

# Package: KbMvtSkew (via r-universe)

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**Type** Package

**Title** Khattree-Bahuguna's Univariate and Multivariate Skewness

**Version** 1.1.0

**Maintainer** Zhixin Lun <zhixin.lun@cuanschutz.edu>

**Description** Computes Khattree-Bahuguna's univariate and multivariate skewness, principal-component-based Khattree-Bahuguna's multivariate skewness. It also provides several measures of univariate or multivariate skewnesses including, Pearson's coefficient of skewness, Bowley's univariate skewness and Mardia's multivariate skewness. See Khattree, R. and Bahuguna, M. (2019) <[doi:10.1007/s41060-018-0106-1](https://doi.org/10.1007/s41060-018-0106-1)>.

**License** GPL (>= 3)

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**Imports** stats

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**Author** Zhixin Lun [aut, cre] (ORCID:  
<<https://orcid.org/0000-0002-8980-1554>>), Ravindra Khattree  
[aut] (ORCID: <<https://orcid.org/0000-0002-9305-2365>>)

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|            |                                     |
|------------|-------------------------------------|
| BowleySkew | <i>Bowley’s Univariate Skewness</i> |
|------------|-------------------------------------|

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**Description**

Compute Bowley’s Univariate Skewness.

**Usage**

BowleySkew(x)

**Arguments**

x                      a vector of original observations.

**Details**

Bowley’s skewness is defined in terms of quantiles as

$$\hat{\gamma} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

where  $Q_i$  is the  $i$ th quartile  $i = 1, 2, 3$  of the data.

**Value**

BowleySkew gives the Bowley’s univariate skewness of the data.

**References**

Bowley, A. L. (1920). *Elements of Statistics*. London : P.S. King & Son, Ltd.

**Examples**

```
# Compute Bowley's univariate skewness

set.seed(2019)
x <- rnorm(1000) # Normal Distribution
BowleySkew(x)

set.seed(2019)
y <- rlnorm(1000, meanlog = 1, sdlog = 0.25) # Log-normal Distribution
BowleySkew(y)
```

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data-Chatterjee*Chatterjee, Hadi and Price Data*

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**Description**

Chatterjee, Hadi and Price Data

**Usage**

```
data(Chatterjee)
```

**Format**

The format is a dataframe of 40 observations and 7 variables.

**Source**

The data come from Chatterjee, Hadi and Price (2000).

**References**

Chatterjee, S., Hadi, A. S., and Price, B. (2000). *Regression Analysis by Example*. Hoboken: Wiley.

**Examples**

```
data(Chatterjee)
```

---

data-OlymWomen

1984 Los Angeles Olympic records data of track events for women

---

### Description

Data are time records from 1984 Olympic track events for women from 55 countries: 100-meter, 200-meter, 400-meter, 800-meter, 1500-meter, 3000-meter, and Marathon. The corresponding variables are named as m100, m200, m400, m800, m1500, m3000, and marathon. The time measurements are recorded in seconds.

### Usage

```
data(OlymWomen)
```

### Format

The format is a dataframe of 55 observations and 8 variables.

### Source

The data come from Khatree and Naik (2000, pp. 511-512).

### References

Khatree, R. and Naik, D. (2000). *Multivariate Data Reduction and Discrimination with SAS® Software*. Cary, NC: SAS Institute Inc.

### Examples

```
data(OlymWomen)
```

---

data-SwissHead

*Measurements of Heads of Swiss Soldiers*


---

### Description

Data are measurements, in millimeters, of the heads of 200 Swiss soldiers.

### Usage

```
data(SwissHead)
```

### Format

The format is a dataframe of 200 observations and 6 variables.

## Source

The data come from Flurry and Riedwyl (1988).

## References

Flurry, B. and Riedwyl, H. (1988). *Multivariate Statistics: A Practical Approach*. London: Chapman and Hall.

## Examples

```
data(SwissHead)
```

---

kbMvtSkew

*Khattree-Bahuguna's Multivariate Skewness*


---

## Description

Compute Khattree-Bahuguna's Multivariate Skewness.

## Usage

```
kbMvtSkew(x)
```

## Arguments

`x` a matrix of original observations.

## Details

Let  $\mathbf{X} = (X_1, \dots, X_p)'$  be the multivariate random vector and  $(X_{i_1}, X_{i_2}, \dots, X_{i_p})'$  be one of the  $p!$  permutations of  $(X_1, \dots, X_p)'$ . We predict  $X_{i_j}$  conditionally on subvector  $(X_{i_1}, \dots, X_{i_{j-1}})$  and compute the corresponding residual  $V_{i_j}$  through a linear regression model for  $j = 2, \dots, p$ . For  $j = 1$ , we define  $V_{i_1} = X_{i_1} - \bar{X}_{i_1}$ , where  $\bar{X}_{i_1}$  is the mean of  $X_{i_1}$ . For  $j \geq 2$ , we have

$$\begin{aligned} \hat{X}_{i_2} &= \hat{\beta}_0 + \hat{\beta}_1 X_{i_1}, & V_{i_2} &= X_{i_2} - \hat{X}_{i_2} \\ \hat{X}_{i_3} &= \hat{\beta}_0 + \hat{\beta}_1 X_{i_1} + \hat{\beta}_2 X_{i_2}, & V_{i_3} &= X_{i_3} - \hat{X}_{i_3} \\ &\vdots \\ \hat{X}_{i_p} &= \hat{\beta}_0 + \hat{\beta}_1 X_{i_1} + \hat{\beta}_2 X_{i_2} + \dots + \hat{\beta}_{p-1} X_{i_{p-1}}, & V_{i_p} &= X_{i_p} - \hat{X}_{i_p}. \end{aligned}$$

We calculate the sample skewness  $\hat{\delta}_{i_j}$  of  $V_{i_j}$  by the sample Khattree-Bahuguna's univariate skewness formula (see details of [kbSkew](#) that follows) respectively for  $j = 1, \dots, p$  and define  $\hat{\Delta}_i = \sum_{j=1}^p \hat{\delta}_{i_j}$ ,  $i = 1, 2, \dots, P$  for all  $P = p!$  permutations of  $(X_1, \dots, X_p)'$ . The sample Khattree-Bahuguna's multivariate skewness is defined as

$$\hat{\Delta} = \frac{1}{P} \sum_{i=1}^P \hat{\Delta}_i.$$

Clearly,  $0 \leq \hat{\Delta} \leq \frac{p}{2}$ .

**Value**

kbMvtSkew computes the Khattree-Bahuguna's multivairate skewness for a  $p$ -dimensional data.

**References**

Khattree, R. and Bahuguna, M. (2019). An alternative data analytic approach to measure the univariate and multivariate skewness. *International Journal of Data Science and Analytics*, Vol. 7, No. 1, 1-16.

Lun, Z. and Khattree, R. (2025). A Comment on a Skewness Coefficient. *Mathematical Methods of Statistics*, Vol. 34, No. 3, 301-304.

**See Also**

[kbSkew](#) for Khattree-Bahuguna's univariate skewness.

**Examples**

```
# Compute Khattree-Bahuguna's multivairate skewness

data(OlymWomen)
kbMvtSkew(OlymWomen[, c("m800", "m1500", "m3000", "marathon")])
```

---

kbSkew

*Khattree-Bahuguna's Univariate Skewness*


---

**Description**

Compute Khattree-Bahuguna's Univariate Skewness.

**Usage**

```
kbSkew(x)
```

**Arguments**

x a vector of original observations.

**Details**

Given a univariate random sample of size  $n$  consist of observations  $x_1, x_2, \dots, x_n$ , let  $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$  be the order statistics of  $x_1, x_2, \dots, x_n$  after being centered by their mean. Define

$$y_i = \frac{x_{(i)} + x_{(n-i+1)}}{2}$$

and

$$w_i = \frac{x_{(i)} - x_{(n-i+1)}}{2}$$

The sample Khattree-Bahuguna's univariate skewness is defined as

$$\hat{\delta} = \frac{\sum y_i^2}{\sum y_i^2 + \sum w_i^2}.$$

It can be shown that  $0 \leq \hat{\delta} \leq \frac{1}{2}$ . Values close to zero indicate, low skewness while those close to  $\frac{1}{2}$  indicate the presence of high degree of skewness.

### Value

kbSkew gives the Khattree-Bahuguna's univariate skewness of the data.

### References

Khattree, R. and Bahuguna, M. (2019). An alternative data analytic approach to measure the univariate and multivariate skewness. *International Journal of Data Science and Analytics*, Vol. 7, No. 1, 1-16.

Lun, Z. and Khattree, R. (2025). A Comment on a Skewness Coefficient. *Mathematical Methods of Statistics*, Vol. 34, No. 3, 301-304.

### Examples

```
# Compute Khattree-Bahuguna's univariate skewness

set.seed(2019)
x <- rnorm(1000) # Normal Distribution
kbSkew(x)

set.seed(2019)
y <- rlnorm(1000, meanlog = 1, sdlog = 0.25) # Log-normal Distribution
kbSkew(y)
```

---

MardiaMvtSkew

*Mardia's Multivariate Skewness*


---

### Description

Compute Mardia's Multivariate Skewness.

### Usage

```
MardiaMvtSkew(x)
```

### Arguments

x                      a matrix of original observations.

## Details

Given a  $p$ -dimensional multivariate random vector with mean vector  $\boldsymbol{\mu}$  and positive definite variance-covariance matrix  $\boldsymbol{\Sigma}$ , Mardia's multivariate skewness is defined as

$$\beta_{1,p} = E[(\mathbf{X}_1 - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathbf{X}_2 - \boldsymbol{\mu})]^3,$$

where  $\mathbf{X}_1$  and  $\mathbf{X}_2$  are independently and identically distributed copies of  $\mathbf{X}$ . For a multivariate random sample of size  $n$ ,  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ , its sample version is defined as

$$\hat{\beta}_{1,p} = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n [(\mathbf{x}_i - \bar{\mathbf{x}})' \mathbf{S}^{-1} (\mathbf{x}_j - \bar{\mathbf{x}})]^3,$$

where the sample mean  $\bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^n \mathbf{x}_i$  and the sample variance-covariance matrix  $\mathbf{S} = \frac{1}{n} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})'$ . It is assumed that  $n \geq p$ .

## Value

MardiaMvtSkew gives the sample Mardia's multivariate skewness.

## References

Mardia, K.V. (1970). Measures of multivariate skewness and kurtosis with applications. *Biometrika*, 57(3), 519–530.

## Examples

```
# Compute Mardia's multivariate skewness

data(OlymWomen)
MardiaMvtSkew(OlymWomen[, c("m800", "m1500", "m3000", "marathon")])
```

---

pcKbSkew

*Principal-component-based Khattree-Bahuguna's Multivariate Skewness*

---

## Description

Compute Principal-component-based Khattree-Bahuguna's Multivariate Skewness.

## Usage

```
pcKbSkew(x, cor = FALSE)
```

## Arguments

|     |                                                                                                                                                                             |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x   | a matrix of original scale observations.                                                                                                                                    |
| cor | a logical value indicating whether the calculation should use the correlation matrix (cor = TRUE) or the covariance matrix (cor = FALSE). The default value is cor = FALSE. |

### Details

Let  $\mathbf{X} = X_1, \dots, X_p$  be a  $p$ -dimensional multivariate random vector. We compute the sample skewness for  $p$  principal components of  $\mathbf{X}$  respectively by the sample Khattree-Bahuguna's univariate skewness formula (see details of [kbSkew](#) that follows). Let  $\eta_1, \eta_2, \dots, \eta_p$  be the  $p$  univariate skewnesses for  $p$  principal components. Principal-component-based Khattree-Bahuguna's multivariate skewness for a sample is then defined as

$$\eta = \sum_{i=1}^p \eta_i.$$

Clearly,  $0 \leq \eta \leq \frac{p}{2}$ .

### Value

`pcKbSkew` gives the sample principal-component-based Khattree-Bahuguna's multivariate skewness.

### References

Khattree, R. and Bahuguna, M. (2019). An alternative data analytic approach to measure the univariate and multivariate skewness. *International Journal of Data Science and Analytics*, Vol. 7, No. 1, 1-16.

### See Also

[kbSkew](#) for Khattree-Bahuguna's univariate skewness.

### Examples

```
# Compute principal-component-based Khattree-Bahuguna's multivariate skewness

data(OlymWomen)
pcKbSkew(OlymWomen[, c("m800", "m1500", "m3000", "marathon")])
```

---

PearsonSkew

*Pearson's coefficient of skewness*

---

### Description

Compute Pearson's coefficient of skewness.

### Usage

```
PearsonSkew(x)
```

### Arguments

`x` a vector of original observations.

### Details

Pearson's coefficient of skewness is defined as

$$\gamma_1 = \frac{E[(X - \mu)^3]}{(\sigma^3)}$$

where  $\mu = E(X)$  and  $\sigma^2 = E[(X - \mu)^2]$ . The sample version based on a random sample  $x_1, x_2, \dots, x_n$  is defined as

$$\hat{\gamma}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})^3}{ns^3}$$

where  $\bar{x}$  is the sample mean and  $s$  is the sample standard deviation of the data, respectively.

### Value

PearsonSkew gives the sample Pearson's univariate skewness.

### References

Pearson, K. (1894). Contributions to the mathematical theory of evolution. *Philos. Trans. R. Soc. Lond.* A 185, 71-110.

Pearson, K. (1895). Contributions to the mathematical theory of evolution II: skew variation in homogeneous material. *Philos. Trans. R. Soc. Lond.* A 86, 343-414.

### Examples

```
# Compute Pearson's univariate skewness

set.seed(2019)
x <- rnorm(1000) # Normal Distribution
PearsonSkew(x)

set.seed(2019)
y <- rlnorm(1000, meanlog = 1, sdlog = 0.25) # Log-normal Distribution
PearsonSkew(y)
```

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