# Package: MFT (via r-universe)

August 26, 2024

Type Package

Title The Multiple Filter Test for Change Point Detection

Version 2.0

Date 2019-03-11

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**Description** Provides statistical tests and algorithms for the detection of change points in time series and point processes particularly for changes in the mean in time series and for changes in the rate and in the variance in point processes. References - Michael Messer, Marietta Kirchner, Julia Schiemann, Jochen Roeper, Ralph Neininger and Gaby Schneider (2014), A multiple filter test for the detection of rate changes in renewal processes with varying variance <doi:10.1214/14-AOAS782>. Stefan Albert, Michael Messer, Julia Schiemann, Jochen Roeper, Gaby Schneider (2017), Multi-scale detection of variance changes in renewal processes in the presence of rate change points <doi:10.1111/jtsa.12254>. Michael Messer, Kaue M. Costa, Jochen Roeper and Gaby Schneider (2017), Multi-scale detection of rate changes in spike trains with weak dependencies <doi:10.1007/s10827-016-0635-3>. Michael Messer, Stefan Albert and Gaby Schneider (2018), The multiple filter test for change point detection in time series <doi:10.1007/s00184-018-0672-1>. Michael Messer, Hendrik Backhaus, Albrecht Stroh and Gaby Schneider (2019+) Peak detection in time series.

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RoxygenNote 6.1.1

NeedsCompilation no

Repository CRAN

**Date/Publication** 2019-03-11 20:42:55 UTC

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# Description

Naive routine to remove trend from the data.

# Usage

```
MFT.filterdata(x, filterwidth = NULL, filtersigma = NULL)
```

# Arguments

X	numeric vector, input sequence of random variables.
filterwidth	postive interger, $<$ length(x)/2, number of data points left and right of the current value that are taken into account for Gaussian smoothing.
filtersigma	numeric, > 0, standard deviation of Gassian kernel.

# Value

invisible	
xfiltered	filtered data (for filtering the first and last (filterwidth many) data points of the original series cannot be evaluated and are omited)
xraw	orignal data, but the first and last (filterwidth many) data point are omitted
xtrend	trend that is removed by filtering. That is xfiltered = xraw - xtrend
X	orignal data
filterwidth	number of data points left and right of the current value that are taken into account for Gaussian smoothing
filtersigma	standard deviation of the Gaussian kernel

# Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

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#### References

Michael Messer, Hendrik Backhaus, Albrecht Stroh and Gaby Schneider (2019+). Peak detection in times series

# See Also

```
MFT.peaks, plot.MFT, summary.MFT, MFT.rate, MFT.variance, MFT.mean
```

# **Examples**

```
set.seed(0)
# Normally distributed sequence with negative trend
x <- rnorm(1000,mean=seq(5,0,length.out=1000))
MFT.filterdata(x)
MFT.filterdata(x,filterwidth=200,filtersigma=200)</pre>
```

MFT.mean

MFT.mean

# Description

The multiple filter test for mean change detection in time series or sequences of random variables.

#### Usage

```
MFT.mean(X, autoset.H = TRUE, S = NULL, E = NULL, H = NULL,
    alpha = 0.05, method = "asymptotic", sim = 10000,
    rescale = FALSE, Q = NA, perform.CPD = TRUE, print.output = TRUE)
```

# **Arguments**

Χ	numeric vector, input sequence of random variables
autoset.H	logical, automatic choice of window size H
S	numeric, start of time interval, default: NULL, if NULL then 1 is chosen
Е	numeric, end of time interval, default: NULL, if NULL then length(X) is chosen, needs $E > S$ .
Н	vector, window set H, all elements must be increasing, the largest element must be $=<$ (T/2). H is automatically set if autoset.H = TRUE
alpha	numeric, in (0,1), significance level
method	either "asymptotic" or "fixed", defines how threshold Q is derived, default: "asymptotic", If "asymptotic": Q is derived by simulation of limit process L (Brownian motion); possible set number of simulations (sim), If "fixed": Q may be set manually (Q)
sim	integer, $> 0$ , No of simulations of limit process (for approximation of Q), default = $10000$

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rescale logical, if TRUE statistic G is rescaled to statistic R, default = FALSE

Q numeric, rejection threshold, default: Q is simulated according to sim and alpha.

perform. CPD logical, if TRUE change point detection algorithm is performed

print.output logical, if TRUE results are printed to the console

#### Value

invisible

M test statistic

Q rejection threshold

method how threshold Q was derived, see 'Arguments' for detailed description

sim number of simulations of the limit process (approximation of Q)

rescale states whether statistic G is rescaled to R

CP set of change points estmated by the multiple filter algorithm, increasingly or-

dered in time

means estimated mean values between adjacent change points

S start of time interval

E end of time interval

Tt length of time interval

H window set

alpha significance level

perform. CPD logical, if TRUE change point detection algorithm was performed

tech.var list of technical variables with processes X and G\_ht or R\_ht

type type of MFT which was performed: "mean"

# Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Michael Messer, Stefan Albert and Gaby Schneider (2018). The multiple filter test for change point detection in time series. Metrika <doi:10.1007/s00184-018-0672-1>

#### See Also

```
plot.MFT, summary.MFT, MFT.rate, MFT.variance, MFT.peaks
```

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#### **Examples**

```
# Normal distributed sequence with 3 change points of the mean (at n=100, 155, 350)
set.seed(50)
X1 <- rnorm(400,0,1); X2 <- rnorm(400,3,1); X3 <- rnorm(400,5,1); X4 <- rnorm(600,4.6,1)
X <- c(X1[1:100],X2[101:155],X3[156:350],X4[351:600])
mft <- MFT.mean(X)
plot(mft)
# Set additional parameters (window set)
mft2 <- MFT.mean(X,autoset.H=FALSE,H=c(80,160,240))
plot(mft2)</pre>
```

MFT.m\_est

MFT.m\_est

# **Description**

Naive routine for the estimation of the order of serial correlation (m-dependence) in point processes.

## Usage

```
MFT.m_est(Phi, n = 200, maxlag = 10, alpha = 0.05, plot = TRUE)
```

# **Arguments**

Phi	point process, vector of time stamps
n	positive integer, number of life times used in segments for estimation of serial correlation
maxlag	non-negative integer, maximal lag up to which serial correlations are calculated
alpha	numeric, in (0,1), significance level
plot	logical, if TRUE, estimation procedure is plotted

# Value

m\_est non-negative integer, estimated order of serial correlation (m-dependence)

# Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Michael Messer, Kaue M. Costa, Jochen Roeper and Gaby Schneider (2017). Multi-scale detection of rate changes in spike trains with weak dependencies. Journal of Computational Neuroscience, 42 (2), 187-201. <doi:10.1007/s10827-016-0635-3>

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#### See Also

```
MFT.rate, plot.MFT, summary.MFT, MFT.variance, MFT.mean, MFT.peaks
```

#### **Examples**

```
# 1. Independent life times (m=0)
set.seed(117)
n <- 5000
Phi1 <- cumsum(rexp(n, 3.5))
Phi2 <- cumsum(rexp(n,5))
Phi3 <- cumsum(rexp(n,2))
Phi <- c(Phi1[Phi1<=200], Phi2[Phi2>200 & Phi2<400], Phi3[Phi3>400 & Phi3<700])
MFT.m_est(Phi)
# 2. Point process simulated according to model
\# X_i = a_0 X_i + a_1 X_{i-1} + ... a_m X_{i-m}
# with life times X_i gamma-distributed, 2 change points and true m = 3.
set.seed(210)
Tt <- 3000
m <- 3
a < -c(1,0.5,0.25,0.125)
mu <- c(0.5,1,2)/(sum(a))
sigmaX \leftarrow sqrt(0.225/(sum(a^2)))
shape <- mu^2/sigmaX^2; rate <- mu/sigmaX^2</pre>
len <- 10000
# build auxiliary processes
X1 <- rgamma(len,rate=rate[1],shape=shape[1]); M1 <- embed(X1,m+1)</pre>
v1 <- cumsum(as.vector(M1 %*% a)); v1 <- v1[v1<Tt]
X2 <- rgamma(len,rate=rate[2],shape=shape[2]); M2 <- embed(X2,m+1)</pre>
v2 <- cumsum(as.vector(M2 %*% a)); v2 <- v2[v2<Tt]</pre>
X3 <- rgamma(len,rate=rate[3],shape=shape[3]); M3 <- embed(X3,m+1)</pre>
v3 <- cumsum(as.vector(M3 %*% a)); v3 <- v3[v3<Tt]</pre>
# build final point process with cps at 100 and 200
Phi <- c(v1[v1<Tt/3], v2[v2>Tt/3 & v2<(2/3)*Tt], v3[v3>(2/3)*Tt])
# estimate m
MFT.m_est(Phi)
```

MFT.peaks

MFT.peaks

#### **Description**

The multiple filter test for peak detection in time series or sequences of random variables

# Usage

```
MFT.peaks(x, autoset.H = TRUE, S = NULL, E = NULL, H = NULL,
alpha = 0.05, method = "asymptotic", sim = 10000, Q = NA,
blocksize = NA, two.sided = FALSE, perform.CPD = TRUE,
print.output = TRUE)
```

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#### **Arguments**

numeric vector, input sequence of random variables autoset.H logical, automatic choice of window size H S numeric, start of time interval, default: NULL, if NULL then 1 is chosen Ε numeric, end of time interval, default: NULL, if NULL then length(X) is chosen, needs E > SН vector, window set H, the smallest element must >= 3 be and the largest =< (T/2). H is automatically set if autoset.H = TRUE alpha numeric, in (0,1), significance level method either "asymptotic", "bootstrap" or "fixed", defines how threshold Q is derived, default: "asymptotic", If "asymptotic": Q is derived by simulation of limit process L (Gaussian process); possible set number of simulations (sim), If "bootstrap": Q is derived by (Block)-Bootstrapping; possibly set number of simulations (sim) and blocksize (blocksize), If "fixed": Q may be set manually (Q) integer, > 0, No of simulations of limit process (for approximation of Q), default sim = 10000Q numeric, rejection threshold, default: Q is simulated according to sim and alpha blocksize NA or integer >= 1, if method == 'bootstrap', blocksize determines the size of blocks (number of life times) for bootstrapping two.sided logical, if TRUE a two sided test is performed and also negative peaks are considered in peak detection perform.CPD logical, if TRUE change point detection algorithm is performed logical, if TRUE results are printed to the console print.output

#### Value

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	1 V 13	111	nu

type

М test statistic Q rejection threshold method how threshold Q was derived, see 'Arguments' for detailed description number of simulations of the limit process (approximation of Q) sim blocksize size of blocks (number of life times) for bootstrapping (approximation of Q) CP set of change points estmated by the multiple filter algorithm, increasingly ordered in time S start of time interval Ε end of time interval Τt length of time interval Н window set alpha significance level two.sided logigal, if TRUE also negative peaks are considered perform.CPD logical, if TRUE change point detection algorithm was performed tech.var list of technical variables with processes x and D ht

type of MFT which was performed: "peaks"

MFT.rate

#### Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Michael Messer, Hendrik Backhaus, Albrecht Stroh and Gaby Schneider (2019+). Peak detection in times series

#### See Also

```
MFT.filterdata, plot.MFT, summary.MFT, MFT.mean, MFT.rate, MFT.variance
```

#### **Examples**

MFT.rate

MFT.rate

# **Description**

The multiple filter test for rate change detection in point processes on the line.

#### Usage

```
MFT.rate(Phi, m = 0, cutout = TRUE, autoset.d_H = TRUE, S = NULL,
    E = NULL, d = NULL, H = NULL, alpha = 0.05,
    method = "asymptotic", sim = 10000, rescale = FALSE, Q = NA,
    blocksize = NA, perform.CPD = TRUE, print.output = TRUE)
```

#### **Arguments**

Phi	numeric vector of increasing events, input point process
m	non-negative integer, dependence parameter: serial corellation rho up to order m estimated
cutout	logical, if TRUE for every point, for which the estimated rho becomes negative, the h-neighborhood of G (resp. R) is set to zero. This might only occur, if $m > 0$

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toset.d_H	logical, automatic choice of window size H and step size d
	numeric, start of time interval, default: Smallest multiple of d that lies beyond min(Phi)
	numeric, end of time interval, default: Smallest multiple of d that lies beyond $max(Phi)$ , needs $E > S$ .
	numeric, $> 0$ , step size delta at which processes are evaluated. d is automatically set if autoset.d_H = TRUE
	vector, window set H, all elements must be increasing ordered multiples of d, the smallest element must be $>=$ d and the largest $=<$ (T/2). H is automatically set if autoset.d_H = TRUE
pha	numeric, in $(0,1)$ , significance level
thod	either "asymptotic", "bootstrap" or "fixed", defines how threshold Q is derived, default: "asymptotic", If "asymptotic": Q is derived by simulation of limit process L (Brownian motion); possible set number of simulations (sim), If "bootstrap": Q is derived by (Block)-Bootstrapping; possibly set number of simulations (sim) and blocksize (blocksize), If "fixed": Q may be set manually (Q)
m	integer, $> 0$ , No of simulations of limit process (for approximation of Q), default = $10000$
scale	logical, if TRUE statistic G is rescaled to statistic R, default = FALSE
	numeric, rejection threshold, default: Q is simulated according to sim and alpha.
ocksize	NA or integer >= 1, if method == 'bootstrap', blocksize determines the size of blocks (number of life times) for bootstrapping
rform.CPD	logical, if TRUE change point detection algorithm is performed
int.output	logical, if TRUE results are printed to the console
m scale ocksize rform.CPD	vector, window set H, all elements must be increasing ordered multiples the smallest element must be >= d and the largest =< (T/2). H is automated if autoset.d_H = TRUE  numeric, in (0,1), significance level either "asymptotic", "bootstrap" or "fixed", defines how threshold Q is default: "asymptotic", If "asymptotic": Q is derived by simulation of limic cess L (Brownian motion); possible set number of simulations (sim), If strap": Q is derived by (Block)-Bootstrapping; possibly set number of sitions (sim) and blocksize (blocksize), If "fixed": Q may be set manually (integer, > 0, No of simulations of limit process (for approximation of Q), of a simulation of the statistic G is rescaled to statistic R, default = FALSE numeric, rejection threshold, default: Q is simulated according to sim and NA or integer >= 1, if method == 'bootstrap', blocksize determines the statistic (number of life times) for bootstrapping logical, if TRUE change point detection algorithm is performed

# Value

invisible	
М	test statistic
Q	rejection threshold
method	how threshold Q was derived, see 'Arguments' for detailed description
sim	number of simulations of the limit process (approximation of Q)
blocksize	size of blocks (number of life times) for bootstrapping (approximation of Q)
rescale	states whether statistic G is rescaled to R
m	order of respected serial correlation (m-dependence)
СР	set of change points estmated by the multiple filter algorithm, increasingly ordered in time
rate	estimated mean rates between adjacent change points
S	start of time interval
Е	end of time interval
Tt	length of time interval

MFT.rate

Н	window set
d	step size delta at which processes were evaluated
alpha	significance level
cutout	states whether cutout was used (see 'Arguments')
perform.CPD	logical, if TRUE change point detection algorithm was performed
tech.var	list of technical variables with processes Phi and G_ht or R_ht
type	type of MFT which was performed: "rate"

#### Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Michael Messer, Marietta Kirchner, Julia Schiemann, Jochen Roeper, Ralph Neininger and Gaby Schneider (2014). A multiple filter test for the detection of rate changes in renewal processes with varying variance. The Annals of Applied Statistics 8(4): 2027-67 <doi:10.1214/14-AOAS782>

Michael Messer, Kaue M. Costa, Jochen Roeper and Gaby Schneider (2017). Multi-scale detection of rate changes in spike trains with weak dependencies. Journal of Computational Neuroscience, 42 (2), 187-201. <doi:10.1007/s10827-016-0635-3>

#### See Also

```
MFT.variance, MFT.m_est, plot.MFT, summary.MFT, MFT.mean, MFT.peaks
```

#### **Examples**

```
# Rate change detection in Poisson process
# with three change points (at t = 250, 600 and 680)
set.seed(0)
Phi1 <- runif(rpois(1,lambda=390),0,250)
Phi2 <- runif(rpois(1,lambda=380),250,600)
Phi3 <- runif(rpois(1,lambda=200),600,680)
Phi4 <- runif(rpois(1,lambda=400),680,1000)
Phi <- sort(c(Phi1,Phi2,Phi3,Phi4))
mft <- MFT.rate(Phi)
plot(mft)</pre>
```

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MFT.variance MFT.variance
---------------------------

# Description

The multiple filter test for variance change detection in point processes on the line.

# Usage

```
MFT.variance(Phi, rcp = NULL, autoset.d_H = TRUE, S = NULL,
E = NULL, d = NULL, H = NULL, alpha = 0.05,
method = "asymptotic", sim = 10000, Q = NA, perform.CPD = TRUE,
print.output = TRUE)
```

# Arguments

numeric vector of increasing events, input point process
vector, rate CPs of Phi (if MFT for the rates is used: as CP[,1]), default: constant rate
logical, automatic choice of window size H and step size d
numeric, start of time interval, default: Smallest multiple of d that lies beyond $\min(Phi)$
numeric, end of time interval, default: Smallest multiple of d that lies beyond $\max(\text{Phi})$ , needs $E > S$
numeric, $>$ 0, step size delta at which processes are evaluated. d is automatically set if autoset.d_H = TRUE
vector, window set H, all elements must be increasing ordered multiples of d, the smallest element must be $>=$ d and the largest $=<$ (T/2). H is automatically set if autoset.d_H = TRUE
numeric, in $(0,1)$ , significance level
either "asymptotic", or "fixed", defines how threshold $Q$ is derived, default: "asymptotic". If "asymptotic": $Q$ is derived by simulation of limit process $Q$ (Brownian motion); possible set number of simulations (sim). If "fixed": $Q$ may be set manually $Q$
integer, $> 0$ , No of simulations of limit process (for approximation of Q), default = $10000$
numeric, rejection threshold, default: Q is simulated according to sim and alpha
logical, if TRUE change point detection algorithm is performed
logical, if TRUE results are printed to the console

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#### Value

M test statistic

varQ rejection threshold

method how threshold Q was derived, see 'Arguments' for detailed description

sim number of simulations of the limit process (approximation of Q)

CP set of change points estmated by the multiple filter algorithm, increasingly or-

dered in time

var estimated variances between adjacent change points

S start of time interval
E end of time interval
Tt length of time interval

H window set

d step size delta at which processes were evaluated

alpha significance level

perform. CPD logical, if TRUE change point detection algorithm was performed

tech.var list of technical variables with processes Phi and G\_ht

type type of MFT which was performed: "variance"

# Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Stefan Albert, Michael Messer, Julia Schiemann, Jochen Roeper and Gaby Schneider (2017) Multiscale detection of variance changes in renewal processes in the presence of rate change points. Journal of Time Series Analysis, <doi:10.1111/jtsa.12254>

#### See Also

```
MFT.rate, plot.MFT, summary.MFT, MFT.mean, MFT.peaks
```

# **Examples**

```
# Rate and variance change detection in Gamma process
# (rate CPs at t=30 and 37.5, variance CPs at t=37.5 and 52.5)
set.seed(51)
mu <- 0.03; sigma <- 0.01
p1 <- mu^2/sigma^2; lambda1 <- mu/sigma^2
p2 <- (mu*0.5)^2/sigma^2; lambda2 <- (mu*0.5)/sigma^2
p3 <- mu^2/(sigma*1.5)^2; lambda3 <- mu/(sigma*1.5)^2
p4 <- mu^2/(sigma*0.5)^2; lambda4 <- mu/(sigma*0.5)^2
Phi <- cumsum(c(rgamma(1000,p1,lambda1),rgamma(500,p2,lambda2),
rgamma(500,p3,lambda3),rgamma(300,p4,lambda4)))</pre>
```

plot.MFT

```
# rcp <- MFT.rate(Phi)$CP[,1] # MFT for the rates
rcp <- c(30,37.5) # but here we assume known rate CPs
mft <- MFT.variance(Phi,rcp=rcp) # MFT for the variances
plot(mft)</pre>
```

plot.MFT plot.MFT

# **Description**

Plot method for class 'mft'.

## Usage

```
## S3 method for class 'MFT'
plot(x, col = NULL, ylab1 = NULL, ylab2 = NULL,
   cex.legend = 1.2, cex.diamonds = 1.4, main = TRUE, plot.Q = TRUE,
   plot.M = TRUE, plot.h = TRUE, breaks = NULL, wid = NULL, ...)
```

#### **Arguments**

X	object of class MFT
col	"gray" or vector of colors of length(H). Colors for (G_ht) plot, default: NULL -> rainbow colors from blue to red
ylab1	character, ylab for 1. graphic
ylab2	character, ylab for 2. graphic
cex.legend	numeric, size of annotations in plot
cex.diamonds	numeric, size of diamonds that indicate change points
main	logical, indicates if title and subtitle are plotted
plot.Q	logical, indicates if rejection threshold Q is plotted
plot.M	logical, indicates if test statistic M is plotted
plot.h	logical, indicates if a legend for the window set H is plotted
breaks	integer, >0, number of breaks in rate histogram
wid	integer, >0, width of bars in variance histogram
	additional parameters

# Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Michael Messer, Marietta Kirchner, Julia Schiemann, Jochen Roeper, Ralph Neininger and Gaby Schneider (2014). A multiple filter test for the detection of rate changes in renewal processes with varying variance. The Annals of Applied Statistics 8(4): 2027-67 <doi:10.1214/14-AOAS782>

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#### See Also

```
MFT.rate, MFT.variance, MFT.mean, MFT.peaks, summary.MFT
```

#### **Examples**

```
# Rate change detection in Poisson process
# with three change points (at t = 250, 600 and 680)
set.seed(0)
Phi1 <- runif(rpois(1,lambda=390),0,250)
Phi2 <- runif(rpois(1,lambda=380),250,600)
Phi3 <- runif(rpois(1,lambda=200),600,680)
Phi4 <- runif(rpois(1,lambda=400),680,1000)
Phi <- sort(c(Phi1,Phi2,Phi3,Phi4))
mft <- MFT.rate(Phi)
plot(mft)</pre>
```

summary.MFT

summary.MFT

# Description

Summary method for class 'mft'.

#### Usage

```
## S3 method for class 'MFT'
summary(object, ...)
```

# Arguments

```
object of class MFT additional parameters
```

## Author(s)

Michael Messer, Stefan Albert, Solveig Plomer and Gaby Schneider

#### References

Michael Messer, Marietta Kirchner, Julia Schiemann, Jochen Roeper, Ralph Neininger and Gaby Schneider (2014). A multiple filter test for the detection of rate changes in renewal processes with varying variance. The Annals of Applied Statistics 8(4): 2027-67 <doi:10.1214/14-AOAS782>

#### See Also

```
MFT.rate, MFT.variance, MFT.mean, MFT.peaks, plot.MFT
```

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# Examples

```
# Rate change detection in Poisson process
# with three change points (at t = 250, 600 and 680)
set.seed(0)
Phi1 <- runif(rpois(1,lambda=390),0,250)
Phi2 <- runif(rpois(1,lambda=380),250,600)
Phi3 <- runif(rpois(1,lambda=200),600,680)
Phi4 <- runif(rpois(1,lambda=400),680,1000)
Phi <- sort(c(Phi1,Phi2,Phi3,Phi4))
mft <- MFT.rate(Phi)
summary(mft)</pre>
```

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