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Description Level-wise statistical methods for fuzzy random variables, implementing the Puri-Ralescu framework. Provides level-wise empirical distribution functions, moments, covariance and correlation, a level-wise Kolmogorov-Smirnov goodness-of-fit test, an approximate fuzzy normal distribution with simulation, and level-wise central limit theorem and law of large numbers inference. The package only depends on base R.

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`alpha_levels` *Alpha grid of a fuzzy object*

Description

Alpha grid of a fuzzy object

Usage

```
alpha_levels(x)
```

Arguments

`x`	A `FuzzyNumber` or `FuzzySample`.

Value

Numeric vector of alpha levels.

Examples

```
alpha_levels(triangular_fuzzy(1, 2, 4))[1:3]
```

`as_fuzzy_sample` *Coerce to a FuzzySample*

Description

Coerces from a list of `FuzzyNumber`, an array, a data frame (with columns `id`, `alpha`, `lower`, `upper`) or a single `FuzzyNumber` (giving a sample of size 1).

Usage

```
as_fuzzy_sample(x, alpha = NULL, ...)

## Default S3 method:
as_fuzzy_sample(x, alpha = NULL, ...)

## S3 method for class 'FuzzyNumber'
as_fuzzy_sample(x, alpha = NULL, ...)

## S3 method for class 'data.frame'
as_fuzzy_sample(x, alpha = NULL, ...)
```

Arguments

`x`	Object to coerce.
`alpha`	Alpha grid (for array/data-frame input).
`...`	Unused.

Value

A `FuzzySample`.

Examples

```
fs <- as_fuzzy_sample(triangular_fuzzy(0, 1, 2))
n_obs(fs)
```

clt — *Central limit theorem for a fuzzy random sample*

Description

Uses the level-wise central limit theorem (Puri & Ralescu, 1986): at each level `alpha` the sample-mean endpoints are approximately normal, so the fuzzy sample mean `Xbar` is approximately a `fuzzy_normal` with mean `expectation(x)` and level- wise covariance `variance(x) / n`.

Usage

```
clt(x, alpha = NULL, ...)
```

Arguments

`x`	A `FuzzySample`.
`alpha`	Numeric levels (default: those of `x`).
`...`	Unused.

Value

An object of class `c("FuzzyCLT","FuzzyNormal")`. Its `cov` is the covariance of `Xbar` (i.e. `Sigma / n`); `sigma` is the limiting covariance `Sigma`.

Examples

```
set.seed(7)
s <- rnorm_fuzzy(500, triangular_fuzzy(0, 1, 2), diag(2) * 0.25)
cl <- clt(s); summary(cl)
simulate(cl, nsim = 4)
```

correlation — *Correlation of two fuzzy random variables (sample)*

Description

Level-wise Pearson correlation between the lower endpoints (`cor(X^-(alpha), Y^-(alpha))`) and between the upper endpoints (`cor(X^+(alpha), Y^+(alpha))`), plus the scalar reduction

$$\rho_{FHS}(X,Y) = \tfrac{1}{2}\int_0^1 \left[r_{--}(a) + r_{++}(a)\right] da.$$

Usage

```
correlation(x, y = NULL, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
correlation(x, y = NULL, alpha = NULL, ...)
```

Arguments

x, y	Two FuzzySample objects.
alpha	Optional alpha grid.
...	Unused.

Value

An object of class "FuzzyCorrelation" (with a scalar slot).

Examples

```
set.seed(5)
x <- FuzzySample(replicate(50, triangular_fuzzy(0, 1, 2), simplify = FALSE))
y <- FuzzySample(replicate(50, triangular_fuzzy(0, 1, 2), simplify = FALSE))
correlation(x, y)
```

---

covariance — *Covariance of two fuzzy random variables (sample)*

Description

Level-wise covariance. At each level alpha the covariance of X and Y is the 2 x 2 matrix

```
Cov(X^-, Y^-)   Cov(X^-, Y^+)
 Cov(X^+, Y^-)   Cov(X^+, Y^+).
```

When y is omitted, x is paired with itself and the result is the variance. A scalar reduction through the Feng–Hu–Shu integral

$$\mathrm{Cov}_{FHS}(X,Y) = \tfrac{1}{2}\int_0^1 \left[\mathrm{Cov}(X^-(a),Y^-(a)) + \mathrm{Cov}(X^+(a),Y^+(a))\right] da$$

is also returned.

Usage

```
covariance(x, y = NULL, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
covariance(x, y = NULL, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
cov(x, y = NULL, ...)
```

Arguments

x, y	Two FuzzySample objects (or x alone for the variance).
alpha	Optional alpha grid.
...	Unused.

Value

A FuzzyCovariance object.

Examples

```
set.seed(3)
x <- FuzzySample(replicate(30, triangular_fuzzy(0, 1, 2), simplify = FALSE))
y <- FuzzySample(replicate(30, triangular_fuzzy(1, 2, 3), simplify = FALSE))
covariance(x, y)
```

---

cut_at *Cut of a fuzzy number at a single level*

Description

Cut of a fuzzy number at a single level

Usage

```
cut_at(x, alpha, ...)

## S3 method for class 'FuzzyNumber'
cut_at(x, alpha = 0.5, ...)
```

Arguments

x	A FuzzyNumber.
alpha	A single level in [0, 1].
...	Unused.

Value

A named numeric vector c(lower = ..., upper = ...).

Examples

```
cut_at(triangular_fuzzy(1, 2, 4), 0.5)
```

cuts — *Alpha-level cuts of a fuzzy number*

Description

Alpha-level cuts of a fuzzy number

Usage

```
cuts(x, alpha = NULL, ...)

## S3 method for class 'FuzzyNumber'
cuts(x, alpha = NULL, ...)
```

Arguments

x	A FuzzyNumber (or other compatible object).
alpha	Optional alpha grid at which to evaluate the cuts.
...	Unused.

Value

A numeric matrix with columns lower and upper.

Examples

```
fn <- triangular_fuzzy(1, 2, 4)
cuts(fn, alpha = c(0, 0.5, 1))
```

expectation — *Expectation of a fuzzy random variable (sample)*

Description

Level-wise expectation. For a sample the estimator is the level-wise mean of the alpha-cuts:

$$[E[X]]_\alpha = \Big[\frac{1}{n}\sum_i X_i^-(\alpha),\ \frac{1}{n}\sum_i X_i^+(\alpha)\Big].$$

Usage

```
expectation(x, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
expectation(x, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
mean(x, ...)
```

Arguments

x	A FuzzySample.
alpha	Optional alpha grid on which to evaluate.
...	Unused.

Value

A FuzzyNumber, the sample expectation.

Examples

```
set.seed(2)
s <- FuzzySample(replicate(40, triangular_fuzzy(-1, 0, 1), simplify = FALSE))
ex <- expectation(s); plot(ex, main = "sample mean")
```

---

fuzzy_ecdf *Empirical distribution function of a fuzzy random sample*

Description

Level-wise empirical cumulative distribution functions of the alpha-cut endpoints.

Usage

```
fuzzy_ecdf(x, alpha = NULL, at = NULL, ...)

## S3 method for class 'FuzzySample'
fuzzy_ecdf(x, alpha = NULL, at = NULL, ...)
```

Arguments

x	A FuzzySample.
alpha	Numeric vector of levels (default: the levels stored in x).
at	Numeric grid on which to evaluate the CDFs (default: 201 points over the data range).
...	Unused.

Value

An object of class "FuzzyECDF".

Examples

```
set.seed(8)
s <- FuzzySample(replicate(40, triangular_fuzzy(0, 1, 2), simplify = FALSE))
fe <- fuzzy_ecdf(s, alpha = c(0, 0.25, 0.5, 0.75, 1))
plot(fe)
```

fuzzy_from_membership *A fuzzy number from a membership function evaluated on a grid*

Description

Reconstructs the alpha-cuts of a (normal, quasi-concave) fuzzy number from a membership function evaluated on a grid. The level set `{x : mu(x) >= alpha}` is assumed to be a single interval.

Usage

```
fuzzy_from_membership(x, mu, alpha = NULL)
```

Arguments

`x`	Sorted numeric grid of the support (unsorted grids are sorted).
`mu`	Membership values in `[0, 1]` evaluated at `x`.
`alpha`	Optional alpha grid.

Value

An object of class `"FuzzyNumber"`.

Examples

```
x  <- seq(-3, 3, length.out = 201)
mu <- pmax(1 - abs(x), 0)            # triangular membership
fn <- fuzzy_from_membership(x, mu)
plot(fn)
```

fuzzy_normal *Construct a fuzzy normal distribution*

Description

A fuzzy normal distribution is the family, over levels `alpha`, of bivariate-normal laws on the alpha-cut endpoints with fuzzy mean `mean` and level-wise covariance `cov`.

Usage

```
fuzzy_normal(mean, cov = NULL, n = NULL)
```

Arguments

`mean`	A `FuzzyNumber` (the fuzzy mean).
`cov`	A `2 x 2` covariance matrix, a length-4 vector, or a `FuzzyCovariance` whose levels match `mean`.
`n`	Optional sample size associated with the mean (used by `clt`).

Value

An object of class "FuzzyNormal".

Examples

```
fn <- fuzzy_normal(triangular_fuzzy(0, 1, 2), diag(2))
simulate(fn, nsim = 5)
```

FuzzyNumber *A fuzzy number given by its alpha-level cuts*

Description

Build a FuzzyNumber object from its alpha-level envelopes. When alpha is not supplied it is taken as a regularly spaced grid on [0, 1] with one node per supplied endpoint.

Usage

```
FuzzyNumber(alpha = NULL, lower, upper)
```

Arguments

alpha	Numeric vector in [0, 1], strictly increasing.
lower	Numeric; lower envelope, non-decreasing in alpha.
upper	Numeric; upper envelope, non-increasing in alpha.

Value

An object of class "FuzzyNumber".

Examples

```
fn <- FuzzyNumber(alpha = c(0, 0.5, 1), lower = c(1, 2, 2), upper = c(4, 2, 2))
cuts(fn)
```

FuzzyRandom *Statistics of Fuzzy Random Variables: a Puri-Ralescu Framework*

Description

FuzzyRandom is a computationally self-contained implementation of the level-wise ("Puri-Ralescu") theory of statistics for fuzzy random variables.

Details

Fuzzy numbers are represented through their alpha-level sets [X^-(alpha), X^+(alpha)], and every statistical operator is defined level-wise, so that the **expectation**, **variance**, **covariance**, **empirical distribution**, **law of large numbers** and **central limit theorem** are all handled in a single coherent framework.

The key design choices are:

- A *fuzzy number* is a list of alpha levels and the matching non-decreasing lower / non-increasing upper envelopes (class FuzzyNumber).
- A *sample of fuzzy random variables* is an (n, m, 2) array of cuts (class FuzzySample), m being the number of levels.
- The covariance of two fuzzy random variables is returned as a level-wise 2 x 2 matrix of covariances between the lower and upper endpoints (class FuzzyCovariance); scalar reductions are provided through the Feng-Hu-Shu integral and scalar product.
- The central limit theorem yields the *fuzzy normal* distribution (class FuzzyNormal), whose level cuts are bivariate-normal intervals.

The package only depends on base R (stats, graphics).

References

Puri, M. L. and Ralescu, D. A. (1986). Fuzzy random variables. *Journal of Mathematical Analysis and Applications*, **114**(2), 409-422.

Klement, E. P., Puri, M. L. and Ralescu, D. A. (1986). Limit theorems for fuzzy random variables. *Proceedings of the Royal Society A*, **407**, 171-182.

Feng, Y., Hu, L. and Shu, H. (2002). The variance and covariance of fuzzy random variables and their applications. *Fuzzy Sets and Systems*, **128**, 335-345.

FuzzySample — *Build a sample of fuzzy random variables*

Description

Build a sample of fuzzy random variables

Usage

```
FuzzySample(x, alpha = NULL)
```

Arguments

x	Either a list of FuzzyNumber objects, an (n, m, 2) numeric array of cuts (dimension 3 = c("lower", "upper")), or an existing FuzzySample.
alpha	Alpha grid shared by all observations. Required when x is an array with an unlabelled third dimension; otherwise inferred.

Value

An object of class "FuzzySample".

Examples

```
set.seed(1)
s <- FuzzySample(list(
  triangular_fuzzy(0, 1, 2),
  triangular_fuzzy(1, 2, 3),
  trapezoidal_fuzzy(0.5, 1, 3, 4)))
n_obs(s)
```

is_fuzzy_number — *Whether an object is a (valid) fuzzy number*

Description

Whether an object is a (valid) fuzzy number

Usage

```
is_fuzzy_number(x)
```

Arguments

x	Any R object.

Value

Logical; TRUE if x is a valid FuzzyNumber.

Examples

```
is_fuzzy_number(triangular_fuzzy(0, 1, 2))
is_fuzzy_number(1:3)
```

---

is_fuzzy_sample *Whether an object is a FuzzySample*

Description

Whether an object is a FuzzySample

Usage

```
is_fuzzy_sample(x)
```

Arguments

x	Any R object.

Value

Logical.

Examples

```
is_fuzzy_sample(FuzzySample(list(triangular_fuzzy(0, 1, 2))))
```

---

ks_test *One- and two-sample fuzzy Kolmogorov-Smirnov tests*

Description

Performs, at each level alpha, a Kolmogorov-Smirnov test on the lower and upper endpoints of the alpha-cuts. With p0 a one-sample test against the CDF p0 is run; otherwise a two-sample test against the second sample y is run. The result is a family of level-wise tests consistent with the Puri-Ralescu level-wise approach.

Usage

```
ks_test(x, ...)

## S3 method for class 'FuzzySample'
ks_test(x, y = NULL, p0 = NULL, alpha = NULL, ...)
```

Arguments

x	A FuzzySample.
...	Additional arguments passed on to ks.test.
y	A second FuzzySample for a two-sample test (default NULL).
p0	Null CDF for the one-sample test: a character string naming a CDF in stats (e.g. "pnorm") or a function p0(q).
alpha	Numeric vector of levels (default: those of x).

Value

An object of class "FuzzyKS".

Examples

```
set.seed(9)
s <- FuzzySample(replicate(50, triangular_fuzzy(-1, 0, 1), simplify = FALSE))
ks_test(s, p0 = "pnorm")
```

---

lln — *Level-wise sample-mean convergence (LLN)*

Description

Following Puri & Ralescu (1986), statistics on fuzzy random variables are evaluated level-wise on the alpha-cuts. This function computes the fuzzy sample mean for an increasing sequence of sample sizes and measures, at a fixed level, the sup-norm distance between the size-s mean and the full-sample mean. By the level-wise (fuzzy) law of large numbers this distance shrinks to zero with the sample size (at rate 1/sqrt(s) under the level-wise CLT).

Usage

```
lln(x, sizes = NULL, alpha = NULL, levels = c(0.5, 1), ...)
```

Arguments

x	A FuzzySample.
sizes	Integer vector of sample sizes (default: a grid from 5 up to x$n).
alpha	Optional alpha grid on which to evaluate the means.
levels	Numeric levels at which to measure convergence, used by the summary, as.data.frame and plot methods (default c(0.5, 1)).
...	Unused.

Value

An object of class "FuzzyLLN" with components sizes, means, full and alpha.

Examples

```
set.seed(23)
s <- rnorm_fuzzy(400, triangular_fuzzy(0, 1, 2), diag(2) * 0.25)
w <- lln(s)
plot(w)
summary(w)
```

`n_cuts` *Number of alpha levels of a fuzzy object*

Description

Number of alpha levels of a fuzzy object

Usage

```
n_cuts(x)
```

Arguments

`x`	A `FuzzyNumber` or `FuzzySample`.

Value

Length-one integer.

Examples

```
n_cuts(triangular_fuzzy(1, 2, 4))
```

`n_obs` *Number of observations in a sample*

Description

Number of observations in a sample

Usage

```
n_obs(x)
```

Arguments

`x`	A `FuzzySample`.

Value

Length-one integer.

Examples

```
s <- FuzzySample(list(triangular_fuzzy(0, 1, 2), triangular_fuzzy(1, 2, 3)))
n_obs(s)
```

---

rnorm_fuzzy — *Sample fuzzy random variables from a fuzzy normal distribution*

Description

Draws n fuzzy numbers whose alpha-cut endpoints have the bivariate-normal level-wise distribution N_2(mu_alpha, Sigma_alpha).

Usage

```
rnorm_fuzzy(n, mean, cov = NULL, valid = TRUE, seed = NULL)

## S3 method for class 'FuzzyNormal'
simulate(object, nsim = 1, ...)

## S3 method for class 'FuzzyCLT'
simulate(object, nsim = 1, ...)
```

Arguments

n	Sample size.
mean	A FuzzyNumber (the fuzzy mean) or a FuzzyNormal.
cov	A 2 x 2 covariance matrix, a length-4 vector, or a FuzzyCovariance. Ignored if mean is a FuzzyNormal.
valid	Logical; whether to project draws to valid fuzzy numbers.
seed	Optional integer seed (NULL leaves the RNG alone).
object	A FuzzyNormal (or FuzzyCLT) object.
nsim	Number of simulations.
...	Unused.

Details

When valid = TRUE (the default) every draw is projected onto the cone of valid, nested alpha-cuts (lower <= upper; lower non-decreasing and upper non-increasing in alpha) so that it is a proper fuzzy number. When valid = FALSE the raw, unprojected level-wise bivariate-normal cuts are returned – useful for checking the level-wise central limit theorem, where projection would bias the marginals.

Value

A `FuzzySample` of `n` fuzzy random variables.

Examples

```
set.seed(123)
s <- rnorm_fuzzy(80, triangular_fuzzy(0, 1, 2), diag(2) * 0.25)
plot(s, main = "fuzzy normal sample")
```

`spectral_covariance` *Scalar covariance (Feng–Hu–Shu integral)*

Description

`spectral_covariance` returns the scalar covariance

$$\mathrm{Cov}_{FHS}(X,Y) = \tfrac{1}{2}\int_0^1 \left[\mathrm{Cov}(X^-(a),Y^-(a)) + \mathrm{Cov}(X^+(a),Y^+(a))\right] da.$$

Usage

```
spectral_covariance(x, y = NULL, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
spectral_covariance(x, y = NULL, alpha = NULL, ...)
```

Arguments

`x, y`	Two `FuzzySample` objects (or `x` alone).
`alpha`	Optional alpha grid.
`...`	Unused.

Value

A length-one numeric.

Examples

```
set.seed(6)
x <- FuzzySample(replicate(30, triangular_fuzzy(0, 1, 2), simplify = FALSE))
spectral_covariance(x)
```

spectral_variance *Scalar variance (Feng–Hu–Shu integral)*

Description

`spectral_variance` returns `spectral_covariance(x, x)`.

Usage

```
spectral_variance(x, alpha = NULL, ...)
```

Arguments

`x`	A `FuzzySample`.
`alpha`	Optional alpha grid.
`...`	Unused.

Value

A length-one numeric.

Examples

```
set.seed(7)
x <- FuzzySample(replicate(30, triangular_fuzzy(0, 1, 2), simplify = FALSE))
spectral_variance(x)
```

trapezoidal_fuzzy *Trapezoidal fuzzy number*

Description

The trapezoidal fuzzy number with support `[a, d]` and core `[b, c]` (a <= b <= c <= d).

Usage

```
trapezoidal_fuzzy(a, b, c, d, alpha = NULL)
```

Arguments

a, b, c, d	Scalars with a <= b <= c <= d.
`alpha`	Optional alpha grid.

Value

An object of class `"FuzzyNumber"`.

Examples

```
plot(trapezoidal_fuzzy(1, 2, 3, 5))
```

`triangular_fuzzy` *Triangular fuzzy number*

Description

The triangular fuzzy number with support `[a, c]` and core `b` (`a <= b <= c`). Its alpha-cuts are `[a + (b-a)*alpha, c - (c-b)*alpha]`.

Usage

```
triangular_fuzzy(a, b, c, alpha = NULL)
```

Arguments

`a, b, c`	Scalars with `a <= b <= c`.
`alpha`	Optional alpha grid (see `FuzzyNumber`).

Value

An object of class `"FuzzyNumber"`.

Examples

```
plot(triangular_fuzzy(1, 2, 4))
```

`variance` *Variance of a fuzzy random variable (sample)*

Description

Level-wise variance, i.e. `covariance` of a sample with itself. Returns the level-wise `2 x 2` matrix `[[Var(X^-), Cov(X^-,X^+)], [Cov(X^+,X^-), Var(X^+)]` at each level, together with the scalar Feng–Hu–Shu variance

$$\mathrm{Var}_{FHS}(X) = \tfrac{1}{2}\int_0^1 \left[\mathrm{Var}(X^-(a)) + \mathrm{Var}(X^+(a))\right] da.$$

Usage

```
variance(x, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
variance(x, alpha = NULL, ...)

## S3 method for class 'FuzzySample'
var(x, ...)
```

Arguments

x	A FuzzySample.
alpha	Optional alpha grid.
...	Unused.

Value

A FuzzyCovariance object (with type = "variance").

Examples

```
set.seed(4)
s <- FuzzySample(replicate(30, triangular_fuzzy(0, 1, 2), simplify = FALSE))
variance(s)
```

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