

# Package: LongMemoryTS (via r-universe)

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**Type** Package

**Title** Long Memory Time Series

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**Description** Long Memory Time Series is a collection of functions for estimation, simulation and testing of long memory processes, spurious long memory processes and fractionally cointegrated systems.

**License** GPL-2

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**Imports** Rcpp, stats, longmemo, partitions, fracdiff, mvtnorm

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**Author** Christian Leschinski [aut, cre], Michelle Voges [ctb], Kai Wenger [ctb]

**Maintainer** Christian Leschinski <christian\_leschinski@gmx.de>

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ARRLS.sim

*Simulation of Autoregressive Random Level Shift processes.***Description**

Simulation of a AR-RLS process as discussed in and Xu and Perron (2014).

**Usage**

```
ARRLS.sim(T, phi, sig.shifts, prob, sig.noise = 0, const = 0,
          trend = 0, burnin = 100)
```

**Arguments**

|            |  |
|------------|--|
| T          | length of the desired series.  |
| phi        | autoregressive parameter that determines the persistence of the shifts. For phi=1 the process is a "stationary RLS" and for phi=0 the process is a non stationary RLS. |
| sig.shifts | standard deviation of the shifts.  |
| prob       | shift probability. For rare shifts $p^*/T$ , where $p^*$ is the expected number of shifts in the sample.   |
| sig.noise  | standard deviation of the noise component. Default is sig.noise=0.   |
| const      | mean of the process. Default is const=0.   |
| trend      | trend of the process. Default is trend=0.  |
| burnin     | length of the burnin period used. Default is burnin=100.   |

**Details**

add details here

**Author(s)**

Christian Leschinski

**References**

Xu, J. and Perron, P. (2014): Forecasting return volatility: Level shifts with varying jump probability and mean reversion. *International Journal of Forecasting*, 30, pp. 449-463.

**Examples**

```
ts.plot(ARRLS.sim(T=500,phi=0.5, sig.shift=1, prob=0.05), ylab=expression(X[t]))
```

---

cross.Peri

*Cross periodogram of vector valued time series X and Y*

---

**Description**

Calculates the cross periodogram of the vector valued time series X and Y.

**Usage**

```
cross.Peri(X, Y)
```

**Arguments**

|   |              |
|---|--------------|
| X | data matrix. |
| Y | data matrix. |

**Examples**

```

T<-500
d<-c(0.4, 0.2, 0.3)
data<-FI.sim(T, q=3, rho=0, d=d)
X<-data[,1:2]
Y<-data[,3]
cper<-cross.Peri(X, Y)
pmax<-max(Re(cper), Im(cper))
pmin<-min(Re(cper), Im(cper))
plot(Re(cper[1,,]), type="h", ylim=c(pmin,pmax))
lines(Im(cper[1,,]), col=2)
plot(Re(cper[2,,]), type="h", ylim=c(pmin,pmax))
lines(Im(cper[2,,]), col=2)

```

---

|        |   |
|--------|---|
| ddiffw | <i>Helper function that returns AR-representation of FI(d)-process.</i> |
|--------|---|

---

**Description**

returns the first n coefficients in the AR-infinity representation of an FI(d) process

**Usage**

```
ddiffw(n, d)
```

**Arguments**

|   |                                       |
|---|---------------------------------------|
| n | number of coefficients to be returned |
| d | memory parameter                      |

---

|     |  |
|-----|--|
| ELW | <i>Exact local Whittle estimator of the fractional difference parameter d for stationary and non-stationary long memory.</i> |
|-----|--|

---

**Description**

ELW implements the exact local Whittle estimator of Shimotsu and Phillips (2005) that is consistent and asymptotically normal as long as the optimization range is less than  $9/2$ , so that it is possible to estimate the memory of stationary as well as non-stationary processes.

**Usage**

```
ELW(data, m, mean.est = c("mean", "init", "weighted", "none"))
```

**Arguments**

|          |   |
|----------|---|
| data     | data vector of length T.  |
| m        | bandwidth parameter specifying the number of Fourier frequencies. used for the estimation usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| mean.est | specifies the form of mean correction. One of <code>c("mean", "init", "weighted", "none")</code> .  |

**Author(s)**

Christian Leschinski

**References**

Shimotsu, K. and Phillips, P. C. B. (2005): Exact Local Whittle Estimation Of Fractional Integration. The Annals of Statistics, Vol. 33, No. 4, pp. 1890 - 1933

**Examples**

```
library(fracdiff)
T<-1000
d<-0.8
series<-cumsum(fracdiff.sim(T,d=(d-1))$series)
ts.plot(series)
ELW(series, m=floor(1+T^0.7))$d
```

---

|       |   |
|-------|---|
| ELW2S | <i>Two-Step Exact local Whittle estimator of fractional integration with unknown mean and time trend.</i> |
|-------|---|

---

**Description**

ELW2S implements the two-step ELW estimator of Shimotsu (2010) that is consistent and asymptotically normal in the range from  $-1/2$  to  $2$ .

**Usage**

```
ELW2S(data, m, trend_order = 0, taper = c("Velasco", "HC"))
```

**Arguments**

|             |   |
|-------------|---|
| data        | data vector of length T.  |
| m           | bandwidth parameter specifying the number of Fourier frequencies. used for the estimation usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| trend_order | specifies the form of detrending: 0 for a constant, only, 1 for a linear trend, and so on.  |
| taper       | string from <code>c("Velasco", "HC")</code> specifying the tapered form of the local Whittle estimator used in the first step.                            |

**Author(s)**

Christian Leschinski

**References**

Shimotsu, K. (2010): Exact Local Whittle Estimation Of Fractional Integration with Unknown Mean and Time Trend. *Econometric Theory*, Vol. 26, pp. 501 - 540.

**Examples**

```
library(fracdiff)
T<-1000
d<-0.8
trend<-(1:T)/T
series<-cumsum(fracdiff.sim(T,d=(d-1))$series)
ts.plot(series)
ELW2S(series, m=floor(1+T^0.7), trend_order=0)$d
series2<-series+2*trend
ELW2S(series2, m=floor(1+T^0.7), trend_order=1)$d
series3<-series+2*trend+2*trend^2
ELW2S(series3, m=floor(1+T^0.7), trend_order=2)$d
```

---

F.hat

*Empirical cummulative spectral distribution function*


---

**Description**

Calculates the empirical cummulative spectral distribution function from the cross periodogram of the vector valued time series X and Y.

**Usage**

```
F.hat(X, Y, k, l)
```

**Arguments**

|   |   |
|---|---|
| X | data matrix.  |
| Y | data matrix.  |
| k | integer that determines the order number of the first Fourier frequency used. |
| l | integer that determines the order number of the last Fourier frequency used.  |

**Examples**

```
T<-500
d<-c(0.4, 0.2, 0.3)

data<-FI.sim(T, q=3, rho=0, d=d)
X<-data[,1:2]
Y<-data[,3]
F.hat(X, Y, 1, floor(T/2))
```

---

fBM

*Fractional Brownian Motion / Bridge of Type I or II.*

---

**Description**

fBM simulates a fractional Brownian motion / bridge of type I or II.

**Usage**

```
fBM(n, d, type = c("I", "II"), bridge = FALSE)
```

**Arguments**

|        |   |
|--------|---|
| n      | number of increments in the fractional Brownian motion.   |
| d      | memory parameter $-0.5 < d < 0.5$ . Note that $d = H - 1/2$ .   |
| type   | either "I" or "II", to define the type of motion.   |
| bridge | either TRUE or FALSE, to specify whether a fractional Brownian motion or bridge should be returned. Default is FALSE so that the function returns a fractional Brownian motion. |

**Author(s)**

Kai Wenger

**References**

Marinucci, D., Robinson, P. M. (1999). Alternative forms of fractional Brownian motion. *Journal of statistical planning and inference*, 80(1-2), 111 - 122.

Davidson, J., Hashimzade, N. (2009). Type I and type II fractional Brownian motions: A reconsideration. *Computational statistics & data analysis*, 53(6), 2089-2106.

Bardet, J.-M. et al. (2003): *Generators of long-range dependent processes: a survey. Theory and applications of long-range dependence*, pp. 579 - 623, Birkhauser Boston.

**Examples**

```
n<-1000
d<-0.4
set.seed(1234)
motionI<-fBM(n,d, type="I")
set.seed(1234)
motionII<-fBM(n,d, type="II")
ts.plot(motionI, ylim=c(min(c(motionI,motionII)), max(motionI,motionII)))
lines(motionII, col=2)
```

FCI\_CH03

*Rank estimation in fractionally cointegrated systems.***Description**

FCI\_CH03 Rank estimation in fractionally cointegrated systems by Chen, Hurvich (2003). Returns estimated cointegrating rank.

**Usage**

```
FCI_CH03(X, diff_param = 1, m_peri, m)
```

**Arguments**

|            |   |
|------------|---|
| X          | vector of length T.   |
| diff_param | integer specifying the order of differentiation in order to ensure stationarity of data, where diff_param-1 are the number of differences. Default is diff_param=1. |
| m_peri     | fixed positive integer for averaging the periodogram, where m_peri>(nbr of series + 3)  |
| m          | bandwidth parameter specifying the number of Fourier frequencies used for the estimation, usually $\text{floor}(1+T^{\delta})$ , where $0<\delta<1$ .               |

**Author(s)**

Christian Leschinski

**References**

Chen, W. W. and Hurvich, C. M. (2003): Semiparametric estimation of multivariate fractional cointegration. Journal of the American Statistical Association, Vol. 98, No. 463, pp. 629 - 642.

**Examples**

```
T<-1000
series<-FI.sim(T=T, q=3, rho=0.4, d=c(0.1,0.2,0.4), B=rbind(c(1,0,-1),c(0,1,-1),c(0,0,1)))
FCI_CH03(series,diff_param=1, m_peri=25, m=floor(1+T^0.65))
```



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|          |  |
|----------|--|
| FCI_CH06 | <i>Residual-based test for fractional cointegration (Chen, Hurvich (2006))</i> |
|----------|--|

---

### Description

FCI\_CH06 Semiparametric residual-based test for fractional cointegration by Chen, Hurvich (2003). Returns test statistic, critical value and testing decision. Null hypothesis: no fractional cointegration.

### Usage

```
FCI_CH06(X, m_peri, m, alpha = 0.05, diff_param = 1)
```

### Arguments

|            |  |
|------------|--|
| X          | data matrix.   |
| m_peri     | fixed positive integer for averaging the periodogram, where $m\_peri > (\text{nbr of series} + 3)$   |
| m          | bandwidth parameter specifying the number of Fourier frequencies used for the estimation, usually $\text{floor}(1 + T^\delta)$ , where $0 < \delta < 1$ .  |
| alpha      | desired significance level. Default is $\alpha = 0.05$ .   |
| diff_param | integer specifying the order of differentiation in order to ensure stationarity of data, where $\text{diff\_param} - 1$ are the number of differences. Default is $\text{diff\_param} = 1$ for no differences. |

### Author(s)

Christian Leschinski

### References

Chen, W. W. and Hurvich, C. M. (2006): Semiparametric estimation of fractional cointegrating subspaces. The Annals of Statistics, Vol. 34, No. 6, pp. 2939 - 2979.

### Examples

```
T<-1000
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.1,0.4), B=rbind(c(1,-1),c(0,1)))
FCI_CH06(series, diff_param=1, m_peri=25, m=floor(T^0.65))
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.4,0.4))
FCI_CH06(series, diff_param=1, m_peri=25, m=floor(T^0.65))
```

FCI\_MV04

*Test for fractional cointegration (Marmol, Velasco (2004))***Description**

FCI\_MV04 Semiparametric test for fractional cointegration by Marmol, Velasco (2004). Returns test statistic, critical value and testing decision. Null hypothesis: no fractional cointegration.

**Usage**

```
FCI_MV04(X, type = c("none", "const", "trend"), N, m, alpha = 0.05)
```

**Arguments**

|       |  |
|-------|--|
| X     | data matrix.   |
| type  | string that is either "none", "const", or "trend" and determines the form of linear regression.  |
| N     | bandwidth parameter specifying the number of Fourier frequencies used for the beta estimation, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ .             |
| m     | bandwidth parameter specifying the number of Fourier frequencies used for the memory parameter estimation, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| alpha | desired significance level. Default is $\alpha = 0.05$ .   |

**Author(s)**

Christian Leschinski, Michelle Voges

**References**

Marmol, F. and Velasco, C. (2004): Consistent testing of cointegrating relationships. *Econometrica*, Vol. 72, No. 6, pp. 1809 - 1844.

**Examples**

```
T<-500
series<-FI.sim(T=T, q=2, rho=0.1, d=c(0.6,1), B=rbind(c(1,-1),c(0,1)))
FCI_MV04(series, type="const", N=floor(T^(0.75)), m=floor(T^(2/3)))
series<-FI.sim(T=T, q=2, rho=0.1, d=c(0.8,0.8))
FCI_MV04(series, type="const", N=floor(T^(0.75)), m=floor(T^(2/3)))
```

FCI\_N10

*Nonparametric test for fractional cointegration (Nielsen (2010))***Description**

FCI\_CH06 Nonparametric test and rank estimation for fractional cointegration by Nielson (2010). Returns either test statistic, critical value and testing decision (null hypothesis: no fractional cointegration) or the estimated cointegrating rank.

**Usage**

```
FCI_N10(X, d1 = 0.1, m, mean_correct = c("mean", "init", "weighted",
    "none"), type = c("test", "rank"), alpha = 0.05)
```

**Arguments**

|              |   |
|--------------|---|
| X            | data matrix.  |
| d1           | fixed order of integration, default is $d1=0.1$ as recommended by Nielsen (2010), no critical values for other choices available.   |
| m            | bandwidth parameter specifying the number of Fourier frequencies used for the memory estimation required for the asymptotic distribution, usually $\text{floor}(1+T^\delta)$ , where $0 < \delta < 1$ . |
| mean_correct | specifies the form of mean correction in the memory estimation.   |
| type         | string that determines whether the test or the rank estimation procedure is applied.  |
| alpha        | desired significance level. Default is $\alpha=0.05$ .  |

**Author(s)**

Christian Leschinski, Michelle Voges

**References**

Nielsen, M. O. (2010): Nonparametric cointegration analysis of fractional systems with unknown integration orders. *Journal of Econometrics*, Vol. 155, No. 2, pp. 170 - 187.

**Examples**

```
T<-1000
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.1,0.9), B=rbind(c(1,-1),c(0,1)))
FCI_N10(series, m=floor(T^0.75), type="test")
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.9,0.9))
FCI_N10(series, m=floor(T^0.75), type="test")
series<-FI.sim(T=T, q=3, rho=0.4, d=c(0.2,0.2,1), B=rbind(c(1,-0.5,-0.3),c(0,1,-0.4),c(0,0,1)))
FCI_N10(series,m=floor(T^0.75), type="rank")
```

---

|          |  |
|----------|--|
| FCI_NS07 | <i>Rank estimation in fractionally cointegrated systems by Nielsen, Shimotsu (2007).</i> |
|----------|--|

---

### Description

FCI\_NS07 Rank estimation in fractionally cointegrated systems by Nielsen, Shimotsu (2007). Returns estimated cointegrating rank,  $r=0,\dots,\text{dim}-1$ .

### Usage

```
FCI_NS07(X, m, m1, mean_correct = c("mean", "init", "weighted", "none"),
         v_n = m^(-0.3))
```

### Arguments

|              |   |
|--------------|---|
| X            | data matrix.  |
| m            | bandwidth parameter specifying the number of Fourier frequencies used for the estimation of G, usually $\text{floor}(1+T^\delta)$ , where $0<\delta<1$ and $m1>m$ .         |
| m1           | bandwidth parameter specifying the number of Fourier frequencies used for the memory estimation, usually $\text{floor}(1+T^{\delta_1})$ , where $0<\delta_1<1$ and $m1>m$ . |
| mean_correct | specifies the form of mean correction in the memory estimation.   |
| v_n          | bandwidth parameter. Nielsen and Shimotsu (2007) use $m^{-0.3}$ in their simulation studies, which is the default value. $m^{-b}$ mit $0<b<0.5$ can be used.                |

### Author(s)

Christian Leschinski, Michelle Voges

### References

Nielsen, M. O. and Shimotsu, K. (2007): Determining the cointegrating rank in nonstationary fractional systems by the exact local Whittle approach. Journal of Econometrics, Vol. 141, No. 2, pp. 574 - 596.

### Examples

```
T<-1000
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.1,0.4), B=rbind(c(1,-1),c(0,1)))
FCI_NS07(series, m1=floor(1+T^0.75), m=floor(1+T^0.65))
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.9,0.9))
FCI_NS07(series, m1=floor(1+T^0.75), m=floor(1+T^0.65))
```

FCI\_R08

*Hausman-type test for fractional cointegration (Robinson (2008))***Description**

FCI\_R08 Semiparametric Hausmann-type test for fractional cointegration by Robinson (2008). Returns test statistic, critical value, testing decision and type. Null hypothesis: no fractional cointegration.

**Usage**

```
FCI_R08(X, m, type = c("", "*", "**"), alpha = 0.05, a.vec = NULL)
```

**Arguments**

|       |   |
|-------|---|
| X     | data matrix.  |
| m     | bandwidth parameter specifying the number of Fourier frequencies used for the estimation, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ .       |
| type  | determines the implementation of the test statistic: "" - real-valued, "*" - complex-valued, or "**" - complex-valued allowing for different memory parameters. |
| alpha | desired significance level. Default is $\alpha = 0.05$ .  |
| a.vec | weighting scheme for averaging univariate memory estimates, default is simple arithmetic mean.  |

**Author(s)**

Christian Leschinski, Michelle Voges

**References**

Robinson, P. (2008): Diagnostic testing for cointegration. *Journal of Econometrics*, Vol. 143, No. 1, pp. 206 - 225.

**Examples**

```
T<-1000
series<-FI.sim(T=T, q=2, rho=0.9, d=c(0.1,0.4), B=rbind(c(1,-1),c(0,1)))
FCI_R08(series, m=floor(T^0.75), type="*")
series<-FI.sim(T=T, q=2, rho=0.9, d=c(0.4,0.4))
FCI_R08(series, m=floor(T^0.75), type="*")
```

---

|            |   |
|------------|---|
| FCI_SRFB18 | <i>Frequency-domain test for fractional cointegration (Souza, Reise, Franco, Bondon (2018))</i> |
|------------|---|

---

### Description

FCI\_CH06 Semiparametric frequency-domain test for fractional cointegration by Souza, Reise, Franco, Bondon (2018). Returns test statistic, critical value, testing decision and estimate of the cointegrating strength. Null hypothesis: no fractional cointegration.

### Usage

```
FCI_SRFB18(X, d, m, r, alpha = 0.05)
```

### Arguments

|       |   |
|-------|---|
| X     | bivariate data matrix.  |
| d     | known common memory parameter. However, simulations indicate that consistent memory estimation does not invalidate the test.                              |
| m     | bandwidth parameter specifying the number of Fourier frequencies used for the estimation, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| r     | integer trimming parameter, $r > 0$ .   |
| alpha | desired significance level. Default is $\alpha = 0.05$ .  |

### Author(s)

Michelle Voges

### References

Souza, I. V. M., Reisen, V. A., Franco, G. d. C. and Bondon, P. (2018): The estimation and testing of the cointegration order based on the frequency domain. *Journal of Business & Economic Statistics*, Vol. 36, No. 4, pp. 695 - 704.

### Examples

```
T<-1000
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.1,0.7), B=rbind(c(1,-1),c(0,1)))
FCI_SRFB18(series, d=0.7, m=floor(T^0.75), r=1)
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.4,0.4))
FCI_SRFB18(series, d=0.4, m=floor(T^0.75), r=1)
```

---

|           |   |
|-----------|---|
| FCI_WWC15 | <i>Semiparametric test for fractional cointegration (Wang, Wang, Chan (2015))</i> |
|-----------|---|

---

### Description

FCI\_WWC15 Semiparametric implementation of the testing strategy for fractional cointegration by Wang, Wang, Chan (2015). Returns test statistic, critical value and testing decision. Null hypothesis: no fractional cointegration.

### Usage

```
FCI_WWC15(X, m, mean_correct = c("init", "mean", "weighted", "none"),
          alpha = 0.05)
```

### Arguments

|              |   |
|--------------|---|
| X            | bivariate data matrix.  |
| m            | bandwidth parameter specifying the number of Fourier frequencies used for the estimation, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| mean_correct | specifies the form of mean correction in the memory estimation.   |
| alpha        | desired significance level. Default is $\alpha = 0.05$ .  |

### Author(s)

Christian Leschinski, Michelle Voges

### References

Wang, B., Wang, M. and Chan, N. H. (2015): Residual-based test for fractional cointegration. *Economics Letters*, Vol. 126, pp. 43 - 46.

Hualde, J. (2013): A simple test for the equality of integration orders. *Economics Letters*, Vol. 119, No. 3, pp. 233 - 237.

### Examples

```
T<-1000
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.1,0.8), B=rbind(c(1,1),c(0,1)))
FCI_WWC15(series, m=floor(1+T^0.65))
series<-FI.sim(T=T, q=2, rho=0.4, d=c(0.8,0.8))
FCI_WWC15(series, m=floor(1+T^0.65))
```

---

|           |   |
|-----------|---|
| FCI_ZRY18 | <i>Rank estimation in fractionally cointegrated systems (Zhang, Robinson, Yao (2018))</i> |
|-----------|---|

---

### Description

FCI\_CH06 SRank estimation in fractionally cointegrated systems (Zhang, Robinson, Yao (2018)). Returns estimated cointegrating rank,  $r=0, \dots, \text{dim}-1$ .

### Usage

```
FCI_ZRY18(X, lag_max, lag_max2 = 20, c0 = 0.3)
```

### Arguments

|          |   |
|----------|---|
| X        | data matrix.  |
| lag_max  | number of lags in autocovariance matrix of data for eigenvector estimation.   |
| lag_max2 | number of residual autocorrelations that are averaged, default is $m=20$ as recommended by Zhang, Robinson, Yao (2018).       |
| c0       | threshold to compare averaged residual autocorrelation to, default is $c0=0.3$ as recommended by Zhang, Robinson, Yao (2018). |

### Author(s)

Michelle Voges

### References

Zhang, R., Robinson, P. and Yao, Q. (2018): Identifying cointegration by eigenanalysis. Journal of the American Statistical Association (forthcoming).

### Examples

```
T<-1000
series<-FI.sim(T=T, q=3, rho=0.4, d=c(0.2,0.2,1), B=rbind(c(1,0,-1),c(0,1,-1),c(0,0,1)))
FCI_ZRY18(series, lag_max=5, lag_max2=20, c0=0.3)
series<-FI.sim(T=T, q=3, rho=0.4, d=c(1,1,1))
FCI_ZRY18(series, lag_max=5, lag_max2=20, c0=0.3)
```



---

`fdiff`*Fast fractional differencing procedure of Jensen and Nielsen (2014).*

---

**Description**

Takes the d-th difference of the series.

**Usage**

```
fdiff(x, d)
```

**Arguments**

|                |   |
|----------------|---|
| <code>x</code> | series to be differenced  |
| <code>d</code> | memory parameter indicating order of the fractional difference. |

**Details**

This code was first published on the [university webpage of Morten Nielsen](#) and is redistributed here with the author's permission.

**Author(s)**

Jensen, A. N. and Nielsen, M. O.

**References**

Jensen, A. N. and Nielsen, M. O. (2014): A fast fractional difference algorithm, *Journal of Time Series Analysis* 35(5), pp. 428-436.

**Examples**

```
acf(fdiff(x=rnorm(500), d=0.4))
```

---

`FDLS`*Narrow band estimation of the cointegrating vector.*

---

**Description**

Semiparametric estimator for the cointegrating vector as suggested by Robinson (1994) and discussed by Robinson and Marinucci (2003) and Christensen and Nielsen (2006), among others.

**Usage**

```
FDLS(X, Y, m)
```

**Arguments**

|   |   |
|---|---|
| X | data matrix.  |
| Y | data matrix.  |
| m | bandwidth parameter specifying the number of Fourier frequencies. used for the estimation of d, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |

**Details**

add details here. Assumes that there is no long-run coherence between the errors and the regressors. Consistency and Normality, Stationarity, assumptions,...

**References**

Christensen, B. J. and Nielsen, M. O. (2006): Asymptotic normality of narrow-band least squares in the stationary fractional cointegration model and volatility forecasting. *Journal of Econometrics*, 133, pp. 343-371.

Robinson, P. M., (1994): Semiparametric analysis of long-memory time series. *Annals of Statistics*, 22, pp. 515-539.

Robinson, P. M. and Marinucci, D. (2003): Semiparametric frequency domain analysis of fractional cointegration. In: Robinson, P. M. (Ed.), *Time Series with Long Memory*, Oxford University Press, Oxford, pp. 334-373.

**Examples**

```
T<-500
d<-0.4
beta<-1

data<-FI.sim(T, q=2, rho=0, d=c(d,0))
xt<-data[,1]
et<-data[,2]
yt<-beta*xt+et
FDLS(xt,yt,m=floor(1+T^0.4))

data<-FI.sim(T, q=2, rho=0.8, d=c(d,0))
xt<-data[,1]
et<-data[,2]
yt<-beta*xt+et
FDLS(xt,yt,m=floor(1+T^0.4))
```

---

 FI.sim

---

*Simulate multivariate fractional white noise.*


---

**Description**

FI.sim Simulates a

**Usage**

```
FI.sim(T, q, rho, d, B = diag(q), var = 1, burnin = 250)
```

**Arguments**

|        |  |
|--------|--|
| T      | positive integer determining the length of the simulated series.   |
| q      | positive integer determining the dimension of the simulated series.  |
| rho    | real value between 0 and 1 that determines correlation between the innovations.                            |
| d      | vector of memory parameters with length q.   |
| B      | qxq matrix specifying cointegrating relations. By default <code>diag(q)</code> .                           |
| var    | positive real value that determines the variance of the innovations. Default value is <code>var=1</code> . |
| burnin | positive integer determining the length of the burnin period. Default is <code>burnin=250</code> .         |

**Examples**

```
T=1000
series<-FI.sim(T=T,q=2,rho=0.7,d=c(0.4,0.4))
ts.plot(series, col=1:2)
cor(series)

series<-FI.sim(T=T,q=2,rho=0,d=c(0.1,0.4), B=rbind(c(1,-1),c(0,1)))
ts.plot(series, col=1:2)
```

---

 FMNBLS

*Fully Modified Narrow Band Least Squares (FMNBLS) estimation of the cointegrating vector.*

---

**Description**

Semiparametric estimator for the cointegrating vector as suggested by Nielsen and Frederiksen (2011). Refines the FDLS estimator by allowing for long run coherence between the regressors and the errors.

**Usage**

```
FMNBLS(X, Y, m0, m1, m2, m3, method = c("local.W", "Hou.Perron", "ELW"))
```

**Arguments**

|    |                      |
|----|----------------------|
| X  | data matrix.         |
| Y  | data matrix.         |
| m0 | bandwidth parameter. |
| m1 | bandwidth parameter. |
| m2 | bandwidth parameter. |

m3                    bandwidth parameter.  
 method                one from method=c("local.W", "Hou.Perron", "ELW"), to determine which semiparametric long memory estimator is to be used.

### Details

add details here. Especially on the selection of all these bandwidth parameters. careful: it is not clear, whether HP an be used here.

### References

Nielsen and Frederiksen (2011): Fully modified narrow-band least squares estimation of weak fractional cointegration. The Econometrics Journal, 14, pp. 77-120.

### See Also

[FDLS](#), [local.W](#), [Hou.Perron](#), [ELW](#)

### Examples

```
T<-500
d<-0.4
beta<-1

m0<-m3<-floor(T^0.4)
m1<-floor(T^0.6)
m2<-floor(T^0.8)

data<-FI.sim(T, q=2, rho=0.8, d=c(d,0))
xt<-data[,1]
et<-data[,2]
yt<-beta*xt+et
FDLS(xt,yt,m=m0)
FMNBS(xt,yt,m0=m0, m1=m1, m2=m2, m3=m3)
```

---

G.hat

*Estimation of G matrix for multivariate long memory processes.*

---

### Description

G.hat Estimates the matrix G of a multivariate long memory process based on an estimate of the vector of memory parameters. The assumed spectral density is that of Shimotsu (2007).

### Usage

G.hat(X, d, m)

**Arguments**

|   |  |
|---|--|
| X | data matrix with T observations of q-dimensional process.  |
| d | q-dimensional data vector.   |
| m | bandwith parameter specifying the number of Fourier frequencies. used for the estimation usually $\text{floor}(1+T^{\delta})$ , where $0<\delta<1$ . |

**References**

Shimotsu, K. (2007): Gaussian semiparametric estimation of multivariate fractionally integrated processes. *Journal of Econometrics*, Vol. 137, No. 2, pp. 277 - 310.

**Examples**

```
T<-500
d1<-0.4
d2<-0.2
data<-FI.sim(T, q=2, rho=0, d=c(d1,d2))
G.hat(X=data, d=c(d1,d2), m=floor(1+T^0.6))
#diagonal elements should equal 1/(2*pi)
```

gph

*GPH estimator of fractional difference parameter d.***Description**

gph log-periodogram estimator of Geweke and Porter-Hudak (1983) (GPH) and Robinson (1995a) for memory parameter d.

**Usage**

```
gph(X, m, l = 1)
```

**Arguments**

|   |  |
|---|--|
| X | vector of length T.  |
| m | bandwith parameter specifying the number of Fourier frequencies. used for the estimation usually $\text{floor}(1+T^{\delta})$ , where $0<\delta<1$ . |
| l | trimming parameter that determines with which Fourier frequency to start. Default value is l=1.  |

**Details**

add details here.

## References

- Robinson, P. M. (1995): Log-periodogram regression of time series with long range dependence. *The Annals of Statistics*, Vol. 23, No. 5, pp. 1048 - 1072.
- Geweke, J. and Porter-Hudak, S. (1983): The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4, 221-238.

## Examples

```
library(fracdiff)
T<-500
m<-floor(1+T^0.8)
d=0.4
series<-fracdiff.sim(n=T, d=d)$series
gph(X=series,m=m)
```

---

GSE

*Multivariate local Whittle estimation of long memory parameters.*

---

## Description

GSE Estimates the memory parameter of a vector valued long memory process.

## Usage

```
GSE(X, m = m, l = 1)
```

## Arguments

- |   |   |
|---|---|
| X | data matrix with T observations of q-dimensional process.   |
| m | bandwidth parameter specifying the number of Fourier frequencies used for the estimation. Usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| l | integer that specifies the number of Fourier frequencies (l-1) that are trimmed.  |

## References

- Shimotsu, K. (2007): Gaussian semiparametric estimation of multivariate fractionally integrated processes. *Journal of Econometrics*, Vol. 137, No. 2, pp. 277 - 310.

## Examples

```
T<-500
d1<-0.4
d2<-0.2
data<-FI.sim(T, q=2, rho=0.5, d=c(d1,d2))
ts.plot(data, col=1:2)
GSE(data, m=floor(1+T^0.7))
```

---

|           |  |
|-----------|--|
| GSE_coint | <i>Multivariate local Whittle estimation of long memory parameters and cointegrating vector.</i> |
|-----------|--|

---

**Description**

GSE\_coint is an extended version of GSE that allows the joint estimation of the memory parameters and the cointegration vector for a vector valued process.

**Usage**

```
GSE_coint(X, m = m, elements, l = 1)
```

**Arguments**

|          |  |
|----------|--|
| X        | data matrix with T observations of q-dimensional process.  |
| m        | bandwith parameter specifying the number of Fourier frequencies used for the estimation. Usually $\text{floor}(1+T^\delta)$ , where $0 < \delta < 1$ . |
| elements | vector specifying which elements of the observation vector are cointegrated.   |
| l        | integer that specifies the number of Fourier frequencies (l-1) that are trimmed.   |

**Examples**

```
#
# Cointegration:
#
T<-500
m<-floor(T^0.75)
series<-FI.sim(T=T,q=2,rho=0,d=c(0.1,0.4), B=rbind(c(1,-1),c(0,1)))
ts.plot(series, col=1:2)
GSE_coint(X=series,m=m, elements=c(1,2))
```

---

|            |   |
|------------|---|
| Hou.Perron | <i>Modified local Whittle estimator of fractional difference parameter d.</i> |
|------------|---|

---

**Description**

Hou.Perron Modified semiparametric local Whittle estimator of Hou and Perron (2014). Estimates memory parameter robust to low frequency contaminations.

**Usage**

```
Hou.Perron(data, m)
```

**Arguments**

|      |  |
|------|--|
| data | data vector of length T.   |
| m    | bandwidth parameter specifying the number of Fourier frequencies used for the estimation usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |

**Details**

add details here

**Author(s)**

Christian Hendrik Leschinski

**References**

Hou, J., Perron, P. (2014): Modified local Whittle estimator for long memory processes in the presence of low frequency (and other) contaminations. *Journal of Econometrics*, Vol. 182, No. 2, pp. 309 - 328.

**Examples**

```
library(fracdiff)
T<-1000
d<-0
mean<-c(rep(0,T/2),rep(2,T/2))
FI<-fracdiff.sim(n=T, d=d)$series
series<-mean+FI
ts.plot(series)
lines(mean, col=2)
local.W(series, m=floor(1+T^0.65))
Hou.Perron(series, m=floor(1+T^0.65))
```

---

11. VARFIMA

*Log-likelihood function of a VARFIMA(1,1) in final equations form.*

---

**Description**

11. VARFIMA returns the value of the log-likelihood function for a given sample and parameter vector.

**Usage**

```
11.VARFIMA(theta, data, q, approx = 100, pre.sample = matrix(0, approx,
q), rep = FALSE)
```



**Arguments**

|            |  |
|------------|--|
| theta      | parameter vector.  |
| data       | data matrix with T observations of q-dimensional process.  |
| q          | dimension of the process.  |
| approx     | order of the AR-approximation that is supposed to be used. Default is approx=100.                |
| pre.sample | if likelihood is conditioned on previous observations pre.sample is an additional sample matrix. |
| rep        | determines whether the parameter vector is printed.  |

**References**

Lutkepohl, H. (2007): New introduction to multiple time series analysis. Springer.

---

|         |  |
|---------|--|
| local.W | <i>Local Whittle estimator of fractional difference parameter d.</i> |
|---------|--|

---

**Description**

local.W Semiparametric local Whittle estimator for memory parameter d following Robinson (1995). Returns estimate and asymptotic standard error.

**Usage**

```
local.W(data, m, int = c(-0.5, 2.5), taper = c("none", "Velasco",
"HC"), diff_param = 1, l = 1)
```

**Arguments**

|            |  |
|------------|--|
| data       | vector of length T.  |
| m          | bandwith parameter specifying the number of Fourier frequencies. used for the estimation usually $\text{floor}(1+T^\delta)$ , where $0 < \delta < 1$ .   |
| int        | admissible range for d. Restricts the interval of the numerical optimization.  |
| taper      | string that is either "none", "Velasco", or "HC" and determines whether the standard local Whittle estimator of Robinson (1995), the tapered version of Velasco (1999), or the differenced and tapered estimator of Hurvich and Chen (2000) is used. |
| diff_param | integer specifying the order of differentiation for the estimator of Hurvich and Chen (2000). Default is diff_param=1.   |
| l          | integer that determines how many frequencies (l-1) are trimmed out if taper="none" is selected. Default is l=1.  |

## References

- Robinson, P. M. (1995): Gaussian Semiparametric Estimation of Long Range Dependence. The Annals of Statistics, Vol. 23, No. 5, pp. 1630 - 1661.
- Velasco, C. (1999): Gaussian Semiparametric Estimation for Non-Stationary Time Series. Journal of Time Series Analysis, Vol. 20, No. 1, pp. 87-126.
- Hurvich, C. M., and Chen, W. W. (2000): An Efficient Taper for Potentially Overdifferenced Long-Memory Time Series. Journal of Time Series Analysis, Vol. 21, No. 2, pp. 155-180.

## Examples

```
library(fracdiff)
T<-1000
d<-0.4
series<-fracdiff.sim(n=T, d=d)$series
local.W(series,m=floor(1+T^0.65))
```

---

LongMemoryTS

*LongMemoryTS: Long Memory Time Series*

---

## Description

The LongMemoryTS package is a collection of functions for estimation, simulation and testing of long memory processes, spurious long memory processes, and fractionally cointegrated systems.

## Author(s)

Christian Hendrik Leschinski <christian\_leschinski@gmx.de>, Michelle Voges, Kai Wenger

## References

- Bai, J. and Perron, P. (1998): Estimating and Testing Linear Models With Multiple Structural Changes. Econometrica, Vol. 66, No. 1, pp. 47 - 78.
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- Chen, W. W. and Hurvich, C. M. (2003): Semiparametric estimation of multivariate fractional cointegration. Journal of the American Statistical Association, Vol. 98, No. 463, pp. 629 - 642.
- Chen, W. W. and Hurvich, C. M. (2006): Semiparametric estimation of fractional cointegrating subspaces. The Annals of Statistics, Vol. 34, No. 6, pp. 2939 - 2979.
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- Davidson, J., Hashimzade, N. (2009). Type I and type II fractional Brownian motions: A reconsideration. Computational Statistics & Data Analysis, No. 53, Vol. 6, pp. 2089 - 2106.

- Frederiksen, P., Nielsen, F. S., and Nielsen, M. O. (2012): Local polynomial Whittle estimation of perturbed fractional processes. *Journal of Econometrics*, Vol. 167, No.2, pp. 426-447.
- Geweke, J. and Porter-Hudak, S. (1983): The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4, 221-238.
- Hou, J., Perron, P. (2014): Modified local Whittle estimator for long memory processes in the presence of low frequency (and other) contaminations. *Journal of Econometrics*, Vol. 182, No. 2, pp. 309 - 328.
- Hualde, J. (2013): A simple test for the equality of integration orders. *Economics Letters*, Vol. 119, No. 3, pp. 233 - 237.
- Hurvich, C. M., and Chen, W. W. (2000): An Efficient Taper for Potentially Overdifferenced Long-Memory Time Series. *Journal of Time Series Analysis*, Vol. 21, No. 2, pp. 155-180.
- Jensen, A. N. and Nielsen, M. O. (2014): A fast fractional difference algorithm. *Journal of Time Series Analysis* 35(5), pp. 428-436.
- Lavielle, M. and Moulines, E. (2000): Least Squares Estimation of an Unknown Number of Shifts in a Time Series. *Journal of Time Series Analysis*, Vol. 21, No. 1, pp. 33 - 59.
- Lutkepohl, H. (2007): *New introduction to multiple time series analysis*. Springer.
- Marinucci, D., Robinson, P. M. (1999). Alternative forms of fractional Brownian motion. *Journal of Statistical Planning and Inference*, Vol. 80 No. 1-2, pp. 111 - 122.
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- Nielsen, M. O. and Frederiksen (2011): Fully modified narrow-band least squares estimation of weak fractional cointegration. *The Econometrics Journal*, 14, pp. 77-120.
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- Robinson, P. M., (1994): Semiparametric analysis of long-memory time series. *Annals of Statistics*, 22, pp. 515-539.
- Robinson, P. M. (1995): Log-periodogram regression of time series with long range dependence. *The Annals of Statistics*, Vol. 23, No. 5, pp. 1048 - 1072.
- Robinson, P. M. (1995): Gaussian Semiparametric Estimation of Long Range Dependence. *The Annals of Statistics*, Vol. 23, No. 5, pp. 1630 - 1661.
- Robinson, P. (2008): Diagnostic testing for cointegration. *Journal of Econometrics*, Vol. 143, No. 1, pp. 206 - 225.
- Robinson, P. M. and Marinucci, D. (2003): Semiparametric frequency domain analysis of fractional cointegration. In: Robinson, P. M. (Ed.), *Time Series with Long Memory*, Oxford University Press, Oxford, pp. 334-373.

Robinson, P. M. and Yajima, Y. (2002): Determination of cointegrating rank in fractional systems. *Journal of Econometrics*, Vol. 106, No.2, pp. 217-241.

Shimotsu, K. (2007): Gaussian semiparametric estimation of multivariate fractionally integrated processes. *Journal of Econometrics*, Vol. 137, No. 2, pp. 277 - 310.

Shimotsu, K. (2010): Exact Local Whittle Estimation Of Fractional Integration with Unknown Mean and Time Trend. *Econometric Theory*, Vol. 26, pp. 501 - 540.

Shimotsu, K. and Phillips, P. C. B. (2005): Exact Local Whittle Estimation Of Fractional Integration. *The Annals of Statistics*, Vol. 33, No. 4, pp. 1890-1933.

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Wang, B., Wang, M. and Chan, N. H. (2015): Residual-based test for fractional cointegration. *Economics Letters*, Vol. 126, pp. 43 - 46.

Xu, J. and Perron, P. (2014): Forecasting return volatility: Level shifts with varying jump probability and mean reversion. *International Journal of Forecasting*, 30, pp. 449-463.

Zhang, R., Robinson, P. and Yao, Q. (2018): Identifying cointegration by eigenanalysis. *Journal of the American Statistical Association* (forthcoming).

### See Also

[ARRLS.sim](#), [ELW](#), [ELW2S](#), [F.hat](#), [FCI\\_CH03](#), [FCI\\_CH06](#), [FCI\\_MV04](#), [FCI\\_N10](#), [FCI\\_NS07](#), [FCI\\_R08](#), [FCI\\_SRFB18](#), [FCI\\_WWC15](#), [FCI\\_ZRY18](#), [FDLS](#), [FI.sim](#), [FMNBLS](#), [G.hat](#), [GSE](#), [GSE\\_coint](#), [Hou.Perron](#), [LPWN](#), [MLWS](#), [McC.Perron](#), [Peri](#), [Qu.test](#), [T.rho](#), [T0stat](#), [VARFIMA.est](#), [VARFIMA.sim](#), [W\\_multi](#), [cross.Peri](#), [ddiffw](#), [fBM](#), [fdiff](#), [gph](#), [ll.VARFIMA](#), [local.W](#), [partition.X](#), [pre.White](#), [rank.est](#)

---

LPWN

*Local polynomial Whittle plus noise estimator*

---

### Description

LPWN calculates the local polynomial Whittle plus noise estimator of Frederiksen et al. (2012).

### Usage

LPWN(data, m, R\_short = 0, R\_noise = 0)

**Arguments**

|         |  |
|---------|--|
| data    | data vector  |
| m       | bandwith parameter specifying the number of Fourier frequencies.                             |
| R_short | number of (even) polynomial terms used for approximation of spectral density at the origin.  |
| R_noise | number of (even) polynomial terms used for approximation of dependence in perturbation term. |

**Details**

add details here.

**References**

Frederiksen, P., Nielsen, F. S., and Nielsen, M. O. (2012): Local polynomial Whittle estimation of perturbed fractional processes. *Journal of Econometrics*, Vol. 167, No.2, pp. 426-447.

**Examples**

```
library(fracdiff)
T<-2000
d<-0.2
series<-fracdiff.sim(n=T, d=d, ar=0.6)$series+rnorm(T)
LPWN(series, m=floor(1+T^0.8), R_short=1, R_noise=0)
```

---

|            |  |
|------------|--|
| McC.Perron | <i>GPH estimation of long memory parameter robust to low frequency contaminations.</i> |
|------------|--|

---

**Description**

McC.Perron trimmed and adaptive log-periodogram estimators of McCloskey and Perron (2013, ET) for robust estimation of the memory parameter  $d$ .

**Usage**

```
McC.Perron(X, m, epsilon = 0.05, method = c("adaptive", "trimmed"),
  K1 = 1)
```

**Arguments**

|         |  |
|---------|--|
| X       | vector of length T.  |
| m       | bandwith parameter specifying the number of Fourier frequencies. used for the estimation usually $\text{floor}(1+T^\delta)$ , where $0 < \delta < 1$ . |
| epsilon | small constant that determines the choice of the trimming parameter $l$ used by the gph estimator. Default is $\text{epsilon}=0.05$ .                  |

method either "adaptive" or "trimmed" for the corresponding estimator. Confer McCloskey and Perron (2013, ET) for details. Default is method="adaptive".

K1 proportionality factor for bandwidth selection. Default is K1=1.

### Details

add details here. Recommendation of McCloskey, A. and Perron, P. (2013): Use trimmed version of estimator if there is reason to assume that shifts are present and use adaptive with  $\epsilon=0.05$  and  $m=T^{0.8}$  if you are agnostic about the presence of shifts.

### References

Robinson, P. M. (1995): Log-periodogram regression of time series with long range dependence. The Annals of Statistics, Vol. 23, No. 5, pp. 1048 - 1072.

McCloskey, A. and Perron, P. (2013): Memory parameter estimation in the presence of level shifts and deterministic trends. Econometric Theory, 29, pp. 1196-1237.

### Examples

```
library(fracdiff)
T<-1000
m<-floor(1+T^0.8)
d=0.4
series<-fracdiff.sim(n=T, d=d)$series
McC.Perron(series,m)
```

---

MLWS

*MLWS test for multivariate spurious long memory.*

---

### Description

Multivariate local Whittle Score type test for the null hypothesis of true long memory against the alternative of spurious long memory suggested by Sibbertsen, Leschinski and Holzhausen (2018).

### Usage

```
MLWS(X, m, epsilon = c(0.02, 0.05), coint.elements = NULL, B = NULL,
      prewhite = c("none", "uni", "multi"), eta = rep(1/sqrt(min(dim(X))),
      min(dim(X))), rep = FALSE, approx = 100, split = 1,
      T_limdist = 1000, M_limdist = 5000)
```

### Arguments

X data matrix

m bandwidth parameter specifying the number of Fourier frequencies used for the estimation usually  $\text{floor}(1+T^{\delta})$ , where  $0.5 < \delta < 0.8$  for consistency.

|                             |  |
|-----------------------------|--|
| <code>epsilon</code>        | trimming parameter <code>epsilon=0.05</code> by default. Determines minimum number of Fourier frequencies used in test statistic. For $T > 500$ it is recommended to use <code>epsilon=0.02</code> . Confer Sibbertsen, Leschinski, Holzhausen (2018) for further details.   |
| <code>coint.elements</code> | Vector specifying which elements in the vector series are in a cointegrating relationship. By default NULL. Cf details.  |
| <code>B</code>              | cointegrating matrix, if known. Default is <code>B=NULL</code> .   |
| <code>prewhite</code>       | specifies the form of pre-whitening applied. One of <code>c("none", "uni", "multi")</code> . If <code>uni</code> is selected the univariate a univariate of maximal order $(1,d,1)$ is selected using the AIC. If <code>multi</code> is selected <code>VARFIMA_est</code> is used to fit a <code>VARFIMA(1,d,1)</code> in final equations form. Default is <code>none</code> . |
| <code>eta</code>            | vector of weights. Default is <code>rep(1/sqrt(min(dim(X))), min(dim(X)))</code> .   |
| <code>rep</code>            | if <code>prewhite="multi"</code> is selected, <code>rep</code> specifies whether the current parameter values are displayed to the user during optimization procedure. Default is <code>rep=FALSE</code> .   |
| <code>approx</code>         | if <code>prewhite="multi"</code> is selected, <code>approx</code> specifies the order of the AR-approximation used in <code>VARFIMA_est</code> . Default is <code>approx=100</code> .  |
| <code>split</code>          | if <code>prewhite="multi"</code> is selected, <code>split</code> whether the sample should be split into subsamples to speed up the estimation. Default is <code>split=1</code> , so that the whole sample is used.  |
| <code>T_limdist</code>      | number of increments used in simulation if limit distribution. Only relevant for component-wise version of the test. Default is <code>T_limdist=1000</code> .  |
| <code>M_limdist</code>      | number of replications for simulation of the limit distribution. Default is <code>M_limdist=5000</code> .  |

## References

Sibbertsen, P., Leschinski, C. H., Holzhausen, M., (2018): A Multivariate Test Against Spurious Long Memory. *Journal of Econometrics*, Vol. 203, No. 1, pp. 33 - 49.

## Examples

```
T<-500
m<-floor(1+T^0.75)
series<-FI.sim(T=T,q=2,rho=0.7,d=c(0.4,0.2))
ts.plot(series, col=1:2)
MLWS(X=series, m=m, epsilon=0.05)
```

```
shift.series<-series+ARRLS.sim(T=T, phi=0, sig.shift=2, prob=5/T)
ts.plot(shift.series, col=1:2)
MLWS(X=shift.series, m=m, epsilon=0.05)
```

```
T<-500
m<-floor(T^0.75)
series<-FI.sim(T=T,q=2,rho=0,d=c(0.1,0.4), B=rbind(c(1,-1),c(0,1)))
ts.plot(series, col=1:2)
MLWS(series, m=m)
MLWS(series, m=m, coint.elements=c(1,2))
```

---

|             |  |
|-------------|--|
| partition.X | <i>Automated partitioning of estimated vector of long memory parameters into subvectors with equal memory.</i> |
|-------------|--|

---

### Description

partition.X conducts a sequence of tests for the equality of two or more estimated memory parameters to find possible partitions of a vector into subvectors with equal memory parameters. The procedure follows Robinson and Yajima (2002).

### Usage

```
partition.X(data, d.hat, m, m1, alpha = 0.05, report = FALSE)
```

### Arguments

|        |  |
|--------|--|
| data   | (Txq) data matrix  |
| d.hat  | (qx1) vector of d-estimates obtained using a local Whittle method such as that described in Robinson (1995).                                 |
| m      | the bandwidth parameter to be used for estimation of G   |
| m1     | the bandwidth parameter used for estimation of d.vec with $m1 \gg m$   |
| alpha  | the desired significance level for the tests   |
| report | either TRUE or FALSE determining, whether information about the partitioning process should be printed to the user. Default is report=FALSE. |

### Details

add a lot of details.

### References

Robinson, P. M. (1995): Gaussian semiparametric estimation of long rang dependence. The Annals of Statistics, Vol. 23, No. 5, pp. 1630-1661.

Robinson, P. M. and Yajima, Y. (2002): Determination of cointegrating rank in fractional systems. Journal of Econometrics, Vol. 106, No.2, pp. 217-241.

### See Also

[partitions](#), [T.rho](#), [T0stat](#)



**Examples**

```

library(fracdiff)
T<-1000
d1<-0.2
d2<-0.4
X<-cbind(fracdiff.sim(n=T,d=d1)$series,fracdiff.sim(n=T,d=d1)$series,
fracdiff.sim(n=T,d=d2)$series,fracdiff.sim(n=T,d=d2)$series)
alpha<-0.05
m1<-floor(1+T^0.75)
m<-floor(1+T^0.65)
d.hat<-c(local.W(X[,1],m=m1)$d,local.W(X[,2],m=m1)$d,local.W(X[,3],m=m1)$d,local.W(X[,4],m=m1)$d)
partition.X(data=X, d.hat=d.hat, m=m, m1=m1, alpha=0.05, report=TRUE)

```

---

Peri

*Multivariate Periodogram.*


---

**Description**

Peri calculates the periodogram of a multivariate time series.

**Usage**

```
Peri(X)
```

**Arguments**

X (Txq) data matrix.

**Details**

Returns an array of dimension  $c(q, q, \text{floor}(T/2))$ .

**Examples**

```

series<-FI.sim(T=1000,q=2,rho=0.7,d=c(0.4,0.4))
peri<-Peri(series)
par(mfrow=c(2,2))
for(i in 1:2){
for(j in 1:2){
plot(Re(peri[i,j,]), type="h")
lines(Im(peri[i,j,]), col=2)
}}

```

---

|           |   |
|-----------|---|
| pre.White | <i>Pre-whitening for application of semiparametric long memory estimator.</i> |
|-----------|---|

---

### Description

Given a parameter vector `theta` obtained using `VARFIMA_est`, `pre.White` returns the pre-whitened sample.

### Usage

```
pre.White(theta, data, q, approx = 100)
```

### Arguments

|                     |   |
|---------------------|---|
| <code>theta</code>  | estimated parameter vector.   |
| <code>data</code>   | data matrix with T observations of q-dimensional process.                                       |
| <code>q</code>      | dimension of the process.   |
| <code>approx</code> | order of the AR-approximation that is supposed to be used. Default is <code>approx=100</code> . |

### Details

add details here.

### References

Sibbertsen, P., Leschinski, C. H., Holzhausen, M., (2015): A Multivariate Test Against Spurious Long Memory. Hannover Economic Paper.

---

|         |   |
|---------|---|
| Qu.test | <i>Qu test for true long memory against spurious long memory.</i> |
|---------|---|

---

### Description

`Qu.test` Test statistic of Qu (2011) for the null hypothesis of true long memory against the alternative of spurious long memory.

### Usage

```
Qu.test(data, m, epsilon = 0.05)
```

**Arguments**

|         |  |
|---------|--|
| data    | data vector of length T.   |
| m       | bandwith parameter specifying the number of Fourier frequencies used for the estimation usually $\text{floor}(1+T^{\delta})$ , where $0.5 < \delta < 0.8$ for consistency.   |
| epsilon | trimming parameter $\text{epsilon}=0.05$ by default. Determines minimum number of Fourier frequencies used in test statistic. For $T > 500$ it is recommended to use $\text{epsilon}=0.02$ . Confer Qu (2011) for further details. |

**References**

Qu, Z. (2011): A Test Against Spurious Long Memory. Journal of Business and Economic Statistics, Vol. 29, No. 3, pp. 423 - 438.

**Examples**

```
library(fracdiff)
T<-500
m<-floor(1+T^0.75)
series<-fracdiff.sim(n=T,d=0.4)$series
shift.series<-ARRLS.sim(T=500,phi=0.5, sig.shift=0.75, prob=5/T, sig.noise=1)
ts.plot(series, ylim=c(min(min(series),min(shift.series)),max(max(series),max(shift.series))))
lines(shift.series, col=2)
Qu.test(series,m=m, epsilon=0.05)
Qu.test(shift.series,m=m, epsilon=0.05)
```

rank.est

*Cointegration Rank Estimation using Model Selection.***Description**

Model selection procedure to estimate the cointegrating rank based on eigenvalues of correlation matrix P suggested by Robinson and Yajima (2002).

**Usage**

```
rank.est(data, d.hat, m, m1, v_n = m^(-0.3))
```

**Arguments**

|       |   |
|-------|---|
| data  | data matrix of dimension (qxT).   |
| d.hat | the estimated d.vector  |
| m     | bandwith parameter specifying the number of Fourier frequencies. used for the estimation of d, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ .  |
| m1    | the bandwidth parameter used for estimation of d.vec with $m1 \gg m$  |
| v_n   | bandwidth parameter. Nielsen and Shimotsu (2007) use $m^{-0.3}$ in their simulation studies, which s the default value. $m^{-b}$ mit $0 < b < 0.5$ can be used. |

**Details**

add details here.

**References**

Robinson, P. M. and Yajima, Y. (2002): Determination of cointegrating rank in fractional systems. *Journal of Econometrics*, Vol. 106, No.2, pp. 217-241.

Nielsen, M. O. and Shimotsu, K. (2007): Determining the cointegrating rank in nonstationary fractional systems by the exact local Whittle approach. *Journal of Econometrics*, 141, pp. 574-596.

**Examples**

```
library(fracdiff)
T<-2000
d<-0.4
m1<-floor(1+T^0.75)
m<-floor(1+T^0.65)
xt<-fracdiff.sim(n=T, d=d)$series
yt<-xt+rnorm(T)
zt<-xt+rnorm(T)
X<-cbind(xt,yt,zt)
lW.wrap<-function(data,m){local.W(data,m)$d}
d.hat<-apply(X,2,lW.wrap, m=m1)
rank.est(data=X, d.hat, m=m, m1=m1)
```

---

T.rho

*Test for equality of all elements in an estimated d-vector based on pairwise comparisons.*

---

**Description**

T.rho Uses pairwise test as suggested by Robinson and Yajima (2002) to test for the equality of the memory parameters in a vector series.

**Usage**

```
T.rho(data, d.hat, m, m1, alpha = 0.05, s_bar = 1,
      h_n = 1/sqrt(log(max(dim(data))))))
```

**Arguments**

|       |  |
|-------|--|
| data  | data matrix of dimension (qxT).  |
| d.hat | the estimated d.vector   |
| m     | bandwith parameter specifying the number of Fourier frequencies. used for the estimation of G, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ . |
| m1    | the bandwidth parameter used for estimation of d.vec with $m1 \gg m$   |

|       |  |
|-------|--|
| alpha | the desired significance level for the tests   |
| s_bar | number of subvectors to be tested in partitioning procedure. Default is s_bar=1, for independent use.  |
| h_n   | bandwidth parameter. Default is $h_n=1/\sqrt{\log(\max(\dim(\text{data})))}$ which is recommended by Nielsen and Shimotsu (2007) in their simulation study. Robinson and Yajima (2002) argue non-rejection with $h_n=0$ would imply non-rejection with any $h_n>0$ . |

### Author(s)

Christian Leschinski

### References

Robinson, P. M. and Yajima, Y. (2002): Determination of cointegrating rank in fractional systems. *Journal of Econometrics*, Vol. 106, No.2, pp. 217-241.

Nielsen, M. O. and Shimotsu, K. (2007): Determining the cointegrating rank in nonstationary fractional systems by the exact local Whittle approach. *Journal of Econometrics*, 141, pp. 574-596.

### Examples

```
library(fracdiff)
T<-1000
d1<-0.2
d2<-0.4
X<-cbind(fracdiff.sim(n=T,d=d1)$series,fracdiff.sim(n=T,d=d1)$series,
fracdiff.sim(n=T,d=d2)$series,fracdiff.sim(n=T,d=d2)$series)
alpha<-0.05
m1<-floor(1+T^0.75)
m<-floor(1+T^0.65)
lw.wrap<-function(data,m){local.W(data,m)$d}
d.hat<-apply(X,2,lw.wrap, m=m1)
T.rho(data=X, d.hat=d.hat, m=m, m1=m1)
```

---

T0stat

*Test for equality of all elements in an estimated d-vector based.*

---

### Description

T0stat tests equality of all memory parameters simultaneously. Statistic was suggested by Robinson and Yajima (2002). Test statistic was denoted by  $T_0$  in Nielsen and Shimotsu (2007).

### Usage

```
T0stat(data, d.hat, m, m1, alpha = 0.05, s_bar = 1,
h_n = 1/sqrt(log(max(dim(data)))))
```

**Arguments**

|       |   |
|-------|---|
| data  | data matrix of dimension (qxT).   |
| d.hat | the estimated d.vector  |
| m     | bandwith parameter specifying the number of Fourier frequencies. used for the estimation of d, usually $\text{floor}(1+T^{\delta})$ , where $0 < \delta < 1$ .  |
| m1    | the bandwidth parameter used for estimation of d.vec with $m1 \gg m$  |
| alpha | the desired significance level for the tests  |
| s_bar | number of subvectors to be tested in partitioning procedure. Default is $s\_bar=1$ , for independent use.   |
| h_n   | bandwidth parameter. Default is $h\_n=1/\sqrt{\log(\max(\text{dim}(\text{data})))}$ which is recommended by Nielsen and Shimotsu (2007) in their simulation study. Robinson and Yajima (2002) argue non-rejection with $h\_n=0$ would imply non-rejection with any $h\_n > 0$ . |

**Details**

add details here.

**References**

Robinson, P. M. and Yajima, Y. (2002): Determination of cointegrating rank in fractional systems. *Journal of Econometrics*, Vol. 106, No.2, pp. 217-241.

Nielsen, M. O. and Shimotsu, K. (2007): Determining the coinegrating rank in nonstationary fractional systems by the exact local Whittle approach. *Journal of Econometrics*, 141, pp. 574-596.

**Examples**

```
library(fracdiff)
T<-1000
d1<-0.2
d2<-0.4
X<-cbind(fracdiff.sim(n=T,d=d1)$series,fracdiff.sim(n=T,d=d1)$series,
fracdiff.sim(n=T,d=d2)$series,fracdiff.sim(n=T,d=d2)$series)
alpha<-0.05
m1<-floor(1+T^0.75)
m<-floor(1+T^0.65)
lw.wrap<-function(data,m){local.W(data,m)$d}
d.hat<-apply(X,2,lw.wrap, m=m1)
T0stat(data=X, d.hat=d.hat, m=m, m1=m1)
```

---

|             |   |
|-------------|---|
| VARFIMA.est | <i>Maximum likelihood estimation of a VARFIMA(1,1) in final equations form.</i> |
|-------------|---|

---

### Description

VARFIMA.est returns the maximum likelihood estimate of the parameter vector of a VARFIMA(1,1) in final equations form.

### Usage

```
VARFIMA.est(data, approx = 100, split = 1, rep = FALSE)
```

### Arguments

|        |  |
|--------|--|
| data   | data matrix with T observations of q-dimensional process.  |
| approx | order of the AR-approximation that is supposed to be used. Default is approx=100.  |
| split  | to increase the speed the sample can be divided in split parts. Parameter estimation is then carried out separately for each subsample and results are averaged across the subsamples. |
| rep    | is passed to ll_VARFIMA and determines whether the current parameter vector is printed to the user in every iteration of the numerical maximization procedure.                         |

### Details

add details here.

### References

Lutkepohl, H. (2007): New introduction to multiple time series analysis. Springer.

### Examples

```
series<-VARFIMA.sim(phi=0.4, THETA=matrix(c(0,0,0,0),2,2),
d.vec=c(0.4,0.3), T=1000, Sigma=matrix(c(1,0.4,0.4,1),2,2))
ts.plot(series, col=1:2)
acf(series, lag=100)
VARFIMA.est(series, approx=100, rep=FALSE)
```

---

`VARFIMA.sim`*Simulation of a VARFIMA(1,1) in final equations form.*

---

### Description

`VARFIMA.sim` returns a sample from a VARFIMA(1,1)-process.

### Usage

```
VARFIMA.sim(phi, THETA, d.vec, T, Sigma, approx = 100, burnin = 100)
```

### Arguments

|                     |   |
|---------------------|---|
| <code>phi</code>    | AR(1)-parameter.  |
| <code>THETA</code>  | MA(1)-matrix.   |
| <code>d.vec</code>  | vector of memory parameters.  |
| <code>T</code>      | desired sample size.  |
| <code>Sigma</code>  | Variance-Covariance-Matrix of the innovations.  |
| <code>approx</code> | order of the AR-approximation that is supposed to be used. Default is <code>approx=100</code> . |
| <code>burnin</code> | length of the burnin period that is discarded. Default is <code>burnin=100</code> .             |

### Details

add details here.

### References

Lutkepohl, H. (2007): New introduction to multiple time series analysis. Springer.

### Examples

```
series<-VARFIMA.sim(phi=0.4, THETA=matrix(c(0,0,0,0),2,2),  
d.vec=c(0.4,0.3), T=1000, Sigma=matrix(c(1,0.4,0.4,1),2,2))  
ts.plot(series, col=1:2)  
acf(series, lag=100)
```



---

`W_multi`*Helper function for MLWS test for multivariate spurious long memory.*

---

**Description**

Multivariate local Whittle Score type test for the null hypothesis of true long memory against the alternative of spurious long memory suggested by Sibbertsen, Leschinski and Holzhausen (2015).

**Usage**

```
W_multi(X, d_vec, m, epsilon, eta)
```

**Arguments**

|                      |  |
|----------------------|--|
| <code>X</code>       | data matrix  |
| <code>d_vec</code>   | estimated vector of memory parameters.   |
| <code>m</code>       | bandwidth parameter specifying the number of Fourier frequencies used for the estimation usually $\text{floor}(1+T^\delta)$ , where $0.5 < \delta < 0.8$ for consistency.  |
| <code>epsilon</code> | trimming parameter $\text{epsilon}=0.05$ by default. Determines minimum number of Fourier frequencies used in test statistic. For $T > 500$ it is recommended to use $\text{epsilon}=0.02$ . Confer Sibbertsen, Leschinski, Holzhausen (2015) for further details. |
| <code>eta</code>     | weight vector.   |

**Details**

add details here

**References**

Sibbertsen, P., Leschinski, C. H., Holzhausen, M., (2015): A Multivariate Test Against Spurious Long Memory. Hannover Economic Paper.

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