single seasonality.								
rlgtmodel	an rlgt model.								
params	list of parameters of the model (to be fitted).								

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control list of control parameters, i.e. hyperparameter values for the model's prior dis-

tribution. See rlgt.control

samples stanfit object representing the MCMC samples

## Value

an rlgtfit instance

umcsent.example University of Michigan Monthly Survey of Consumer Sentiment &

Google Trends Queries

## **Description**

A dataset containing monthly University of Michigan survey of Consumer Sentiment along a few related google trend queries Jan from 2014 - June 2018. This aims to mimick the dataset from Scott and Varian (2014).

## Usage

```
data("umcsent.example")
```

## **Format**

A data frame with 174 rows and 8 variables with log-transformation

date first date of each month in US calendar format

consumer.sent monthly initial claims of University of Michigan: Consumer Sentiment

search.engine normalized trend queries retreived from gtrendsR API

financial.planning normalized trend queries retreived from gtrendsR API

bus.news normalized trend queries retreived from gtrendsR API

**investing** normalized trend queries retreived from gtrendsR API

energy.utilities normalized trend queries retreived from gtrendsR API

## References

University of Michigan, University of Michigan: Consumer Sentiment [UMCSENT], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UMCSENT, November 17, 2018.

Trends queries from google search engine. https://trends.google.com/trends/?geo=US

An interface for retrieving and displaying the information returned online by Google Trends is provided. Trends (number of hits) over the time as well as geographic representation of the results can be displayed. https://CRAN.R-project.org/package=gtrendsR

Scott, S. L. and Varian, H. R. (2012). Bayesian Variable Selection for Nowcasting Economic Time Series. https://www.aeaweb.org/conference/2013/retrieve.php?pdfid=447

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