

# Package: HDRFA (via r-universe)

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**Type** Package

**Title** High-Dimensional Robust Factor Analysis

**Version** 0.1.5

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**Description** Factor models have been widely applied in areas such as economics and finance, and the well-known heavy-tailedness of macroeconomic/financial data should be taken into account when conducting factor analysis. We propose two algorithms to do robust factor analysis by considering the Huber loss. One is based on minimizing the Huber loss of the idiosyncratic error's L2 norm, which turns out to do Principal Component Analysis (PCA) on the weighted sample covariance matrix and thereby named as Huber PCA. The other one is based on minimizing the element-wise Huber loss, which can be solved by an iterative Huber regression algorithm. In this package we also provide the code for traditional PCA, the Robust Two Step (RTS) method by He et al. (2022) and the Quantile Factor Analysis (QFA) method by Chen et al. (2021) and He et al. (2023).

**License** GPL-2 | GPL-3

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|      |   |
|------|---|
| HPCA | <i>Huber Principal Component Analysis for Large-Dimensional Factor Models</i> |
|------|---|

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### Description

This function is to fit the factor models via the Huber Principal Component Analysis (HPCA) method. One is based on minimizing the Huber loss of the idiosyncratic error's  $\ell_2$  norm, which turns out to do Principal Component Analysis (PCA) on the weighted sample covariance matrix and thereby named as Huber PCA. The other one is based on minimizing the elementwise Huber loss, which can be solved by an iterative Huber regression algorithm.

### Usage

```
HPCA(X, r, Method = "E", tau = NULL, scale_est="MAD", L_init = NULL,
      F_init = NULL, maxiter_HPCA = 100, maxiter_HLM = 100, eps = 0.001)
```

### Arguments

|                           |   |
|---------------------------|---|
| <code>X</code>            | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ .  |
| <code>r</code>            | A positive integer indicating the factor numbers.   |
| <code>Method</code>       | <code>Method="P"</code> indicates minimizing the Huber loss of the idiosyncratic error's $\ell_2$ norm while <code>Method="E"</code> indicates minimizing the elementwise Huber loss. The default is the elementwise Huber loss.                            |
| <code>tau</code>          | Optional user-supplied parameter for Huber loss; default is NULL, and $\tau$ is provided by default.  |
| <code>scale_est</code>    | A parameter for the elementwise Huber loss. <code>scale_est="MAD"</code> indicates robust variance estimation in each iteration, while <code>scale_est="const"</code> indicates fixing user-supplied $\tau$ . The default is <code>scale_est="MAD"</code> . |
| <code>L_init</code>       | User-supplied initial value of loadings; default is the PCA estimator.  |
| <code>F_init</code>       | User-supplied initial value of factors; default is the PCA estimator.   |
| <code>maxiter_HPCA</code> | The maximum number of iterations in the HPCA. The default is 100.   |
| <code>maxiter_HLM</code>  | The maximum number of iterations in the iterative Huber regression algorithm. The default is 100.   |
| <code>eps</code>          | The stopping criterion parameter in the HPCA. The default is 1e-3.  |

**Details**

See He et al. (2023) for details.

**Value**

The return value is a list. In this list, it contains the following:

|      |  |
|------|--|
| Fhat | The estimated factor matrix of dimension $T \times r$ .  |
| Lhat | The estimated loading matrix of dimension $N \times r$ . |
| m    | The number of iterations.                                |

**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

He Y, Li L, Liu D, Zhou W., 2023 Huber Principal Component Analysis for Large-dimensional Factor Models.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

fit=HPCA(X,r,Method = "E")
fit$Fhat;fit$Lhat

fit=HPCA(X,r,Method = "P")
fit$Fhat;fit$Lhat
```

---

HPCA\_FN

*Estimating Factor Numbers via Rank Minimization Corresponding to Huber PCA*

---

**Description**

This function is to estimate factor numbers via rank minimization corresponding to Huber Principal Component Analysis (HPCA).

**Usage**

```
HPCA_FN(X, rmax, Method = "E", tau = NULL, scale_est="MAD", threshold = NULL,
        L_init = NULL, F_init = NULL, maxiter_HPCA = 100, maxiter_HLM = 100,
        eps = 0.001)
```

**Arguments**

|                           |   |
|---------------------------|---|
| <code>X</code>            | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ .  |
| <code>rmax</code>         | The user-supplied maximum factor numbers.   |
| <code>Method</code>       | <code>Method="P"</code> indicates minimizing the Huber loss of the idiosyncratic error's $\ell_2$ norm while <code>Method="E"</code> indicates minimizing the elementwise Huber loss. The default is the elementwise Huber loss.                            |
| <code>tau</code>          | Optional user-supplied parameter for Huber loss; default is NULL, and $\tau$ is provided by default.  |
| <code>scale_est</code>    | A parameter for the elementwise Huber loss. <code>scale_est="MAD"</code> indicates robust variance estimation in each iteration, while <code>scale_est="const"</code> indicates fixing user-supplied $\tau$ . The default is <code>scale_est="MAD"</code> . |
| <code>threshold</code>    | The threshold of rank minimization; default is NULL.  |
| <code>L_init</code>       | User-supplied initial value of loadings in the HPCA; default is the PCA estimator.  |
| <code>F_init</code>       | User-supplied initial value of factors in the HPCA; default is the PCA estimator.   |
| <code>maxiter_HPCA</code> | The maximum number of iterations in the HPCA. The default is 100.   |
| <code>maxiter_HLM</code>  | The maximum number of iterations in the iterative Huber regression algorithm. The default is 100.   |
| <code>eps</code>          | The stopping criterion parameter in the HPCA. The default is $1e-3$ .   |

**Details**

See He et al. (2023) for details.

**Value**

`rhat` The estimated factor number.

**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

He Y, Li L, Liu D, Zhou W., 2023 Huber Principal Component Analysis for Large-dimensional Factor Models.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E
```

```
HPCA_FN(X,8,Method="E")
```

```
HPCA_FN(X,8,Method="P")
```

---

IQR

---

*Iterative Quantile Regression Methods for Quantile Factor Models*


---

### Description

This function is to fit the quantile factor model via the Iterative Quantile Regression (IQR) algorithm.

### Usage

```
IQR(X, r, tau, L_init = NULL, F_init = NULL, max_iter = 100, eps = 0.001)
```

### Arguments

|          |  |
|----------|--|
| X        | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ . |
| r        | A positive integer indicating the factor numbers.  |
| tau      | The user-supplied quantile level.  |
| L_init   | User-supplied initial value of loadings; default is the PCA estimator.                                     |
| F_init   | User-supplied initial value of factors; default is the PCA estimator.                                      |
| max_iter | The maximum number of iterations. The default is 100.  |
| eps      | The stopping criterion parameter. The default is $1e-06$ .   |

### Details

See Chen et al. (2021) and He et al. (2023) for details.

### Value

The return value is a list. In this list, it contains the following:

|      |  |
|------|--|
| Fhat | The estimated factor matrix of dimension $T \times r$ .  |
| Lhat | The estimated loading matrix of dimension $N \times r$ . |
| t    | The number of iterations.                                |

### Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

### References

Chen, L., Dolado, J.J., Gonzalo, J., 2021. Quantile factor models. *Econometrica* 89, 875–910.  
 He Y, Kong X, Yu L, Zhao P., 2023 Quantile factor analysis for large-dimensional time series with statistical guarantee <arXiv:2006.08214>.

**Examples**

```

set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

tau=0.5
fit=IQR(X,r,tau)
fit$Fhat;fit$Lhat

```

---

|        |   |
|--------|---|
| IQR_FN | <i>Estimating Factor Numbers via Rank Minimization Corresponding to IQR</i> |
|--------|---|

---

**Description**

This function is to estimate factor numbers via rank minimization corresponding to Iterative Quantile Regression (IQR).

**Usage**

```

IQR_FN(X, rmax, tau, threshold = NULL, L_init = NULL, F_init = NULL,
       max_iter = 100, eps = 10-6)

```

**Arguments**

|           |  |
|-----------|--|
| X         | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ . |
| rmax      | The user-supplied maximum factor numbers.  |
| tau       | The user-supplied quantile level.  |
| threshold | The threshold of rank minimization; default is NULL.   |
| L_init    | User-supplied initial value of loadings in the IQR; default is the PCA estimator.                          |
| F_init    | User-supplied initial value of factors in the IQR; default is the PCA estimator.                           |
| max_iter  | The maximum number of iterations. The default is 100.  |
| eps       | The stopping criterion parameter of the IQR method. The default is 1e-06.                                  |

**Details**

See Chen et al. (2021) for more details.

**Value**

|      |                              |
|------|------------------------------|
| rhat | The estimated factor number. |
|------|------------------------------|

**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

Chen, L., Dolado, J.J., Gonzalo, J., 2021. Quantile factor models. *Econometrica* 89, 875–910.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

tau=0.5
IQR_FN(X,8,tau)
```

---

 PCA

---

*Principal Component Analysis for Large-Dimensional Factor Models*


---

**Description**

This function is to fit the factor models via Principal Component Analysis (PCA) methods.

**Usage**

```
PCA(X, r, constraint = "L")
```

**Arguments**

|                         |  |
|-------------------------|--|
| <code>X</code>          | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ .   |
| <code>r</code>          | A positive integer indicating the factor numbers.  |
| <code>constraint</code> | The type of identification condition. If <code>constraint="L"</code> , the columns of the estimated loading matrix are orthogonal and <code>constraint="F"</code> indicates the columns of the estimated factor matrix are orthogonal. |

**Details**

See Bai (2003) for details.

**Value**

The return value is a list. In this list, it contains the following:

|                   |  |
|-------------------|--|
| <code>Fhat</code> | The estimated factor matrix of dimension $T \times r$ .  |
| <code>Lhat</code> | The estimated loading matrix of dimension $N \times r$ . |

**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

Bai, J., 2003. Inferential theory for factor models of large dimensions. *Econometrica* 71, 135–171.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

fit=PCA(X,3,"L")
t(fit$Lhat)%*%fit$Lhat/N

fit=PCA(X,3,"F")
t(fit$Fhat)%*%fit$Fhat/T
```

---

|        |   |
|--------|---|
| PCA_FN | <i>Estimating Factor Numbers via Eigenvalue Ratios Corresponding to PCA</i> |
|--------|---|

---

**Description**

This function is to estimate factor numbers via eigenvalue ratios corresponding to Principal Component Analysis (PCA).

**Usage**

```
PCA_FN(X, rmax)
```

**Arguments**

|      |  |
|------|--|
| X    | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ . |
| rmax | The user-supplied maximum factor numbers.  |

**Details**

See Ahn and Horenstein (2013) for details.

**Value**

|      |                               |
|------|-------------------------------|
| rhat | The estimated factor numbers. |
|------|-------------------------------|



**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

Ahn, S.C., Horenstein, A.R., 2013. Eigenvalue ratio test for the number of factors. *Econometrica* 81, 1203–1227.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

PCA_FN(X,8)
```

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|     |   |
|-----|---|
| RTS | <i>Robust Two Step Algorithm for Large-Dimensional Elliptical Factor Models</i> |
|-----|---|

---

**Description**

This function is to fit the large-dimensional elliptical factor models via the Robust Two Step (RTS) algorithm.

**Usage**

```
RTS(X, r)
```

**Arguments**

|   |  |
|---|--|
| X | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ . |
| r | A positive integer indicating the factor numbers.  |

**Details**

See He et al. (2022) for details.

**Value**

The return value is a list. In this list, it contains the following:

|      |  |
|------|--|
| Fhat | The estimated factor matrix of dimension $T \times r$ .  |
| Lhat | The estimated loading matrix of dimension $N \times r$ . |

**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

He, Y., Kong, X., Yu, L., Zhang, X., 2022. Large-dimensional factor analysis without moment constraints. *Journal of Business & Economic Statistics* 40, 302–312.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F*%t(L)+E

fit=RTS(X,3)
fit$Fhat;fit$Lhat
```

---

RTS\_FN

*Estimating Factor Numbers Robustly via Multivariate Kendall's Tau Eigenvalue Ratios*

---

**Description**

This function is to estimate factor numbers robustly via multivariate Kendall's tau eigenvalue ratios.

**Usage**

```
RTS_FN(X, rmax)
```

**Arguments**

|      |  |
|------|--|
| X    | Input matrix, of dimension $T \times N$ . Each row is an observation with $N$ features at time point $t$ . |
| rmax | The user-supplied maximum factor numbers.  |

**Details**

See Yu et al. (2019) for details.

**Value**

|      |                              |
|------|------------------------------|
| rhat | The estimated factor number. |
|------|------------------------------|

**Author(s)**

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

**References**

Yu, L., He, Y., Zhang, X., 2019. Robust factor number specification for large-dimensional elliptical factor model. *Journal of Multivariate analysis* 174, 104543.

**Examples**

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

RTS_FN(X,8)
```

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